Appendix of "No-regret algorithms for online k-submodular maximization"

A Ommited proofs in applications

A.1 Coupled feature selection

It suffices to show that f satisfies the pairwise-monotonicity and orthant submodularity. The pairwise-monotonicity is immediate since f is a monotone function. For the orthant submodularity, we note that f decomposes into two functions: $g(X_1, \dots, X_k) = H(X_1, \dots, X_k)$ and $-\sum_{i \in [k]} \sum_{j \in X_i} H(Y_j \mid Z_i)$. The latter term is simply modular and we can ignore it. For the former term, let us take $X_i \subseteq X'_i$ ($i \in [k]$) and $j \in [n] \setminus (X'_1 \cup \cdots \cup X'_k)$. Let $S = X_1 \cup \cdots \cup X_k$ and $S' = X'_1 \cup \cdots \cup X'_k$. Then, for any i,

$$
\begin{aligned}
&\Delta_{j,i} g(X_1, \dots, X_k) + \Delta_{j,i} g(X'_1, \dots, X'_k) \\
&= H(S \cup j) - H(S) - H(S' \cup j) + H(S') \\
&\geq 0,
\end{aligned}
$$

since the entropy function H is submodular.

A.2 Sensor placement with k different sensors

Again, we show the pairwise-monotonicity and orthant submodularity. For the pairwise-monotonicity, let us fix $(X_1, \dots, X_k) \in (k+1)^V$ and $j \notin \cup_i X_i$. Then, for arbitrary distinct i and i',

$$
\begin{aligned}
&\Delta_{j,i} f(X_1, \dots, X_k) + \Delta_{j,i'} f(X_1, \dots, X_k) \\
&= g(X_i \cup j) - g(X_i) + g(X_{i'} \cup j) - g(X_{i'}) \\
&= g(V \setminus (X_i \cup j)) + g(X_{i'} \cup j) - g(X_i) - g(X_{i'}) \\
&\geq g(V \setminus X_i) + g(X_{i'}) - g(X_i) - g(X_{i'}) \\
&= 0,
\end{aligned}
$$

by the symmetricity and submodularity of g. For the orthant submodularity, let us take $X_i \subseteq Y_i$ ($i \in [k]$) and $j \in [n] \setminus (Y_1 \cup \cdots \cup Y_k)$. Then, for any i,

$$
\begin{aligned}
&\Delta_{j,i} f(X_1, \dots, X_k) + \Delta_{j,i} f(Y_1, \dots, Y_k) \\
&= g(X_i \cup j) - g(X_i) - g(Y_i \cup j) + g(Y_i) \\
&\geq 0,
\end{aligned}
$$

by the submodularity of g.