

Table of Contents

Introduction	1
1 Mathematical preliminaries	5
1.1 Banach spaces	5
1.2 Hilbert spaces	6
1.3 Some useful function spaces	8
1.3.1 Spaces of continuous functions	8
1.3.2 Spaces of integrable functions	9
1.3.3 Sobolev spaces	10
1.4 Analytic functions and harmonic functions	12
1.5 Fourier transform and Laplace transform	14
2 Regularization of moment problems by truncated expansion and by the Tikhonov method	17
2.1 Method of truncated expansion	19
2.1.1 A construction of regularized solutions	19
2.1.2 Convergence of regularized solutions and error estimates	22
2.1.3 Error estimates using eigenvalues of the Laplacian	27
2.2 Method of Tikhonov	30
2.2.1 Case 1: exact solutions in $L^2(\Omega)$	30
2.2.2 Case 2: exact solutions in $L^{\alpha^*}(\Omega)$, $1 < \alpha^* <$	36
2.2.3 Case 3: exact solutions in $H^1(\Omega)$	42
2.3 Notes and remarks	45
3 Backus-Gilbert regularization of a moment problem	51
3.1 Introduction	51
3.2 Backus-Gilbert solutions and their stability	54
3.2.1 Definition of the Backus-Gilbert solutions	54
3.2.2 Stability of the Backus-Gilbert solutions	59
3.3 Regularization via Backus-Gilbert solutions	63
3.3.1 Definitions and notations	64
3.3.2 Main results	73
4 The Hausdorff moment problem: regularization and error estimates	83
4.1 Finite moment approximation of (4.1)	84
4.1.1 Proof of Theorem 4.1	88

4.1.2 Proof of Theorem 4.2.....	89
4.2 A moment problem from Laplace transform	92
4.3 Notes and remarks.....	94
5 Analytic functions: reconstruction and Sinc approximations	99
5.1 Reconstruction of functions in $H^2(U)$: approximation by polynomials.....	99
5.2 Reconstruction of an analytic function: a problem of optimal recovery	106
5.3 Cardinal series representation and approximation: reformulation of moment problems.....	120
5.3.1 Two-dimensional Sinc theory	120
5.3.2 Approximation theorems	123
6 Regularization of some inverse problems in potential theory	131
6.1 Analyticity of harmonic functions.....	131
6.2 Cauchy's problem for the Laplace equation	133
6.3 Surface temperature determination from borehole measurements (steady case)	145
7 Regularization of some inverse problems in heat conduction	147
7.1 The backward heat equation	147
7.2 Surface temperature determination from borehole measurements: a two-dimensional problem	155
7.3 An inverse two-dimensional Stefan problem: identification of boundary values	164
7.4 Notes and remarks.....	169
8 Epilogue	171
References	175
Index	181