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PHYS652

Quantum Decoherence and the Measurement Problem

Introduction

Quantum mechanics has been one of the most useful and successful theories in the history of physics - it has been able to describe the workings of particles, atoms, and molecules with extraordinary accuracy. Insights from quantum theory have made possible the transistor, lasers, and modern chemistry and yet after the greater part of a century there are some basic philosophical questions that remain unanswered. For example, the so-called measurement problem has been a source of endless speculation. The phenomenon of quantum decoherence does not provide a final answer to the measurement problem but in a practical sense lets us push the difficulties farther away.

At issue is the Born probability rule and the associated wavefunction collapse. The standard postulate describes the measurement process in terms of eigenvalues and projection operators:

If the particle is in a state $|\Psi\rangle$, measurement of the variable Ω will yield one of the eigenvalues ω with probability $P(\omega)\propto|\langle\omega|\Psi\rangle|^2$. The state of the system will change from $|\Psi\rangle$ to $|\omega\rangle$ as a result of the measurement.[1]

There are a number of ambiguities in this statement. For one, which physical processes are to be considered measurements? In a hydrogen atom the proton "measures" the electron via the Coulomb attraction but there is no wavefunction collapse. Apparently there is no measurement taking place in this case, at least as far as the Born rule is concerned. Another ambiguity is the preferred basis problem; if a particle is passed through a Stern-Gerlach apparatus and which-path information is measured the wavefunction will collapse into a spin eigenstate. In this case a measurement is definitely made – but how does quantum mechanics decide which operator is being measured? Which basis should be used for the wavefunction collapse?

As usual, there are no shortages of opinion and speculation on such matters.[2] Some people claim that the wavefunction is not real but rather some statistical approximation of something perhaps not yet understood (ensemble interpretation). Some have added non-unitary processes to the wavefunction evolution to cause collapse (objective collapse theories). Some completely reject collapse as being an illusion (Everett's many-worlds interpretation). Dirac stated the agnostic interpretation, "Shut up and calculate." It is this category that decoherence theory best fits into.

Decoherence

Decoherence attempts to explain the transition from quantum to classical by analyzing the interaction of a system with a measuring device or with the environment. It is convenient to imagine a quantum mechanical particle or system of particles as an isolated system floating in empty space. This simplification may be fine in some cases but in the real world there is no such thing as an isolated system. Typically a particle in flight will collide with air molecules or will emit thermal radiation that gets absorbed by the environment. Any interaction with the environment leads to an entanglement between the particle's state and the environment's state. As the entanglement diffuses throughout the environment the total state can no longer be separated into the direct product of a particle state and an environment state. What was once a superposition of particle states becomes a superposition of particle $\otimes$ environment states. At this point the particle ceases to act as if it were in a quantum superposition of states, instead acting as a statistical ensemble of states.

The end result of the decoherence process is that the particle will appear to have collapsed in a manner described by the Born probability law. The operator that Born's law uses for the observable is determined from the interaction Hamiltonian between particle and environment. This process is called einselection (environmentally induced superselection).[3] The basis of the observed operator consists of states that remain stable under interaction with the environment. Essentially this means that the observable commutes with the interaction Hamiltonian ($[\Omega, H_{\text{int}}]=0$).[4] In practice the interaction tends to care most about the position of the particle (for example through the Coulomb force) thus position is most often what gets measured.[5] This explains why position usually appears to be localized.

Decoherence tends to happen on an extremely fast timescale in most situations. The decoherence rate depends on several factors including temperature, uncertainty in position, and number of particles surrounding the system. Temperature affects the rate of blackbody radiation - each radiated photon will interact with the environment. Uncertainty in position tends to create a wide range of interaction energies and thus a rapid spread in vector components. The number of particles in the surroundings affects the rate at which interactions can happen. The rule of thumb is that decoherence occurs when the environment gains enough information to learn something about an observable. In any case it takes only a few interactions before a system has become completely decoherent. A single collision with an air molecule is enough to cause a chain reaction of decoherence as the collision molecule in turn collides with its neighbors.

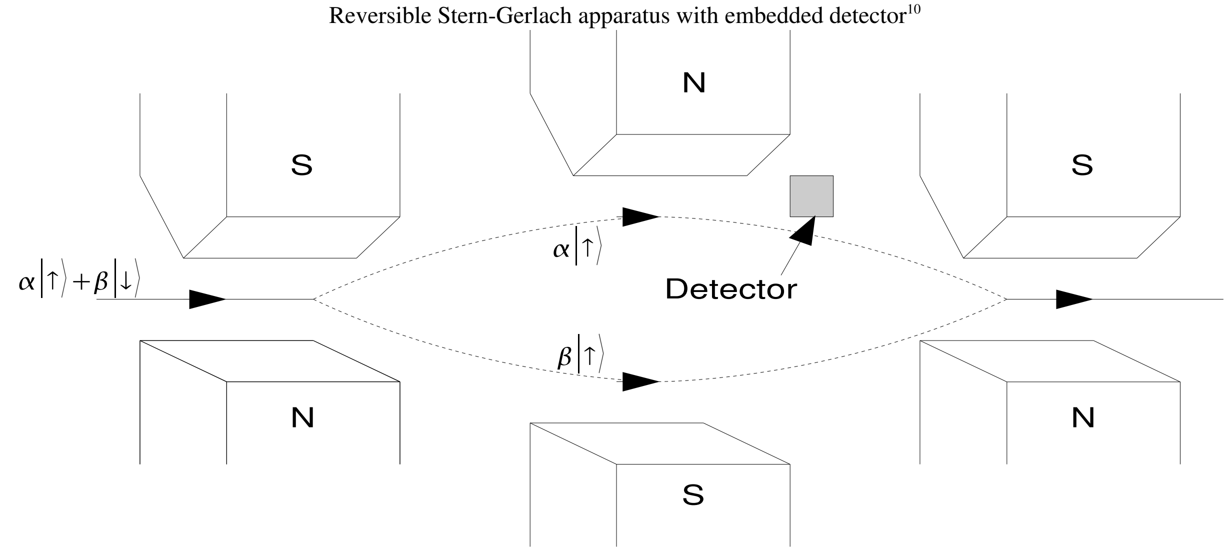
Several recent experiments have measured how much interaction is required for decoherence. The results agree well with theory. Some examples:

- Microwave resonant cavity - the escape of a single photon causes decoherence[6]
- Buckyball in matter wave interferometer - partial decoherence with thermal emission of one or two photons, complete decoherence upon collision with a gas molecule[7]
- Cryogenic Weber bar (gravity wave experiment) - a one ton bar must be treated quantum mechanically since it is cooled to a temperature of 10^{-3}K and has a Δx of 10^{-17}cm[8]

Example: Decoherence in a Stern-Gerlach Experiment

To get an idea of how this works mathematically, consider a spin-1/2 particle as it passes through a Stern-Gerlach apparatus. The interaction Hamiltonian was inspired by an example given in a paper by Tulsi Dass.[9] A formulation using density matrices is more elegant but to avoid introducing a new topic this example will be explained using state vectors.

Reversible Stern-Gerlach apparatus with embedded detector[10]



A spin-1/2 particle (the test particle) is prepared in a state $\alpha|\uparrow\rangle+\beta|\downarrow\rangle$ and is passed into a reversible Stern-Gerlach apparatus. The magnetic field separates the particle's trajectory based upon spin. The spin-up component follows the top track and the spin-down component follows the bottom track. The beams pass near a detector in initial state $|\epsilon_0\rangle$ which acts as a measurement device. The interaction is idealized so that the path of the particle is not affected by the detector, but the state of the detector bears a record of the particle. After the brief interaction with the detector the beams are recombined into a single particle. Note that if the detector was not there then the experiment would have no effect upon the test particle.

In order to carry out the computation we must find a basis for the particle and the detector that diagonalizes the interaction Hamiltonian. If the interaction is assumed to be due to the Coulomb force then the Hamiltonian will be diagonal in the position basis. For the particle that is in flight we choose the basis $\alpha|\uparrow\rangle+\beta|\downarrow\rangle$ which more or less corresponds to position (the purpose of the Stern-Gerlach magnet is to turn spin into position). The orthonormal basis vectors for the detector will be labeled $|\epsilon_j\rangle$ so the initial state is

$$|\epsilon_0\rangle=\sum_j a_j|\epsilon_j\rangle\ .$$

Each configuration of detector particles will have a definite energy of interaction for each path that the test particle can take (top track or bottom track of the Stern-Gerlach apparatus). For simplicity we neglect the particle's Hamiltonian and the detector's Hamiltonian and concentrate only on the interaction between test particle and detector

$$H=H_{particle}+H_{detector}+H_{\text{interaction}}$$
$$H_{\text{int}}=\sum_j (u_j|\uparrow\epsilon_j\rangle\langle\uparrow\epsilon_j|+d_j|\downarrow\epsilon_j\rangle\langle\downarrow\epsilon_j|)\ .$$

In the case of a Coulomb potential the energies are roughly proportional to the number of electrons in the detector and inversely proportional to the distance between the test particle and the detector. The exact details of the energies is however irrelevant to this example. The only assumptions are 1) the initial state of the detector is in a superposition of several basis vectors, and 2) the interaction coefficients u_j and d_j take on a variety of different values. In the case of a position basis it is reasonable to assume that the detector is in a superposition of states since the electrons in its atoms will never be completely localized.

Initially the test particle and the detector are independent so the combined initial state is $(\alpha|\uparrow\rangle+\beta|\downarrow\rangle)\otimes|\epsilon_0\rangle$. The propagator is

$$U(t)|\uparrow\epsilon_0\rangle=e^{-iHt/\hbar}\sum_j c_j|\uparrow\epsilon_j\rangle=\sum_j c_j|\uparrow\epsilon_j\rangle e^{-iu_jt/\hbar} \text{ and}$$
$$U(t)|\downarrow\epsilon_0\rangle=e^{-iHt/\hbar}\sum_j c_j|\downarrow\epsilon_j\rangle=\sum_j c_j|\downarrow\epsilon_j\rangle e^{-id_jt/\hbar}\ .$$

It is easy to see that $[S_z,H_{\text{int}}]=0$ thus the einselected basis consists of the eigenvalues of S_z. Lets pass an eigenstate through the experiment and see what happens. A spin-up particle is run through the apparatus and when it comes out S_z is measured. The detector observes the particle but since the particle is already in an eigenstate it doesn't matter. Thus, we expect to find spin-up every time. The probability of z_+ is

$$P(\uparrow)=|\langle\uparrow|\Psi(t)\rangle|^2=\sum_k|\langle\uparrow\epsilon_k|\Psi(t)\rangle|^2=\sum_k|\langle\uparrow\epsilon_k|U(t)|\uparrow\epsilon_0\rangle|^2\ .$$

Substituting the explicit form of the propagator,

$$P(\uparrow)=\sum_k\left|\langle\uparrow\epsilon_k|\sum_j c_j|\uparrow\epsilon_j\rangle e^{-iu_jt/\hbar}\right|^2\ .$$

Bringing the bra vector into the sum,

$$P(\uparrow)=\sum_k \left|\sum_j c_j \langle \uparrow \epsilon_k | \uparrow \epsilon_j \rangle e^{-i u_j t/\hbar}\right|^2 .$$

Exploiting the orthogonality of basis vectors,

$$P(\uparrow)=\sum_k \left|c_k \langle \uparrow \epsilon_k | \uparrow \epsilon_k \rangle e^{-i u_k t/\hbar}\right|^2=\sum_k |c_k|^2=1 .$$

That case wasn't too exciting. Now lets try a particle that starts in the $|x_+\rangle=\frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle)$ state. When the particle exits the apparatus we will measure S_x. A look at the interaction Hamiltonian shows that $[S_x, H_{\text{int}}]\neq 0$. Therefore, we can expect that there will be some decoherence. If the detector was not there the particle would not be affected and we would get $P(x_+)=1$. The detector has the effect of "observing" the particle. Using a semiclassical analysis we could apply the Born probability law which would project the particle into either a spin-up or spin-down eigenstate. Subsequent measurement of S_x would then give $P(x_+)=1/2$. Using decoherence we will obtain the same result without invoking any observation effect within the apparatus, although we of course have to observe the particle once it exits.

The initial state is the direct product of the particle state and the detector state,

$$|x_+\rangle|\epsilon_0\rangle=\frac{1}{\sqrt{2}}(|\uparrow \epsilon_0\rangle+|\downarrow \epsilon_0\rangle)=\sum_j \left(\frac{1}{\sqrt{2}} c_j |\uparrow \epsilon_j\rangle+\frac{1}{\sqrt{2}} c_j |\downarrow \epsilon_j\rangle\right) .$$

The effect of the propagator is

$$U(t)|x_+ \epsilon_0\rangle=\sum_j \left(\frac{1}{\sqrt{2}} c_j |\uparrow \epsilon_j\rangle e^{-i u_j t/\hbar}+\frac{1}{\sqrt{2}} c_j |\downarrow \epsilon_j\rangle e^{-i d_j t/\hbar}\right) .$$

Now that the particle has exited we measure its spin. The probability of x_+ is

$$P(x_+)=|\langle x_+|\Psi(t)\rangle|^2=\sum_k |\langle x_+ \epsilon_k|\Psi(t)\rangle|^2=\sum_k |\langle x_+ \epsilon_k|U(t)|x_+ \epsilon_0\rangle|^2 .$$

Substituting the explicit expression for the propagator,

$$P(x_+)=\sum_k \left|\langle x_+ \epsilon_k| \sum_j \left(\frac{1}{\sqrt{2}} c_j |\uparrow \epsilon_j\rangle e^{-i u_j t/\hbar}+\frac{1}{\sqrt{2}} c_j |\downarrow \epsilon_j\rangle e^{-i d_j t/\hbar}\right)\right|^2 .$$

Bringing the bra vector inside the sum and exploiting orthogonality of spin,

$$P(x_+)=\sum_k \left|\sum_j \left(\frac{1}{2} c_j \langle \uparrow \epsilon_k | \uparrow \epsilon_j \rangle e^{-i u_j t/\hbar}+\frac{1}{2} c_j \langle \downarrow \epsilon_k | \downarrow \epsilon_j \rangle e^{-i d_j t/\hbar}\right)\right|^2 .$$

Using the orthogonality of basis vectors,

$$P(x_+)=\sum_k \left|\frac{1}{2} c_k e^{-i u_k t/\hbar}+\frac{1}{2} c_k e^{-i d_k t/\hbar}\right|^2=\sum_k \left|\frac{1}{2} c_k (e^{-i u_k t/\hbar}+e^{-i d_k t/\hbar})\right|^2 .$$

Using Euler's formula,

$$P(x_+)=\sum_k \left|\frac{1}{2} c_k 2\cos\frac{(u_k-d_k)t}{\hbar}\right|^2 \quad \text{or}$$

$$P(x_+)=\sum_k |c_k|^2 \cos^2\frac{(u_k-d_k)t}{\hbar} .$$

At this point we call upon our two assumptions - the detector was initially in a superposition of states ($c_k\neq 0$ for many values of k) and the interaction energies u_k and d_k take on a variety of values. Thus after a short period of time the argument of the cosine will take on several essentially random values. When this happens the cosine-squared can be replaced by its average value giving

$$\textit{spread of}\ \frac{(u_k-d_k)t}{\hbar}>\pi \ \Rightarrow\ P(x_+)\approx\sum_k |c_k|^2 \frac{1}{2}=\frac{1}{2} ,$$

which is the desired result. For a Coulomb potential (u_k-d_k) is roughly proportional to the number of electrons in the detector and to the spatial separation between the two beams.

The other extreme is when the interaction time or the interaction energy is zero. In this case the particle is not affected and retains its spin of x_+:

$$\frac{(u_k - d_k)t}{\hbar} \ll \pi \;\Rightarrow\; P(x_+) \approx \sum_k \left|c_k\right|^2 = 1 \ .$$

There are several important things that can be learned from this example. First it is no coincidence that S_z provided the einselected basis. The Stern-Gerlach magnet was used to turn spin into positional displacement and a detector was used that reacted to the position of the test particle. Whenever we set out to test a certain property of a quantum system we do so by constructing a measurement device whose interaction Hamiltonian contains the operator we are measuring. As a consequence we ended up with an interaction Hamiltonian that commutes with S_z.

The second important aspect of this example is that the measurement problem has not been solved completely. Our goal was to avoid using Born's probability law, or at least to make it a result rather than an axiom. There was a partial success in that we did not have to invoke Born's law as the particle passed by the detector, but once the particle left the apparatus we had to "observe" it in the classical sense. Thus decoherence theory has allowed us to not deal with the measurement problem until the very end of the experiment.[11] If we were to continue passing our particle through further experiments we could continue to use similar arguments to again delay the application of Born's law to the very end.

The third point is in regards to the final state of the system. When we measured S_x and found x_+, the total state was projected into an eigenvector of S_x. The particle ends up with a definite value of S_x but the detector is in a superposition state in which it saw the particle take both the upper track and the lower track. This stands in contradiction to the result we would have obtained by applying Born's law at the point when the particle interacts with the detector, in which case there would have been a definite value of S_z at that point. However, this superposition of states will not last for long since the detector itself must eventually interact with the environment at which point it will decohere into a state where it says the particle took either the top track or the bottom track.

Many-Worlds Interpretation (MWI)

In the previous example we were able to view the interaction of the particle with the detector using only Schroedinger's equation for the evolution of the wavefunction. The concept of "observation" and the subsequent wave collapse wasn't necessary until after the particle exited the apparatus. It is natural to question how far this procedure can be taken. Would it be possible to say the state was not observed until the photons hit our optic nerve? And why stop there? Mathematically there is no need to. Hugh Everett proposed the idea that the wavefunction never collapses and that Schroedinger's equation is the end of the story. In this model the observer is allowed to be in a quantum superposition. The principle of decoherence suffices to explain Born's law and the appearance from the observer's point of view of wavefunction collapse.

Everett's interpretation is frequently described in terms of multiple worlds that constantly split every time a quantum decision is made. This is a bit of a simplification but is nevertheless a useful way to visualize the idea. Saying that history unfolds into a continuously branching tree of worlds is akin to saying that time is composed of instants[12] or saying that identical quantum particles are distinguishable if they have enough spatial separation. The unfolding of history probably more closely resembles the spreading over time of a delta function.

Before two quantum mechanical systems come into contact with each other their combined state is just the direct product of their individual states. When an interaction takes place decoherence tends to split the state vector into multiple non-interacting states. The state vector can then be thought of as a sum of components that are drifting farther apart. The total Hilbert space is very large – the number of dimensions is on the order of $\mathbb{R}^N$ where N is the number of particles in the universe. Quantum interference is only manifest between vector components that have non-vanishing scalar product. Nearly every pair of vectors is orthogonal in such a large space, so it is very unlikely therefore that two of these component states will ever interact again.[13] At this point the non-interacting components can be thought of as distinct "worlds" in the Everett interpretation. By dropping the non-local concept of wave collapse Everett's interpretation fits well with the relativistic notion of cause and effect.[14]

The vector norm of each subspace provides a natural definition of the "weight" of each world.[15] If this weight is interpreted as a probability measure then Born's probability law is recovered.[16] In other words, the vast majority of the volume of occupied Hilbert space basis vectors represent configurations of the universe that obey the laws of probability.

Critics claim that such an interpretation is not valid unless a particular basis is given. Grouping regions of Hilbert space into "worlds" requires slicing things up in a rather arbitrary way. Furthermore, the fact that most degrees of freedom are continuous rather than discrete complicates things. Instead of discrete worlds there must be a continuum of similar worlds.[17] Others are not bothered by this and compare the lack of a preferred basis to the lack of preferred orientation or definition of simultaneity in the theory of relativity.[18] Part of the confusion could be due to the fact that the concept of "worlds" is just an approximation.

Everett's many-worlds interpretation has been cause for endless debate. Since there is no way to test its validity this debate is not likely to come to a close. Critics say that MWI is not falsifiable and thus should not be considered a theory. Supporters claim MWI is equivalent to the Copenhagen interpretation but without saying that waves collapse when observed. Since there has been no precise definition of what constitutes "observation" the collapse axiom is not scientific anyway.[19] Supporters of the Copenhagen interpretation claim that MWI violates Occam's razor because it postulates the existence of multiple worlds, only one of which can be observed. Supporters of MWI claim their theory is supported by Occam's razor because it eliminates an axiom (wavefunction collapse) that has no observable effect.[20] Once again Occam's razor becomes a matter of personal taste.

Dynamical Collapse Models

Some who do not like the idea of collapse due to observers try to rig the wavefunction evolution so as to favor reduction of the state vector in a well defined way. One way is to say that the wavefunction, or at least a part of it, periodically gets "hit" in such a way as to cause localization in the position basis.[21] Another way is to add a non-unitary term into Schroedinger's equation,

$$d\Psi(t) = (A\,dt + B\,dW_t)\Psi(t)\ ,$$

where W_t is a Wiener process.[22] The extra term in the equation adds some low level universal noise.[23] The end result is a tendency to collapse into energy eigenstates with probabilities consistent with Born's law.[24]

Applications

The importance of decoherence theory is that it brings some understanding about the process of wave collapse. The tendency of a system to fall out of quantum superposition can be directly calculated. There are significant applications in the field of quantum computing where the goal is to build an entire computing device that can remain in coherent superposition. Decoherence places limits on the amount of time a system can be expected to remain in a superposition, although quantum error correction techniques could possibly provide a workaround.[25]

Although decoherence does not provide the last word in the measurement problem it does bring some light to the matter. The general guideline of $[\Omega, H_{\text{int}}]=0$ provides insight into how a system knows which observable is being measured and also why the position eigenstates seem to be favored in general. It is still not known at which point the wave actually collapses but the arena has been expanded. Environmental entanglement provides a mechanism in which wave collapse can propagate into the system from far away.

1 Shankar, Principles of Quantum Mechanics
2 Dass, Measurements and Decoherence, arXiv:quant-ph/0505070
3 Zurek, Decoherence, Einselection, and the Quantum Origins of the Classical, arXiv:quant-ph/0105127
4 Zurek, Physics Today, October 1991, page 36
5 Dass, Measurements and Decoherence, arXiv:quant-ph/0505070
6 Haroche, Physics Today, July 1998, page 36
7 Hornberger, Uttenthaler, Brezger, Hackermueller, Arndt, Zeilinger, Collisional decoherence observed in matter wave interferometry, arXiv:quant-ph/0303093
Hackermueller, Hornberger, Brezger, Zeilinger, Arndt, Decoherence of matter waves by thermal emission of radiation, arXiv:quant-ph/0402146
8 Zurek, Physics Today, October 1991, page 36
9 Dass, Measurements and Decoherence, arXiv:quant-ph/0505070
10 Zurek, Physics Today, October 1991, page 36
11 Dass, Measurements and Decoherence, arXiv:quant-ph/0505070
12 Wallace, Worlds in the Everett Interpretation, arXiv:quant-ph/0103092v1
13 Price, The Everett FAQ, http://www.hedweb.com/manworld.htm
14 Price, The Everett FAQ, http://www.hedweb.com/manworld.htm
15 Kent, Against Many-Worlds Interpretations, arXiv:gr-qc/9703089v1
16 Price, The Everett FAQ, http://www.hedweb.com/manworld.htm
17 Stapp, The Basis Problem in Many-Worlds Theories, http://www-physics.lbl.gov/~stapp/bp.PDF
18 Wallace, Worlds in the Everett Interpretation, arXiv:quant-ph/0103092v1
19 Price, The Everett FAQ, http://www.hedweb.com/manworld.htm
20 Price, The Everett FAQ, http://www.hedweb.com/manworld.htm
21 Frigg, GRW Theory, http://www.romanfrigg.org/writings/GRW%20Theory.pdf
22 Dass, Measurements and Decoherence, arXiv:quant-ph/0505070
Brody, Hughston, Finite-time stochastic reduction models, arXiv:quant-ph/0503231
23 Dass, Measurements and Decoherence, arXiv:quant-ph/0505070
24 Brody, Hughston, Finite-time stochastic reduction models, arXiv:quant-ph/0503231
25 arXiv:quant-ph/9705032v3 Quantum Computing: Pro and Con