Iterative Model of the Pursuer Trajectory in Space

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Abstract

In this article, it is proposed to consider and discuss the implementation of the method of parallel convergence in space in a computer mathematics system. In this method, the pursuer's speed vector is directed arbitrarily. The pursuer's trajectory gradually approaches the movement in the plane formed by the line connecting the initial positions of the pursuer and the target, and the velocity vector. In this task, the target moves evenly and rectilinearly. The pursuer moves evenly. The points of the pursuer's trajectory are calculated sequentially. They are the intersection points of the plane containing the line of sight, the sphere and the cone. As the pursuer approaches the plane of target's movement, the algorithm for calculating the trajectory points changes. Now the point of the pursuer's trajectory is the result of the intersection of the sphere, the plane of movement of the target and the plane containing the line of sight.

Keywords

Trajectory, pursuit, target, restriction, goal, curvature

1. Introduction

In the method of calculating the trajectory of the pursuer on the plane, the pursuer's velocity vector to a point on the circle of Apollonius.

In Figure 1, the point P is the pursuer's position, and the point T is the target's position. The Apollonius circle is a set of points $\{K\}$, for which it is characteristic that the ratio of distances to two fixed points (points P and K in Figure 1). Inrelation to the pursuit problem, it will look like this:

$$\frac{|PK|}{|TK|} = \frac{|V_P|}{|V_T|},$$

where V_P is the pursuer's speed, V_T is the target's speed.

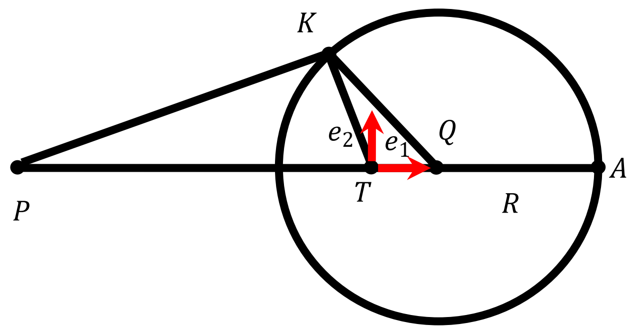

Figure 1: The Circle of Apollonius

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The fixed direction of target's movement allocates a single point K and a single direction of the pursuer's velocity V_P on the Apollonian circle.

Then, for the problem of pursuit on a plane where the pursuer and the target move rectilinearly and uniformly, there is an iterative scheme presented in Figure 2:

$$P_{i+1} = P_i + |V_P| \cdot \frac{P_i K_i}{|P_i K_i|} \cdot \Delta T,$$

where ΔT is the time interval of a discrete pursuit problem.

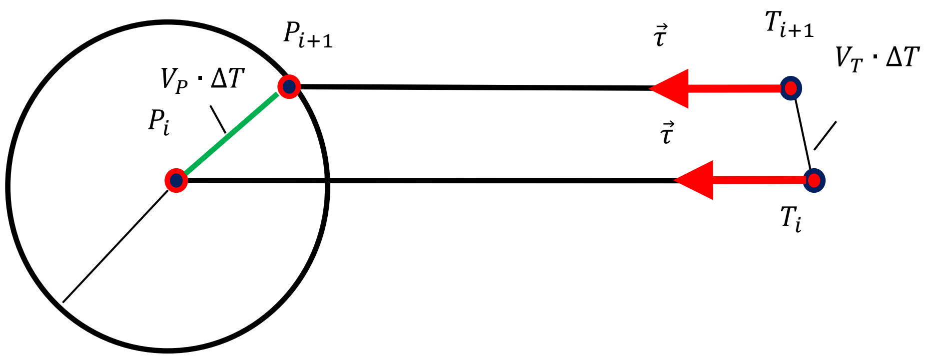

Figure 2: Iterative scheme

Then, for the problem of pursuit on a plane where the pursuer and the target move rectilinearly and uniformly, there is an iterative scheme presented in Figure 2:

$$P_{i+1} = P_i + |V_P| \cdot \frac{P_i K_i}{|P_i K_i|} \cdot \Delta T,$$

where ΔT is the time interval of a discrete pursuit problem.

For the case when the velocity vector of the target T is known, the position of the next step of the target T_{i+1} is predetermined:

$$T_{i+1} = T_i + V_T \cdot \Delta T.$$

The coordinates of the point K_i are the solution of a system of equations with respect to the parameter t:

$$\begin{cases} (K_i - Q_i)^2 = R_i^2 \\ K_i = T_i + V_T \cdot \dfrac{T_{i+1} - T_i}{|T_{i+1} - T_i|} \cdot t \end{cases}.$$

The radius R_i and the center of the Apollonius circle $Q_i$are calculated as follows:

$$R_i = \frac{V_T^2}{V_P^2 - V_T^2} \cdot |T_i - P_i|, Q_i = T_i + \frac{V_T^2}{V_P^2 - V_T^2} \cdot (T_i - P_i).$$

Or, following the iterative scheme shown in Figure 2, the step of the pursuer's trajectory P_{i+1} satisfies the solution of the system of equations (1), with respect to the parameter h:

$$\begin{cases} (P_{i+1} - P_i)^2 = (V_P \cdot \Delta T)^2 \\ P_{i+1} = T_{i+1} + h \cdot \dfrac{P_i - T_i}{|P_i - T_i|} \end{cases}. \quad (1)$$

2. Problem statement

The purpose of this article is to describe a model of the pursuit problem in space, when the velocity vectors of the pursuer and the target, in Figure 3 V_P and V_T, respectively, do not lie in the same plane. The task is to calculate the points of the pursuer's trajectory at a certain trajectory of the target.

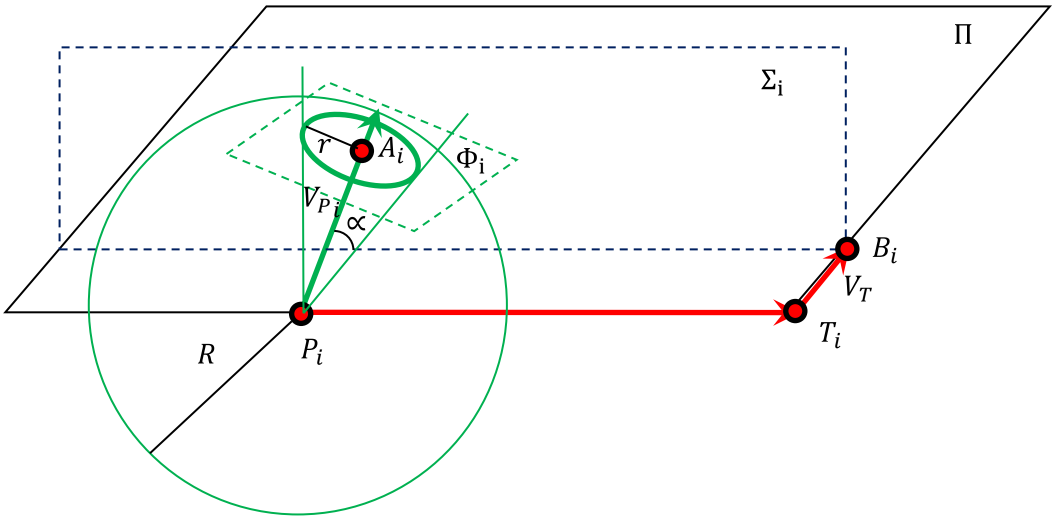

Figure 3: Iterative scheme in space

We will assume that at any time the movement of the target can be represented as rectilinear and uniform.

The plane formed by the line of sight at the time of the start of the pursuit in Figure 3, this is a straight line (PT) and the velocity vector V_T, will be considered as the coordinate plane XY with the origin at point P, the axis of the abscissa directed along a straight line(PT).

It is necessary in the iterative process to ensure that the coordinates of the point of the pursuer are located in the coordinate plane XY in Figure 3 this is the plane Π. In this case, the point of the pursuer's position P belonged to the plane Σ corresponding to the moment of time. The plane Σ contains the next step of the target (point B), is perpendicular to the plane Π and parallel to the line of sight (PT).

In addition, the trajectory of the pursuer must satisfy the curvature restrictions, that is, the radius of curvature of the trajectory cannot be less than a certain threshold value.

3. Geometrical model of parallel approach method in the space

The geometric model for constructing the trajectory of the pursuer can be conditionally divided into three parts. In the first part, the trajectory in space is calculated. In the second part, the trajectory is calculated on the plane. The third describes the smooth transition of the trajectory from space to plane.

3.1. Calculation of the pursuer's trajectory in space

In the kinematic model of parallel convergence in space, considered in this article, the pursuer's trajectory is calculated for two cases. In the first case, the trajectory segment is located in space. In the second case, the task turns into a pursuit on the plane. The pursuer in Figure 3 moves along the plane Π. A smooth transition from space to a plane is also calculated.

The pursuit model is discrete, so the time interval ΔT is introduced, during which the participants of the iterative process make a step. The pursuer, being at the point P_i (Fig. 3), has the opportunity to take a step within the sphere of radius $V_P \cdot \Delta T$ with the center at the point P_i. V_P is the speed module of the uniform pursuer's movement . This possibility is limited by a regular cone with a solution angle $\propto$ and a vertex at the point P_i.

The angle of the cone solution is equal to $\propto = \omega \cdot \Delta T$, ω - is the maximum frequency of angular rotation of the pursuer equal to $\omega = V_P \,/\, R_{min}$, where R_{min} is the minimum radius of curvature of the pursuer's trajectory. In addition, the next point P_{i+1} of the pursuer's position must belong to the plane Σ_i (Figure 3). In the future, when moving to the plane Π (Figure 3), belonging to the plane Σ_i is transformed into an iterative scheme shown in Figure 2.

The axis of the cone is directed along the current velocity vector V_{P_i} of the pursuer leaving the point P_i. Thus, there is a geometric problem of calculating a point belonging to three surfaces: a sphere, a cone and a plane.

The model of the pursuit problem of this article allows you to replace the correct cone with a plane. The intersection line of a regular cone and a sphere belongs to the plane Φ_i. The parameters of the plane Φ_i will be as follows $a_i = {}^{V_{P_i}}\!/_{V_P}$ is the unit vector of the plane normal Φ_i, the velocity vector for the current pursuer's position P_i, V_P is the velocity modulus of the uniform pursuer's movement , $A_i = P_i + a_i \cdot R \cdot cos(\alpha)$, where R is the radius of the sphere equal to the pursuer's step $V_P \cdot \Delta T$ (Figure 3).

The calculation of the intersection points of the planes Φ_i and Σ_i with a sphere of radius R with the center at the point P_i is more preferable than the calculation of the intersection points of the sphere, the cone of the plane from the point of view of computational difficulties.

For the plane Σ_i, the reference point is the point B_i, the normal is the vector b_i (Figure 3):

$$b_i = \left[(P - T) \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right],$$

where P and T are the initial positions of the pursuer and the target.

In the test program, written based on the materials of the article, there is a straight line (2), which is the intersection of the planes Φ_i and Σ_i:

$$L_i(t) = K_i + h \cdot \frac{[a_i \times b_i]}{|[a_i \times b_i]|}, \qquad (2)$$

where K_i is the intersection point of the planes Φ_i, Σ_i and Π (Fig. 3).

Next, we express the point P_{i+1} from the first equation of the system (3) and substitute it into the second equation of the system (3):

$$\begin{cases} P_{i+1} = K_i + h \cdot \dfrac{[a_i \times b_i]}{|[a_i \times b_i]|} \\ (P_{i+1} - P_i)^2 = (V_P \cdot \Delta T)^2 \end{cases} \qquad (3)$$

The found value of h is substituted into the first equation of the system (3), thereby determining the next point P_{i+1} of the pursuer's trajectory.

Thus, the iterative process of calculating the trajectory of the pursuer's movement in space can be considered formed.

3.2. Calculation of the trajectory of the pursuer's movement on the plane

In the case of the transition of the pursuit to a plane, it is necessary to reduce the problem to a parallel convergence, as shown in Figure 1. In this case, the pursuer's speed is always directed to a point on the circle of Apollonius (in Figure 1, this is the point K). Then an iterative scheme is used, represented by the system of equations (1).

If the pursuer's speed when moving to the plane is not directed at the point K, as in Figure 1, then if certain conditions are met in the direction of movement of the pursuer, you can apply the same iterative scheme described by the system of equations (1).

The condition for the movement direction is as follows. It is necessary that the angle γ (Figure 4) between the speeds V_{P_i} and $V_{P_{i+1}}$ was less than or equal to the angle $\propto = \omega \cdot \Delta T$, where ω is the permissible speed of rotation of the pursuer, ΔT is the time period of the iterative process.

If the angle γ is greater than $\propto$, but the pursuer's velocity modulus V_P is greater than the target's velocity modulus V_T, then the following can be proposed as an iterative scheme (Figure 4).

As a one-parameter set of parallel sight lines $(P_i T_i)$ (Figure 4), a set of composite parallel lines $\{L_i(t)\}$ is proposed, which is formed as follows: $L_{i+1}(t) = L_i(t) + (T_{i+1} - T_i)$.

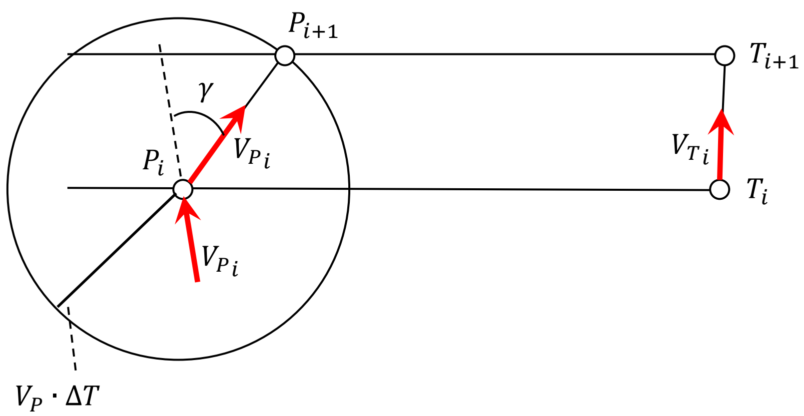

Figure 4: Additions to the iterative scheme

The next pursuer's step P_{i+1} is the point of intersection of a circle of radius $V_P \cdot \Delta T$ centered at the point P_i line $L_{i+1}(t)$ (Figure 5).

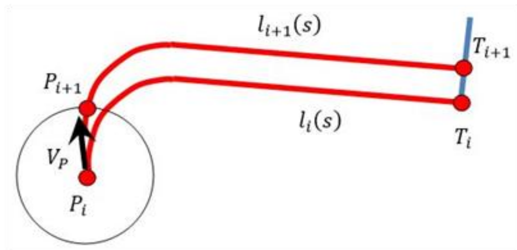

Figure 5: Calculation of the next pursuer's step

The first line of one-parameter subgroups of the set of lines $\{L_i(t)\}$ is formed from a circle of minimum radius R_{min} and direct a tangent line passing through the point T (Figure 6).

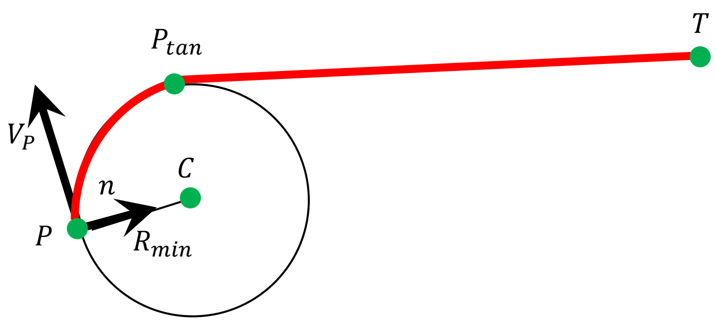

Figure 6: The relative position of the pursuer, the target and the circle

Figure 6 shows that the center of the specified circle is located at the point $C = P + R_{min} \cdot n$. Where P is the initial pursuer's position, R_{min} is the minimum radius of curvature of the pursuer's trajectory, n is a unit vector perpendicular to the pursuer's velocity vectorV_P. A composite line consists of an arc $(\widetilde{PP_{tan}})$ and a rectilinear segment$[P_{tan}T]$, where T is the initial position of the target, and P_{tan} is the point of contact with the circle.

3.3. Criterion for the transition to flat motion

Figure 7 shows the results of calculating the points of the pursuer's trajectory in space without taking into account the transition of the pursuit process to the plane Π.

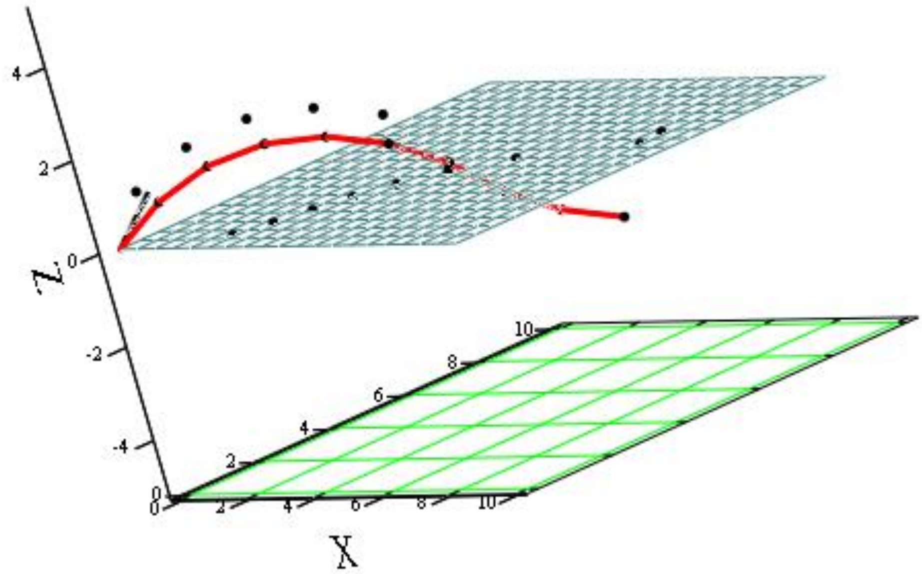

Figure 7: Calculation of the pursuer's trajectory in space

The points $K_i$of the intersection of the plane Π (the XY plane), the plane Σ and the plane Φ (Figure 3) and the intersection points of the cone, sphere and the plane Σ (the plane of parallel convergence) are displayed on the screen.

During the entire iterative process, the mutual location of the points of the pursuer's position P_i and the plane Π of the target's movement is analyzed. Since the plane Π of the target's movement coincides with the coordinate plane XY, it is sufficient to analyze the pursuer's application to the sign. As soon as the application sign changes, it returns to the previous calculated point of the trajectory and the calculation is performed according to another iterative scheme.

It is shown that the application of the point P_{i-1} has a positive value, and the application of the point P_i^* has a negative value. The coordinates of the point P_i^* (Figure 8) are obtained as a result of the intersection of the sphere $S_i(P_i, V_P \cdot \Delta T)$, a cone with an axis of rotation along the vector $V_{P_{i-1}}$with the angle of solution $\propto = \omega \cdot \Delta T$, as in Figure 3, and the plane of parallel motion Σ_i.

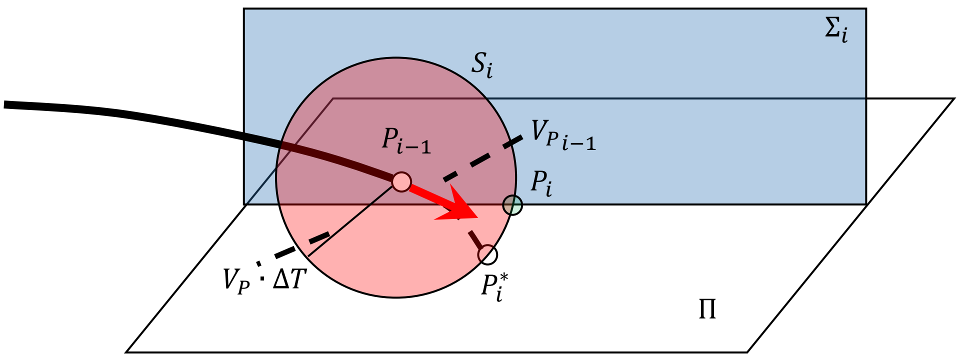

Figure 8: Intersection of the sphere, the plane of target's movement and the plane of parallel convergence

Figure 7 shows the relative position of the initial positions of the pursuer and the target, the velocity vector of the pursuer V_P and the specified circle.

There is a return to the point P_{i-1} and the intersection point P_i with the plane of parallel movement Σ_i and the plane Π of the target movement is searched for.

The test program, written based on the materials of the article, implements exactly such a criterion for switching to a plane.

4. Experimental results

Figure 9 shows the results of the program for calculating the trajectory of an object pursuing a goal moving uniformly and rectilinearly. The trajectory passes from movement in space to movement on the plane.

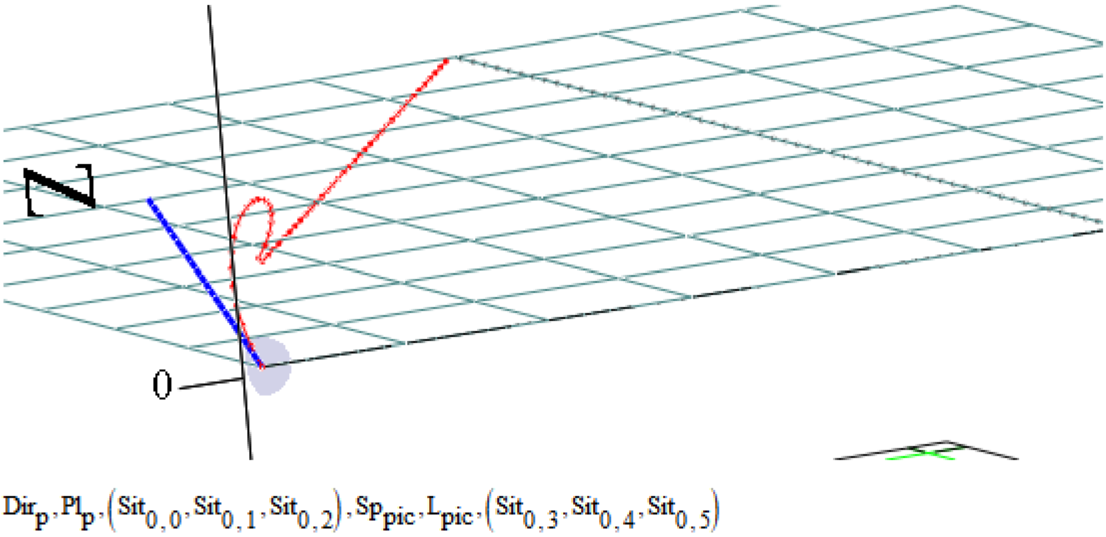

Figure 9: Calculation of the pursuer's trajectory

Figure 9 is supplemented with a link to an animated image, where it is possible to look at the process of persecution.

5. Conclusions

In the proposed model for calculating the trajectory of the pursuer, the plane Π of the target movement is determined by the initial line of sight (PT) and the velocity vector V_T of the target movement. The plane Π in this case is the bounding surface. The test program implements such a model of pursuit: the transition from pursuit in space to pursuit on a plane without going beyond the plane of restriction and with restrictions on the curvature of the pursuer's trajectory.

In the test mode, also during the transition to the plane, the use of the point P_i^* as the center of the sphere intersecting the planes Π and Σ_i was tried, and the construction of the pursuit trajectory from it.

In this article, a kinematic model of the pursuit problem in space was proposed. With the development of technologies, artificial intelligence systems, satellite positioning technologies for moving objects, modeling of pursuit tasks has become important.

There are many tasks and conditions in which modeling of iterative processes is required. The research results may be in demand by developers of unmanned aerial vehicles with elements of artificial intelligence.

6. Acknowledgements

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