Wavelet-Based Multifractal Analysis of Human Balance

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Abstract

Center of pressure (COP) traces have been used to investigate the dynamics of human balance. In this paper we employ a wavelet-based multifractal methodology to identify structural differences in medio-lateral (ML) and antero-posterior (AP) sway between COP traces of healthy and Parkinson's patients. Two statistical techniques are used to summarize the differences in multifractal spectrum (MFS) for both groups. The first technique is a multivariate repeated measures analysis on estimated MF spectra for subjects. The second technique obtains two characteristic measures from each subject's estimated MFS: (i) *location* and (ii) *half-width* of the spectrum. These measures present an intuitive summary of the MFS for each subject, allowing for statistical comparisons between the two groups. Both analyzes lead to significant discrimination between Parkinson versus healthy subject's MFS. We find that COP time series of Parkinson patients exhibit a greater degree of roughness as compared to healthy subjects' COP traces. Furthermore, MFS for Parkinson patients are narrower, suggesting a reduction in complexity as compared to the healthy group. The methodology presented here maybe helpful in development of clinically relevant measures, including the assessment of severity of conditions as the measures developed here correlate with standard severity measures.

Keywords– Center of Pressure, Multifractal Spectrum, Multivariate Repeated Measures Analysis.

Introduction

Outputs of the human postural control system under erect stance are often very irregular. For instance, center-of-pressure trajectories (COP) of subjects under erect stance are very erratic. Despite the irregular behavior of COP trajectories, they have been used as the basis to characterize quantitatively some of the motor impairments associated with idiopathic Parkinson's disease (PD)[10]. In this paper we implement a methodology to further gain insight into the differences in COP trajectories between PD and healthy subjects. While this methodology is applied to COP data in this paper, it is in fact a procedure applicable more generally to time series data.

Recently, researchers have identified interesting properties of COP traces pertaining to their degree of smoothness and correlation structure. They have studied these properties as they relate to the dynamics of the human postural control system and its pathologies. In particular, they have identified long-range dependence, scaling behavior, and fractal-like properties in COP traces[3,7,10,14]. Jim Collins et all. first identified long-range dependence in COP traces and computed their Hurst parameter, a measure of both roughness and correlation in a time series[4]. Others researchers have investigated the likeness of COP displacements to $1/f$-noises, and proposed to estimate the fractal dimension of the traces in order to assess their degree of roughness[3,7]. More recently, wavelet-based techniques have been used to estimate the possibly multiple scaling behavior present in these signals. In particular, Thurner et

all. conclude, using wavelet-variance estimation techniques, that COP traces are a correlated process that presents different scaling regions, suggesting the presence of a multifractal structure[14].

In this paper, we investigate the multifractal nature of COP traces and develop statistical procedures based on the wavelet transform in order to link the classical notion of the multifractal spectrum to measures of potential use in the study of the human postural control system. More precisely, we implement a methodology to further gain insight into the differences in COP trajectories between idiopathic Parkinson's disease patients and otherwise healthy individuals.

As a motivation, consider the two COP trajectories depicted in Figure 1. The top trajectory is the mediolateral (ML) output for a healthy subject, while the bottom trajectory corresponds to a ML output from a Parkinson patient (the experimental details will be given in the Methods section). A striking difference between the two trajectories lies in their smoothness. The top COP output seems to the eye to be a much smoother curve than the bottom one, although its smoothness may not be altogether homogeneous. This characteristic distinction may serve as the basis for a discriminating measure between PD and healthy subjects. In order to construct such a discriminating measure, we are in need of tools that quantify degrees of smoothness in curves. An increasingly popular measure of a signal's degree of smoothness is the so called *multifractal spectrum*.

The multifractal spectrum of a signal or process summarizes the scaling behavior and quantifies the relative degree of regularity present in is[13]. Processes for which the MFS exhibits evidence of potentially intricate, locally varying regularity are termed *multifractal*. These processes are increasingly being used as successful modelling tools in a wide variety of fields including medicine and biology[6,11,16].

The multifractal spectrum consists of two quantities. The first quantity measures the degree of regularity or smoothness of a time series, while the second quantity identifies the prevalence of each degree of regularity present in such process. Formally, one first defines a measure of local smoothness, say α, often based on local scaling properties, and then a set

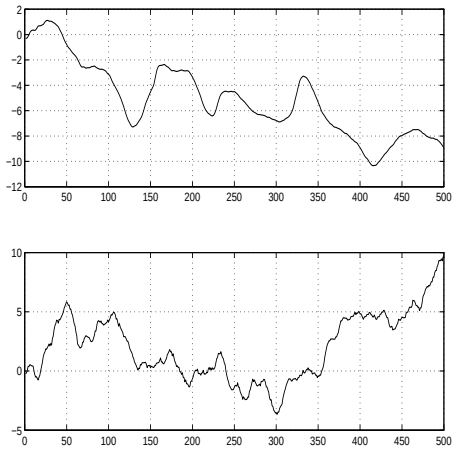

Figure 1: The Top graph is the trace of the mediolateral sway of a healthy subject, while the bottom graph displays the medio-lateral trace of a Parkinson patient. Notice that the Parkinson trace seems rougher.

function $f(\alpha)$ that measures the prevalence of the smoothness α in the signal.

A typical MFS for a multifractal signal is shown in Figure 2. Notice that the MFS of a multifractal signal is generally a convex curve. The most prevalent smooth behavior is α_0, and the range of smoothness is given by the quantities $(\alpha_{min}, \alpha_{max})$. In contrast to a multifractal signal, a *monofractal* signal exhibits a unique smooth behavior α_0; that is, the MFS of a monofractal signal collapses to a single point mass at $\alpha_0 = \alpha_{min} = \alpha_{max}$.

In general, the MFS of a signal provides (i) a measure of the signal's prevalent degree of smoothness as given by the MFS *location* (i.e. the value of α_0), and (ii) a measure of the signal's departure from monofractality as given by the *spread* of the values α_{min} and α_{max}. Information about differences in smoothness between two signals can be obtain by studying both the location and spread of their respective MF spectra.

In this paper, we will study how estimates of both location and spread of the multifractal spectrum of COP traces may lead to discriminating measures be-

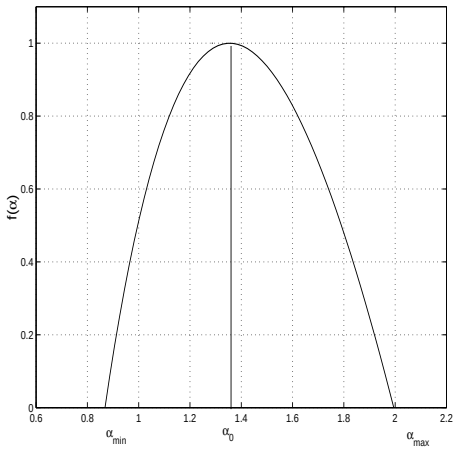

Figure 2: The above graph depicts a typical MFS for a multifractal signal. The maximum of the spectrum is reached at the value α_0. The quantities α_{min} and α_{max} measure the range of smoothness in a signal.

tween PD patients and healthy subjects. For instance, a difference in location would indicate that generally COP traces are smoother in one group as compare to COP traces of the other group. A difference in spread in their respective multifractal spectra would suggest a restricted range of smoothness of the COP traces of one group with respect to the other group.

It is of paramount importance that tools for the analysis of multifractal processes are adapted to cope with these processes' potential non-stationarity, and time-varying statistical properties. To this effect, analytic tools for MFS estimation should preserve *local* features of the analyzed signal.

Wavelet transforms (WT) are well-suited to the study of local scaling behavior and regularity of multifractal processes, since a WT is localized in time and preserves the scaling properties of a given signal. Recently, a number of wavelet-based techniques for estimating a MFS have been introduced by various investigators[1,8,15]. In this paper, we use an estimator based on the discrete wavelet transform, which is analogous to an estimator based on the continuous wavelet transform in Arneodo et all.[1]

One of the main aims in this work is to develop measures that can potentially be used to discriminate between typical COP trajectories from healthy subjects and COP trajectories from subjects with balance pathologies such as those often associated with disease like idiopathic Parkinson. We propose in this work statistical models based on MFS estimation that may serve as the basis for identifying human balance pathologies.

More generally, the techniques developed in this paper can be the basis for tools to investigate and detect pathological patterns in biological data for cases where: (a) the biomedical output under study is a time series, and (b) the presence of the pathological condition manifest itself through a patterned modification of the *smoothness* of the time series (as compared to the non-pathological base case). Furthermore, the techniques presented here provides the investigator with reliable and fast procedures leading to interpretable measures.

Theory

We start with a brief description of the DWT and its properties. Let $\psi_{j,k}(t) = 2^j\psi(2^jt - k)$, where $\psi(\cdot)$ is a wavelet function with compact support and N vanishing moments[5]. Given a process $x(t)$, the wavelet coefficients of the process are defined as

$$W_{j,k} = \int x(t)\psi_{j,k}(t)dt \ , \tag{1}$$

for all integer pairs (j, k). Notice that for large positive j, $W_{j,k}$ contains information in $x(t)$ corresponding to small scales and high frequency; for large negative j, the information pertains to coarse scales and low frequencies. Our interest in this paper will be in the estimation of the fractal spectrum of a time series, based on the wavelet coefficients $\{W_{j,k}\}$ of a sample.

A key feature of the wavelet transform is that local smoothness of a signal can be estimated by studying the decay of the wavelet coefficients. In particular, a measure of local regularity may be defined as[13]

$$\alpha(t_0) = \lim_{k2^{-j}\to t_0} -\frac{1}{j}\log_2|W_{j,k}| \tag{2}$$

where $k2^{-j} \to t_0$ means that $t_0 \in [2^{-j}k, 2^{-j}(k+1)]$ as $j \to \infty$. The quantity $\alpha(t_0)$ is a measure of the smoothness of $x(t)$ around the time t_0.

Multifractal processes exhibit such an intricate pattern of locally varying regularity that estimates based on properties such as (2) alone are often not reliable. In order to overcome this difficulty, the so-called *multifractal formalism* (MFF) was introduced. The MFF is a method by virtue of which both the local regularity and the MFS of a signal can be computed, using a *partition function* relating the two.

The partition function is chosen so as to characterize the scaling of the statistical moments of the wavelet coefficients $W_{j,k}$. Specifically, for any $q \in \Re$, and $j \geq 0$ the partition function is defined as $\tau(q) = \lim_{j\to\infty} \tau(q,j) - 1$ where

$$\tau(q,j) = -\frac{1}{j}\log_2 E[|W_{j,k}|^q], \tag{3}$$

and $E[\cdot]$ denotes statistical expectation. Note that in practice, (3) can be estimated by the $q-th$ sample moment

$$\hat{\tau}(q,j) = -\frac{1}{j}\log_2 \sum_{k=1}^{N2^j} \frac{1}{N2^j}|W_{j,k}|^q. \tag{4}$$

Under some technical conditions, including the convexity of the function $\tau(q,j)$ in q, the MFF states that the partition function $\tau(q)$ relates the local regularity of the signal and the MFS via the Legendre transform

$$f(\alpha) = \min_q(q\alpha - \tau(q)), \tag{5}$$

where the minimum is attained at the value of q for which $\frac{d}{dq}\tau(q) = \alpha$. In this way, one can relate the MFS $f(\alpha)$ with the local regularity α via a parameter q. In particular we can define $\alpha(q) = \lim_{j\to\infty} \frac{d}{dq}\tau(q,j)$, and note that convexity of $\tau(q)$ yields that $\alpha(q) > 0$.[13]

Based on the MFF, for a range of moments $q \in (q_1, q_2)$ a scale dependent estimator of the local regularity can be defined as[12]

$$\hat{\alpha}(q,j) = \frac{\sum_{k=1}^{N2^j} |W_{j,k}|^q \log_2 |W_{j,k}|}{\sum_{k=1}^{N2^j} |W_{j,k}|^q}, \tag{6}$$

and the limiting behavior of $\alpha(q,j)$ can be obtained via a linear regression over a range of scales (j_1, j_2)

$$\hat{\alpha}(q) = \sum_{j=j_1}^{j_2} a_j \hat{\alpha}(q,j). \tag{7}$$

where $a_j = (j - \bar{j})/(\sum_{j=j_1}^{j_2}(j-\bar{j})^2$ are regression weights, and $\bar{j}$ is the average scale. Observe that the estimator $\hat{\alpha}(q,j)$ is a sample-based version of the exact derivative with respect to q of the partition function $\tau(q,j)$ in (3). An estimate of the MFS can then be obtained by

$$\hat{f}(\alpha(q)) = 1 + \sum_{j=j_1}^{j_2} a_j(q\hat{\alpha}(q,j) - \hat{\tau}(q,j)). \tag{8}$$

Using wavelet library functions in standard packages such as Matlab, the above procedure can be easily implemented. It is also worth noting that due to the existence of fast algorithms for the wavelet transform, all the computations required can be implemented very efficiently. In addition, wavelet-based methods yield generally improved estimation of the MFS of signals[1].

In what follows, we will develop a procedure to test for significant differences in MFS of COP trajectories of healthy subjects and idiopathic Parkinson's patients.

Methods

The data for our analysis was gathered in a series of experiments performed by researchers at the Boston University Department of Biomedical Engineering. In the experiments, a Kistler 9287 force platform was used to record the ML and AP displacements of the COP under the feet of the subjects.

COP measurements were taken from 25 subjects, ten healthy and fifteen Parkinson's patient. A series of ten 60-second trials was conducted for each PD and ten 90-second trials for healthy subjects at a sampling frequency rate of 100 Hz. To avoid disparity in the lengths of the time series, only the first 2^{11} data points of the times series were analyzed for

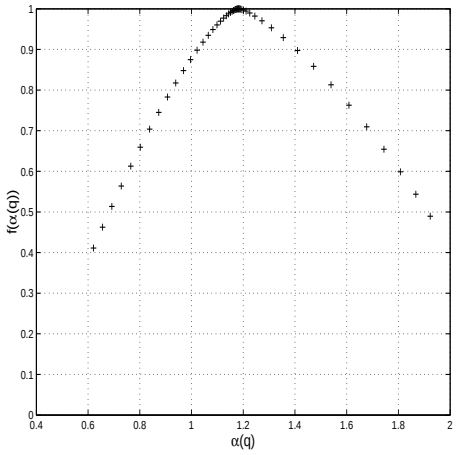

Figure 3: Estimated MFS of medio-lateral COP traces for a healthy patient. The figure displays the averaged spectrum over ten trials using a Daubechies wavelet with 3 vanishing moments and a grid of sample moments $\mathbf{q} = \{-1, -0.95, \ldots, 2\}$. We used scales $(j_1, j_2) = (5, 11)$ for the regression based estimates of both $\alpha(q)$ and $f(\alpha(q))$.

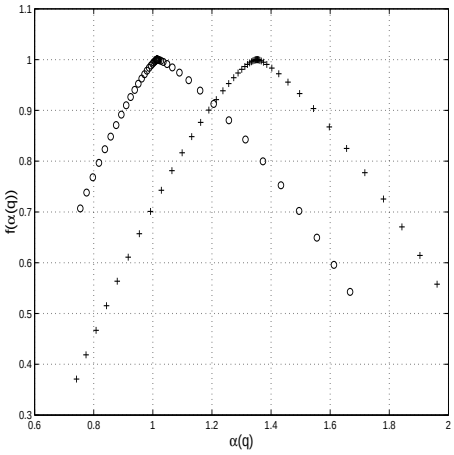

Figure 4: Average spectrum of Medio-lateral COP traces for both Parkinson patients and healthy subjects. The Parkinson MFS is displayed by 'o' and the healthy MFS by '+' (i.e. averaged over all patients and all trials). The MFS estimation was done with identical parameters as in Figure 3.

all subjects. The experimental output to be analyzed consists then of twenty 2^{11}-long time series per subject: ten univariate series containing ML displacement data, and ten univariate series containing AP sway output.

To motivate the statistical procedures below, we will first apply the wavelet-based MFS estimation method explained in the previous section to the individual series, and obtain useful descriptive summaries. In particular, for a grid of values $q = \{-1, -.95, \ldots, 2\}$, we use a fixed range of scales $(j_1 = 5, j_2 = 11)$ to perform the linear regressions in (6) and (8).

Figure 3 depicts the MFS of ML COP trajectory of a healthy subject. The convex shape of the MFS is consistent with evidence of multifractality. A common practice to summarize the difference in MFS in signals from two groups is to plot the group's respective averaged spectrums. Figure 4 shows the averaged MFS over all trials for the PD and healthy subjects. Notice that the PD averaged spectrum is to the left of the averaged spectrum for healthy subjects. This fact suggests that, on the average, COP trajectories of Parkinson's patients exhibit a lower degree of smoothness as compared to healthy subjects' trajectories. A similar pattern is observed for the averaged spectrum of the AP sway.

The averaged spectra in Figure 4 seem to differ not only in location but also in their respective spreads (or curvature). In particular, the averaged MFS for PD seems narrower than the MFS of healthy subjects, suggesting that COP trajectories of healthy subjects have a wider range of smoothness, while COP trajectories of Parkinson's patients exhibit a reduced range in regularity. In other words, COP trajectories of healthy subjects seem to present a higher degree of multifractality.

While the averaged MFS indicates a marked difference between the two groups (in location and possibly spread), we would like to know if such difference is significant in a statistical sense. To answer this question we propose two complimentary methodologies. The first method is based on a multivariate

repeated measures analysis (MRMA) of samples of the estimated spectrum for each subject, and the second is based on a multivariate analysis of variance (MANOVA) of estimated quantities pertaining to both location and spread of the COP multifractal spectrum.

For both analysis we assume that the degree of smoothness of both the ML and AP sway of each subject can be summarized by an idealized MFS. This idealized spectrum is to be estimated by repeated samples from the subject's COP trajectories. Since in practice we only have access to a discrete number of moments q when performing MFS estimation the output of the wavelet-based procedure explained above can be viewed as a discretized estimate of the MFS.

A Multivariate Repeated Measures Model

First observe that the set up of the experiment where data on the COP trajectories were obtained corresponds to a so-called *repeated measures* design where subject's COP trajectories are observed over ten trials. While we could compute estimates for $\alpha(q)$ and $f(\alpha(q))$ on a grid of q's a fine as we desire, sample size considerations lead us to use only a small number of the q's. In particular we will use only five q's, namely, $\mathbf{q} = \{-1, -.5, 0, 0.5, 1\}$. From each subject, we obtain ten estimates of the spectrum;that is, $\alpha(q)$ and $f(\alpha(q))$ are estimated for each trial on ML and AP COP trajectories separately for each given q. Each trial yields a sampled spectrum of points $(\hat{\alpha}(q_m), \hat{f}(\alpha(q_m)))$ for $m = 1, \ldots, 5$. Hence, each trial leads to five bivariate observations. In order to keep the ratio of number of dependent variables to total sample size small, we model exclusively the sampled $\alpha(q)$'s. In summary, the data to analyze comprises a set of five observations per trial per subject, and since we do not assume that observations across trials are independent, the design of the data to be analyzed corresponds to a multivariate repeated measures (MRM) on the observed $\hat{\alpha}(q)$'s.

In accordance to standard practice for MRM data, the model we assume is:

Model 1

$$\begin{aligned}\alpha_{i,t,s(i),m,l} &= \mu_\alpha + Group_i + Trial_t \\ &\quad + Group * Trial_{it} + Q_m \\ &\quad + Group * Q_{im} + Trial * Q_{tm} \\ &\quad + R_{ts(i)m} + \epsilon_{l(ts(i)m)}\end{aligned}$$

for

$$\begin{aligned} i &= 1, 2 \\ t &= 1, \ldots, 10 \\ s(i) &= \begin{cases} 1, \ldots, 10 & \text{if i=1,} \\ 1, \ldots, 15 & \text{if i=2.} \end{cases} \\ m &= 1, \ldots, 5 \\ l &= 1, \ldots, 25 \end{aligned}$$

where the fixed effects are

$$\begin{aligned} \mu_\alpha &= \text{grand mean of } \alpha \\ Group_i &= \begin{cases} 0 & \text{if } i = 1, \\ \text{Parkinson Effect} & \text{if } i = 2. \end{cases} \\ Trial_t &= \text{effect of trial t} \end{aligned}$$

$$\begin{aligned} Group * Trial_{it} &= \text{interaction of } i\text{-th group and } j\text{-th trial} \\ Q_m &= \text{effect of moment } q_m \\ Group * Q_{im} &= \text{interaction of } i\text{-th group and moment } q_m \\ Trial * Q_{tm} &= \text{interaction of } j\text{-th trial and moment } q_m \end{aligned}$$

and all the random effects are subsumed in $R_{ts(i)m}$, and $\epsilon_{l(ts(i)m)}$ is a random error. It is assumed that the vector of random errors is distributed as a multivariate normal variate with mean vector zero and a positive definite variance-covariance matrix. We assume that the variance-covariance matrix is a block diagonal matrix where each block corresponds to each subject's variance-covariance matrix for the multivariate observations across trials. We only assume independence between subjects, and allow for heterogeneity of variances of observations across trials, and

do not assume that the variances are the same in the two groups. In summary, the MRM model presented here is a multivariate normal model where we have allowed for heterogeneity of variances across subjects and groups.

In order to motivate the usefulness of this model, we enumerate the following interpretations for the test of some of the parameters in the model. Testing that the $Group_i$ effect is statistically significant addresses the issue that the estimated $\alpha_{i,j,k,m,l}$'s for the healthy subjects are on the average different from the corresponding estimates for the Parkinson's patients. That is, a significant group effect means that after taking into account other effects such as subject and trial, the two groups are different in terms of their multifractal spectra. A significant Q_m effect would indicate that there is a difference between the estimated $\alpha(q_m)$'s when q_m varies. This fact would result in evidence of multifractality of the original signal since a monofractal signal would yield that the $\alpha(q_m)$'s attain the same values for all m's (i.e. $\alpha(-1) = \ldots = \alpha(0) = \ldots = \alpha(1)$). The $Group * Q_{im}$ interaction has the following important interpretation: if the $Group * Q_{im}$ interaction is significant, this would be an indication that the way the $\alpha(q)$'s are spread around $\alpha(0)$ is different in each group, suggesting a different degree of multifractality for each group.

While the MRM model just explained deals exclusively with the estimated $\alpha(q)$'s, it is important to note that the statistical results will yield information about the whole spectrum. Particularly, differences in $\alpha(0)$ between the groups would indicate a difference in the *location* of the spectrum. Furthermore, both the Q_m effect and the $Group * Q_{im}$ interaction yield information about the *spread* of the spectrum, which translates into a measurement to possibly distinguish between monofractality and multifractality.

In addition to identifying differences in multifractal spectra for typical COP trajectories in both groups of the study in terms of location and spread, it would be useful to consider the *curvature* of the spectra and inquire if differences are also found in this respect. To obtain a measure of the curvature of the estimated spectra for subjects in both groups we need to introduce the estimated values of $f(\alpha(q))$ into the analysis. We do so in the following model.

Location and Half-Width of the MFS

In order to investigate the curvature of the MFS of a signal from its estimate, we propose to measure the left-hand, half-width of the MFS at a pre-specified height. Note that under the underlying assumptions for the existence of a spectrum, we obtain that the MFS is a unimodal, convex curve with support on the positive real numbers. Hence, the left-hand, half-width of the spectrum is a well defined quantity. Observe that under the aforementioned assumptions, we have that

$$0 < \alpha_{\min} \leq \alpha_0$$
$$f(\alpha_0) = 1 \geq f(\alpha_{\min}).$$

We define the d-level half-width of a MFS curve, denoted by $HW(d)$, as follows. Let α_* be the degree of regularity α at which

$$f(\alpha) = d,$$

then

$$HW(d) = \alpha_0 - \alpha_*$$

where α_0 is the value α where the spectrum reaches its maximum. Intuitively the quantity $HW(d)$ gives an idea of the curvature of the left-hand-side of the spectrum. A small value of the d-level half-width would imply a very sharp rise of the MFS between the values α_* and α_0, suggesting that the presence of a more monofractal signal than one exhibiting a larger value for its corresponding spectrum d-level half-width. Hence, the quantity $HW(d)$ computed from the MFS of a signal is interpretable as the *degree of multifractality* of such signal. In practice, it has been widely reported that the right-hand-side of the spectrum is particularly elusive[2]. We propose to concentrate efforts on the half part of the spectrum which is more amenable to stable estimation.

It is important to emphasize that the values $\alpha_{0,s(i)}$, and $HW(d)_{s(i)}$ estimate the two most prominent features of the subjects' MFS: location and spread/curvature. The former measuring the prevalent smoothness of the signal, and the latter serving as a measure of departure from this prevalent behavior.

For each subject, we obtain the estimate $(\hat{\alpha}_{0,s(i)}, \widehat{HW}(d)_{s(i)})$ which are sample averages over the totality of the trials. A statistical test for a group difference between the PD and healthy subjects can then be obtained via a MANOVA on the bivariate estimates $(\hat{\alpha}_{0,s(i)}, \widehat{HW}(d)_{s(i)})$. To that effect we have the model:
Model 2

$$(\hat{\alpha}_{0,s(i)}, \widehat{HW}(d)_{s(i)}) = \mu_{\alpha_0, HW(d)} + Group_i + \epsilon_{s(i)}$$

for

$$\begin{aligned} i &= 1, 2 \\ s(i) &= \begin{cases} 1, \ldots, 10 & \text{if } i = 1, \\ 1, \ldots, 15 & \text{if } i = 2. \end{cases} \end{aligned}$$

where $\mu_{\alpha_0, HW(d)}$ is a grand mean for the bivariate output under consideration, $Group_i$ is the group effect, and ϵ_s is a mean zero bivariate normal random vector. Notice that a significant group effect would indicate that both groups differ in their respective multifractal spectrum with respect to both location and d-level half-width.

Observe that the quantity $HW(d)$ depends of the level d; hence, it is pertinent to study the sensitivity to the choice of level d to use for a given analysis.

An important aspect of this last model is that it leads to estimates of both location and curvature which are readily interpretable.

Results

Analysis of results from Model 1

In order to fit the MRM model suggested in the previous section, we used a maximum likelihood estimation (MLE) procedure called PROC MIX available in the software package SAS for mixed linear models. This procedure allows to test for significance even under heterogeneity of variances. For the statistical test performed, robustness to departure from the normality assumption depends on sample size considerations[9]. In order to preserve the validity of the statistical tests, we keep the ratio of the smallest group to the number of trials high (a ratio of at least 3 is recommended[9]). That is, since we have only 10 subjects in the healthy group, we use data from at most three trials (i.e. $t = 1, 2, 3$).

Antero-Posterior Sway
Tests of Fixed Effects

Source	Pr > F
$GROUP$	0.0001
Q	0.0001
$GROUP * Q$	0.0063
$TRIAL$	0.6343
$GROUP * TRIAL$	0.1658
$TRIAL * Q$	0.3635

Table 1: F Tests for Model 1 (AP sway). Observe that $GROUP$, Q, and $GROUP * Q$ are highly significant, suggesting the groups have different MFS, and that their respective COP traces are multifractal

Conforming to standard practice, we perform an approximate F test to investigate the significance of the fixed effects in the model. Table 1 displays the result of the approximate F tests for the fixed effects of the model applied to the AP data. Notice that $Group_i$, Q_m, and the interaction $GROUP * Q_{im}$ are highly significant. This finding suggests that there is evidence to conclude that COP trajectories for the two groups differ in terms of their respective estimated degrees of smoothness $\alpha(q_m)$. Furthermore, for the two groups, the evidence suggests that the COP trajectories' MF spectra are consistent with multifractal behavior. Table 2 displays the results of the approximate F tests for the analysis of ML output, which yields similar results to that of the AP sway.

Analysis of results from Model 2

In order to implement the second model described in the methods section, we first compute from the data the values corresponding to α_0, and $HW(d)$, which are the values for location and half-width of the estimated MF spectra.

To estimate α_0 for each subject we take the average value of α for which $f(\alpha) = 1$ over all the trials. In order to compute the d-level half-width, it is necessary

Medio-Lateral Sway
Tests of Fixed Effects

Source	Pr > F
$GROUP$	0.0001
Q	0.0001
$GROUP * Q$	0.0035
$TRIAL$	0.9240
$GROUP * TRIAL$	0.7535
$TRIAL * Q$	0.4578

Table 2: F Tests for Model 1 (ML sway). Observe that $GROUP$, Q, and $GROUP * Q$ are significant, suggesting the groups have different MFS, and that their respective COP traces are multifractal

Medio-Lateral Sway
Least Squares Estimates

Group	α_0	$HW(0.95)$
Healthy	1.35373	0.09999872
Parkinson	1.01920	0.06845753
Est. Difference	0.33456	0.03154119
(standard error)	(0.06695)	(0.0047726)

Table 3: LS Means for ML Sway. Observe that both groups differ in terms of the least squares estimates of the location parameter α_0 and d-level half-width of the MFS. We also give an estimate of the group contrast (or estimated difference). The last line gives the standard error for the estimated difference.

to first choose the level $0 \leq d < 1$. We chose $d = 0.95$, and then performed sensitivity analysis to this choice, finding that the test results from the MANOVA did not lead to different conclusions as d was changed from 0.90 to 0.98. In order to obtain an estimate for α_* from the estimated multifractal spectra for each subject, we need to solve the following inverse problem: find the value of α for which $f(\alpha) = 0.95$. Since the estimated MFS is a discrete sample, there is no guarantee that there will be a value in the sample the exactly attains the $f(\alpha)$ value of 0.95. However, we can compute as many moments q as desired in order to achieve some tolerance level. Alternatively, one may interpolate between sampled values of the average MFS using standard spline algorithms. In any case, we take α_* to be the value of α at which $f(\alpha)$ is approximately equal to 0.95 to within a tolerance level.

Figure 5 exemplifies the procedure to estimate both location and half-width. The figure displays the average MFS for a PD patient over all ten trials. The estimate of α_0 is the α value where the curve $f(\alpha)$ attains the value of 1, and the $HW(0.95)$ is the horizontal distance between α_0 and the α at which the curve attains the value of 0.95 (to within a tolerance value of 0.001).

The bivariate quantity $(\hat{\alpha}_{0,s(i)}, \widehat{HW}(d)_{s(i)})$ is then used to compare the two groups in terms of their possible difference in multifractal spectra. Such difference, as it was stated before, is quantified by the joint differences in location and curvature. To visually inspect the difference in both location and curvature exhibited by the MFS of the two groups in consideration, Figure 6 presents a bivariate plot of location and curvature of the MFS for the two groups. Observe that there is a clear separation between the two groups in terms of location and half-width, except for an obvious outlier. Note that a narrower MFS would indicate a less complex signal. This can be interpreted as COP traces of PD patients exhibiting less complexity than COP traces of healthy subjects.

These two graphs provide a simple, interpretable, and visual summary of the fact that PD patients exhibit COP traces which are: (i) less smooth than COP traces of healthy patients, and (ii) exhibit a reduced range in degrees of smoothness as compared to healthy COP traces. That COP traces for PD patients are *rougher* in nature as compared to healthy COP traces is indicated by the estimated regularity parameter α_0, which represents the location of the MFS. Lower values of α indicates a more erratic behavior of COP traces. The half-width of the MFS represents the degree of multifractality, or in other words, how homogeneous the COP trace is in terms of smoothness.

Using SAS to perform a MANOVA on location and half-width for subjects in the two groups we obtain p-values less than 0.0001 for each AP and ML

Antero-Posterior Sway
Least Squares Estimates

Group	α_0	$HW(0.95)$
Healthy	1.28622	0.09427147
Parkinson	0.96856	0.07542800
Est. Difference	0.31764	0.0188422
(standard error)	(0.06482)	(0.006852)

Table 4: LS Means for AP Sway. Observe that both groups differ in terms of the least squares estimates of the location parameter α_0 and d-level half-width of the MFS. We also give an estimate of the group contrast (or estimated difference). The last line gives the standard error for the estimated difference.

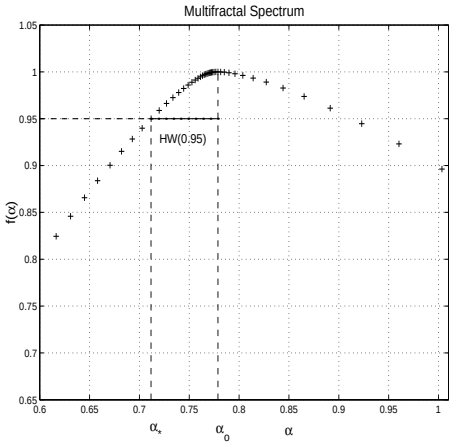

Figure 5: Estimated MFS of COP traces for a Parkinson patient. The figure displays the averaged spectrum over ten trials for a Parkinson patient represented by '+'. The dotted line represents the measure $HW(d) = \alpha_0 - \alpha_*$ for $d = 0.95$.

sway for all MANOVA standard statistics such as Wilk's Lambda, Pillai's trace, and Roy's Greatest Root. This indicates significant differences in the two groups with respect to their MF spectra in terms of location and half-width.

Estimated least squares (LS) means for the AP and ML sway of both groups are displayed in tables 3 and 4, as well as the estimated contrast (or difference) in terms of α_0 and MFS half-width for the two groups.

Multifractal spectra and measures disease severity

In light of the above results, it is important to know how the measures $\alpha(0)$ and the $HW(d)$ correlate with standard measures of severity of balance impairment. Recent development of measures of muscle stiffness from COP traces reveal that roughness of trajectories are correlated to the severity of impairment in the human postural control system. Lauk et all.[10] observe that roughness of AP traces correlates to measures of rigidity, bradykinesia, posture impairment, and others. As indicated in Figure 1 of the cited work, the severity of the affliction seems to be correlated to the degree of smoothness of the AP traces of Parkinson patients. In the spirit of the cited work, we compute Kendal's τ between the location parameter $\alpha(0)$, and $HW(d)$ with relevant measures of balance impairment severity. Kendal's τ, in contrast to the usual Pearson correlation coefficient, is a measure of agreement between two quantities for which we can compute a statistical test without distributional assumptions.

Table 5 shows the correlation measure τ for a diverse subset of clinical scales of human balance with the multifractal spectrum location parameter $\alpha(0)$ and $HW(0.95)$ for both the AP and the ML traces. Observe that in the AP direction, the location parameter $\alpha(0)$ correlates with the clinical measures in the table, except for arm. For the ML direction, all clinical measures correlate with the location parameter. The half-width of the estimated spectrum correlates with all clinical measures in the AP direction except again for arm. No distinct trend emerges in the correlation of clinical measures and the estimated half-width for the spectra of ML traces, except that the clinical measure arm seems to correlate well with both the half-width and location parameter in both the AP and ML directions. An intriguing fact arises at this point, and it would be interesting to investigate further why both the degree of roughness (i.e. $\alpha(0)$) and the degree of multifractality (i.e. $HW(0.95)$) correlate well with the clinical measure arm in the ML direction and not in the AP direction.

Rank Correlation $\alpha(0)$		
	Antero-Posterior	Medio-Lateral
	Kendall's τ	Kendall's τ
Clinical Scale	(p-value)	(p-value)
Rigidity	0.3904 (0.0390)	0.3905 (0.0398)
Bradykinesia	0.3619 (0.0460)	0.4190 (0.0209)
Posture	0.3142 (0.0599)	0.3524 (0.0349)
Hand	0.3143 (0.0920)	0.3333 (0.0740)
Arm	0.0952 (0.6051)	0.3810 (0.0386)
UPDSR	0.3238 (0.0920)	0.3619 (0.0583)

Table 5: Correlation between some clinical scales and location parameter $\alpha(0)$ for Parkinson patients. The correlation value shown is the Kendall's τ, and the p-values correspond to a test with alternative hypothesis $\tau \neq 0$. Note that this test does not make any distributional assumptions on the data.

Rank Correlation $HW(0.95)$		
	Antero-Posterior	Medio-Lateral
	Kendall's τ	Kendall's τ
Clinical Scale	(p-value)	(p-value)
Rigidity	0.4667 (0.0140)	0.2571 (0.1758)
Bradykinesia	0.3048 (0.0929)	0.1714 (0.3446)
Posture	0.1429 (0.3925)	0.1238 (0.4586)
Hand	0.3333 (0.0740)	0.2571 (0.1681)
Arm	0.1905 (0.3011)	0.4380 (0.0174)
UPDSR	0.3238 (0.0902)	0.2476 (0.1951)

Table 6: Correlation between some clinical scales and location parameter $HW(0.95)$ for Parkinson patients. See caption for Table 5.

An investigation in this respect may reveal a difference in the postural control system dynamics for each direction.

Discussion

Using a wavelet-based procedure to estimate the MFS of COP traces from healthy subjects and PD patients, we determined that there is a significant difference in such traces for the two groups in terms of their corresponding degrees of smoothness. We have presented two complimentary statistical procedures to address the issue of comparing multifractal spectra of subjects with a balance impairing disease such as Parkinson's disease to multifractal spectra of COP traces of otherwise healthy subjects. We find via the first procedure, a multivariate repeated measures analysis, that the multifractal spectrum of COP traces for Parkinson patients is statistically different from the MFS of healthy subjects in terms of location and spread. It was also found that the MFS for both groups (and in both AP and ML sway) suggests that COP traces exhibit multifractal behavior. The second procedure, a MANOVA on bivariate estimates of location and half-width of the MFS, suggest that the multifractal spectra of both groups differ not only in location, but also in curvature, as measured by the half-width of the spectrum. Since location and curvature are two defining characteristics of a MFS, we can confidently claim that both groups differ in terms of their estimated multifractal spectra.

As it can be observed in Figures 6 and 7, location and spread of the multifractal spectra of COP traces serve as good discriminating measures between PD patients and healthy subjects.

The fact that both the location and the spread of the MFS of COP traces also correlate with standard measures of disease severity in PD patients suggests that they may be potentially useful as objective measures of severity.

Given the (i) the reliable methodology based on the wavelet transform presented in this paper (among others available), (ii) the efficiency of the algorithms used, (ii) and the potentially useful measures derived, further study seems appropriate to assess the impact of these methods in areas pertaining to identification and diagnosis of balance pathologies.

On a more general note, the methodology developed here can be applied to other biomedical output and signals in the form of time series, with the potential of being similarly successful in identifying and discriminating different degrees of smoothness in signals.

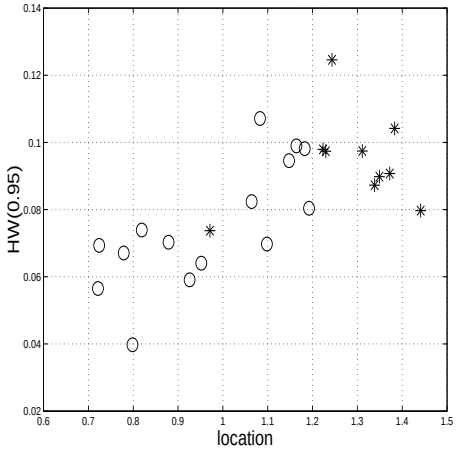

Figure 6: Location and Half-Width of the MFS of AP sway. Observe that overall, Parkinson patients (shown by 'o') exhibit smaller values of location and lower values of HW(0.95) than healthy subjects (denoted by '*'). Notice the good separation between the two groups indicating that these two measures have a high discriminating power.

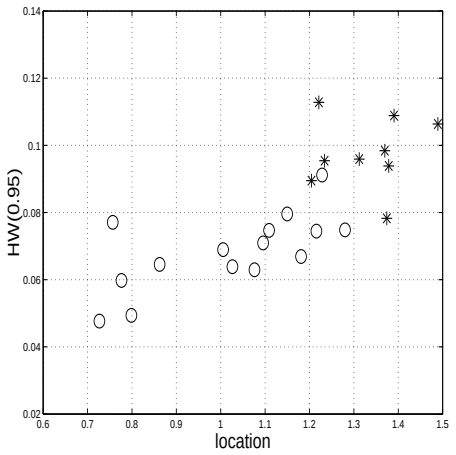

Figure 7: Location and Half-Width of the MFS of ML sway. See caption of Figure 6 for legend.

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