Appendices

A Pseudocode for the auxiliary variable crossovers

Algorithm 1 One-point crossover at point t

function CROSSOVER($(x_{1:T}, y_{1:T}, t)$)
  $u_{1:T} \leftarrow (y_1, \ldots, y_t, x_{t+1}, \ldots, x_T)$
  $v_{1:T} \leftarrow (x_1, \ldots, x_t, y_{t+1}, \ldots, y_T)$
  return$(u_{1:T}, v_{1:T})$
end function

Algorithm 2 Scheme for an auxiliary variable two-point crossover between $\mathbf{x}_i$ and $\mathbf{x}_j$

Pick t uniformly $t \sim U(\{1, \ldots, T\})$
# Flip a coin to decide the direction of crossover
if $u < 0.5$ where $u \sim U(0, 1)$ **then**
  $(\mathbf{u}, \mathbf{v}) \leftarrow$ CROSSOVER$(\mathbf{x}_i, \mathbf{x}_j, t)$
else
  $(\mathbf{v}, \mathbf{u}) \leftarrow$ CROSSOVER$(\mathbf{x}_i, \mathbf{x}_j, t)$
end if
# consider all normal and flipped crossovers of u and v
for $t \in \{1, \ldots, T\}$ **do**
  # Normal crossover of u and v
  $(\mathbf{z}_i, \mathbf{z}_j) \leftarrow$ CROSSOVER$(\mathbf{u}, \mathbf{v}, t)$
  $a_t \leftarrow \pi_i(\mathbf{z}_i)\pi_j(\mathbf{z}_j)$
  # Flipped crossover of u and v
  $(\mathbf{z}_j, \mathbf{z}_i) \leftarrow$ CROSSOVER$(\mathbf{u}, \mathbf{v}, t)$
  $a_{T+t} \leftarrow \pi_i(\mathbf{z}_j)\pi_j(\mathbf{z}_i)$
end for
# Normalise the probabilities
$a_t \leftarrow a_t / \sum_s a_s$
Pick index t_0 with probability proportional to a_{t_0}
$t_0 \sim$ Discrete$(a_1, \ldots, a_T, a_{T+1}, \ldots, a_{2T})$
if $t_0 \leq T$ **then**
  $(\mathbf{x}_i, \mathbf{x}_j) \leftarrow$ CROSSOVER$(\mathbf{x}_i, \mathbf{x}_j, t_0)$
else
  $(\mathbf{x}_j, \mathbf{x}_i) \leftarrow$ CROSSOVER$(\mathbf{x}_i, \mathbf{x}_j, t_0)$
end if

B Supplementary Figures

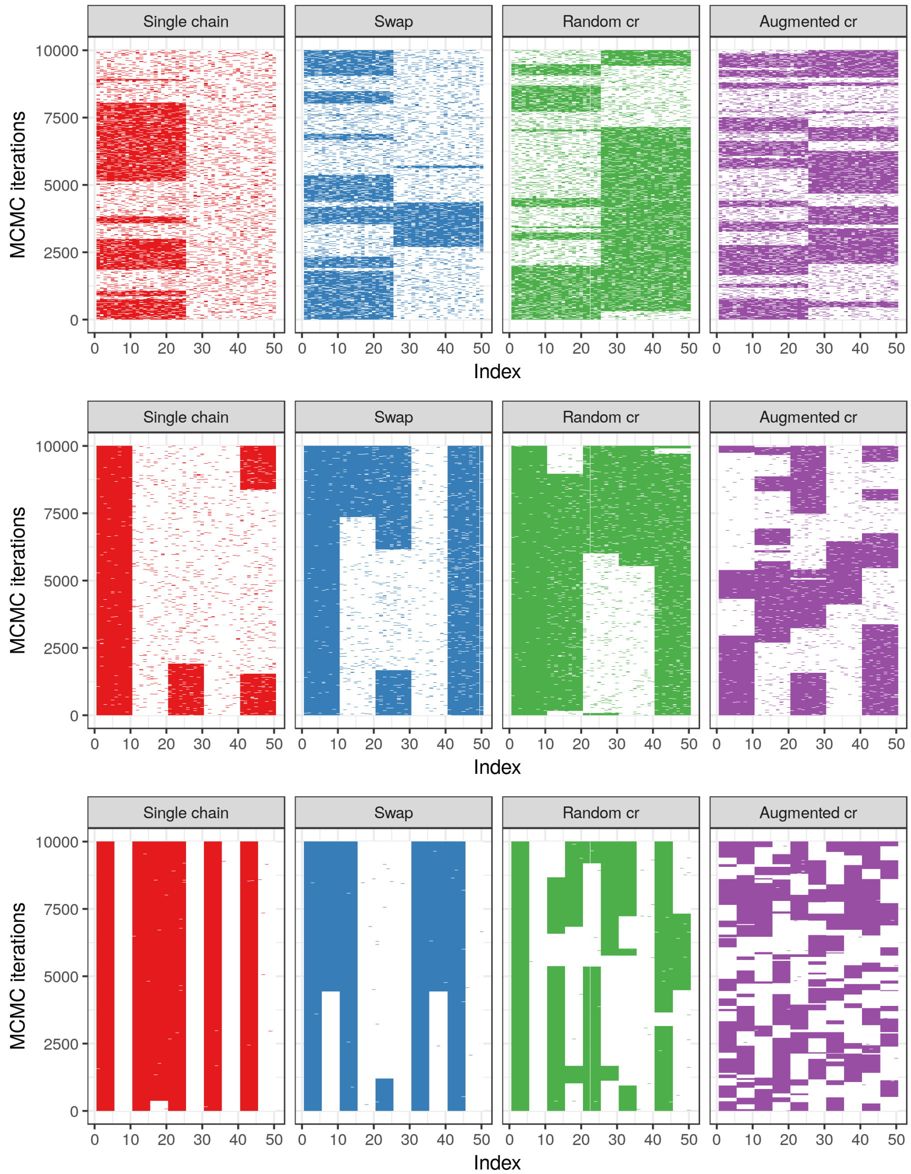

Figure 1: Heatmaps representing the trace plots of $\mathbf{x}$ for the experiment with $B \in \{2, 5, 10\}$ blocks, running a single chain Gibbs sampler (first panel), and its ensemble versions with various exchange moves: swap, random crossover, augmented crossover (in four panels). For each MCMC iteration, the elements of $\mathbf{x}$ have been colour coded: dark = 1, light = 0.