Search for Computationally Universal Cellular Automata Guided by 1/f Noise

Shigeru Ninagawa

Kanazawa Institute of Technology
ninagawa@infor.kanazawa-it.ac.jp

Abstract

The Game of Life (LIFE) is one of the two-dimensional cellular automata (CA) and has a propagating pattern called "glider". LIFE is able to emulate a conventional digital computer by considering a glider as a signal in a digital circuit. From the viewpoint of computability theory, LIFE is called computationally universal. Another distinguishing characteristic of LIFE is 1/f noise. The power spectrum calculated from the time evolution of cells starting from a random initial configuration exhibits 1/f characteristics (Ninagawa et al., 1998). Another example of CA exhibiting both computational universality and 1/f noise is found in elementary CA rule 110. Rule 110 was proved to be computationally universal (Cook, 2004) and exhibits 1/f noise (Ninagawa, 2008). These results suggest a relationship between computational universality and 1/f noise in CA. In this study we search two-dimensional three-state nine-neighbor CA rule space for a rule exhibiting a 1/f spectrum by means of genetic algorithms to find computationally universal rules.

The transition function of a CA is encoded into a 134 ternary digit string. Power spectrum is calculated from the discrete Fourier transform of a time series of states of a site and the power is summed up over all cells in the array. The fitness of a rule is given by the exponent estimated by the least squares fitting of the power spectrum divided by the residual sum of squares. The array consists of 100 * 100 sites and periodic boundary conditions are used. The array is started from a random initial configuration. We randomly generate initial rules whose value of lambda parameter is uniformly distributed between 1/135 and 90/135. We observed the evolution for 7200 and 8000 time steps. Since the calculation of the power spectrum needs a lot of computation time, we carry out a preliminary selection from initial rules to remove rules whose spectrum is far from a 1/f spectrum. In the preliminary selection the power spectrum of the evolution for 1024 time steps are calculated and we pick the rules with the exponent of the power spectrum equal to -0.3 or less. The selected rules are gathered as an initial population of 180 rules. 20 rules with the highest fitness are copied without modification to the next generation. The remaining 160 rules for the next generation are formed by uniform crossovers with a probability of 0.6 between pairs in the population chosen by roulette wheel selection. Every bit of the offspring from each crossover is mutated with a probability of 0.03.

Up to now we have performed the experiments for a total of 18789 generations in 80 runs in 7200 time steps and a total of 7881 generations in 100 runs in 8000 time steps. Although the search is in progress, we have found several rules with 1/f spectrum. Some of these rules exhibit stationary, periodic, and propagating patterns which are necessary for supporting universal computation.

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Journal Article

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