
Contents

Introduction to the Theory of Linear Operators

<i>Alain Joye</i>	1
1 Introduction	1
2 Generalities about Unbounded Operators	2
3 Adjoint, Symmetric and Self-adjoint Operators	5
4 Spectral Theorem	13
4.1 Functional Calculus	15
4.2 L^2 Spectral Representation	22
5 Stone's Theorem, Mean Ergodic Theorem and Trotter Formula	29
6 One-Parameter Semigroups	35
References	40

Introduction to Quantum Statistical Mechanics

<i>Alain Joye</i>	41
1 Quantum Mechanics	42
1.1 Classical Mechanics	42
1.2 Quantization	46
1.3 Fermions and Bosons	53
2 Quantum Statistical Mechanics	54
2.1 Density Matrices	54
3 Boltzmann Gibbs	57
References	67

Elements of Operator Algebras and Modular Theory

<i>Stéphane Attal</i>	69
1 Introduction	70
1.1 Discussion	70
1.2 Notations	71
2 C^* -algebras	71
2.1 First definitions	71
2.2 Spectral analysis	73

XIV Contents

2.3	Representations and states	79
2.4	Commutative C^* -algebras	83
2.5	Appendix	84
3	von Neumann algebras	86
3.1	Topologies on $\mathcal{B}(\mathcal{H})$	86
3.2	Commutant	89
3.3	Predual, normal states	90
4	Modular theory	92
4.1	The modular operators	92
4.2	The modular group	96
4.3	Self-dual cone and standard form	100
	References	105

Quantum Dynamical Systems

<i>Claude-Alain Pillet</i>	107	
1	Introduction	107
2	The State Space of a C^* -algebras	110
2.1	States	110
2.2	The GNS Representation	119
3	Classical Systems	123
3.1	Basics of Ergodic Theory	123
3.2	Classical Koopmanism	127
4	Quantum Systems	130
4.1	C^* -Dynamical Systems	132
4.2	W^* -Dynamical Systems	139
4.3	Invariant States	141
4.4	Quantum Dynamical Systems	142
4.5	Standard Forms	147
4.6	Ergodic Properties of Quantum Dynamical Systems	153
4.7	Quantum Koopmanism	161
4.8	Perturbation Theory	165
5	KMS States	168
5.1	Definition and Basic Properties	168
5.2	Perturbation Theory of KMS States	178
	References	180

The Ideal Quantum Gas

<i>Marco Merkli</i>	183	
1	Introduction	184
2	Fock space	185
2.1	Bosons and Fermions	185
2.2	Creation and annihilation operators	188
2.3	Weyl operators	191
2.4	The C^* -algebras $CAR_F(\mathfrak{H})$, $CCR_F(\mathfrak{H})$	194
2.5	Leaving Fock space	197

3	The CCR and CAR algebras	198
3.1	The algebra CAR(\mathfrak{D})	199
3.2	The algebra CCR(\mathfrak{D})	200
3.3	Schrödinger representation and Stone – von Neumann uniqueness theorem	203
3.4	Q -space representation	207
3.5	Equilibrium state and thermodynamic limit	209
4	Araki-Woods representation of the infinite free Boson gas	213
4.1	Generating functionals	214
4.2	Ground state (condensate)	217
4.3	Excited states	222
4.4	Equilibrium states	224
4.5	Dynamical stability of equilibria	228
	References	233

Topics in Spectral Theory

Vojkan Jakšić	235	
1	Introduction	236
2	Preliminaries: measure theory	238
2.1	Basic notions	238
2.2	Complex measures	238
2.3	Riesz representation theorem	240
2.4	Lebesgue-Radon-Nikodym theorem	240
2.5	Fourier transform of measures	241
2.6	Differentiation of measures	242
2.7	Problems	247
3	Preliminaries: harmonic analysis	248
3.1	Poisson transforms and Radon-Nikodym derivatives	249
3.2	Local L^p norms, $0 < p < 1$	253
3.3	Weak convergence	253
3.4	Local L^p -norms, $p > 1$	254
3.5	Local version of the Wiener theorem	255
3.6	Poisson representation of harmonic functions	256
3.7	The Hardy class $H^\infty(\mathbb{C}_+)$	258
3.8	The Borel transform of measures	261
3.9	Problems	263
4	Self-adjoint operators, spectral theory	267
4.1	Basic notions	267
4.2	Digression: The notions of analyticity	269
4.3	Elementary properties of self-adjoint operators	269
4.4	Direct sums and invariant subspaces	272
4.5	Cyclic spaces and the decomposition theorem	273
4.6	The spectral theorem	273
4.7	Proof of the spectral theorem—the cyclic case	274
4.8	Proof of the spectral theorem—the general case	277

XVI Contents

4.9	Harmonic analysis and spectral theory	279
4.10	Spectral measure for A	280
4.11	The essential support of the ac spectrum	281
4.12	The functional calculus	281
4.13	The Weyl criteria and the RAGE theorem	283
4.14	Stability	285
4.15	Scattering theory and stability of ac spectra	286
4.16	Notions of measurability	287
4.17	Non-relativistic quantum mechanics	290
4.18	Problems	291
5	Spectral theory of rank one perturbations	295
5.1	Aronszajn-Donoghue theorem	296
5.2	The spectral theorem	298
5.3	Spectral averaging	299
5.4	Simon-Wolff theorems	300
5.5	Some remarks on spectral instability	301
5.6	Boole's equality	302
5.7	Poltoratskii's theorem	304
5.8	F. & M. Riesz theorem	308
5.9	Problems and comments	309
	References	311
	Index of Volume I	313
	Information about the other two volumes	
	Contents of Volume II	318
	Index of Volume II	321
	Contents of Volume III	323
	Index of Volume III	327