
Contents

1	A Basic Problem	1
2	Dynamical Systems	5
2.1	Basics of Mechanical Systems	5
2.2	Formal Definitions	10
2.3	Maps	11
2.4	Basic Examples of Maps	12
2.5	More Advanced Examples	17
2.6	Examples of Flows	23
3	Topological Properties	27
3.1	Coding, Kneading	27
3.2	Topological Entropy	30
3.2.1	Topological, Measure, and Metric Spaces	30
3.2.2	Some Examples	30
3.2.3	General Theory of Topological Entropy	32
3.2.4	A Metric Version of Topological Entropy	34
3.3	Attractors	38
4	Hyperbolicity	45
4.1	Hyperbolic Fixed Points	46
4.1.1	Stable and Unstable Manifolds	49
4.1.2	Conjugation	53
4.1.3	Resonances	58
4.2	Invariant Manifolds	60
4.3	Nonwandering Points and Axiom A Systems	63
4.4	Shadowing and Its Consequences	65
4.4.1	Sensitive Dependence on Initial Conditions	69
4.4.2	Complicated Orbits Occur in Hyperbolic Systems	69
4.4.3	Change of Map	70
4.5	Construction of Markov Partitions	72

5	Invariant Measures	79
5.1	Overview	79
5.2	Details	85
5.3	The Perron–Frobenius Operator	89
5.4	The Ergodic Theorem	93
5.5	Convergence Rates in the Ergodic Theorem	103
5.6	Mixing and Decay of Correlations	105
5.7	Physical Measures	114
5.8	Lyapunov Exponents	116
6	Entropy	123
6.1	The Shannon–McMillan–Breiman Theorem	125
6.2	Sinai–Bowen–Ruelle Measures	130
6.3	Dimensions	132
7	Statistics and Statistical Mechanics	141
7.1	The Central Limit Theorem	141
7.2	Large Deviations	147
7.3	Exponential Estimates	149
7.3.1	Concentration	151
7.4	The Formalism of Statistical Mechanics	153
7.5	Multifractal Measures	156
8	Other Probabilistic Results	163
8.1	Entrance and Recurrence Times	163
8.2	Number of Visits to a Set	172
8.3	Extremes	173
8.4	Quasi-Invariant Measures	175
8.5	Stochastic Perturbations	178
9	Experimental Aspects	187
9.1	Correlation Functions and Power Spectrum	188
9.2	Resonances	191
9.3	Lyapunov Exponents	196
9.4	Reconstruction	201
9.5	Measuring the Lyapunov Exponents	205
9.6	Measuring Dimensions	206
9.7	Measuring Entropy	211
9.8	Estimating the Invariant Measure	211
	References	215
	Index	227