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Higgs boson gluon-fusion production at threshold in N^{3}LO QCD

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We present the cross-section for the threshold production of the Higgs boson at hadron-colliders at next-to-next-to-next-to-leading order (N^{3}LO) in perturbative QCD. We present an analytic expression for the partonic cross-section at threshold and the impact of these corrections on the numerical estimates for the hadronic cross-section at the LHC. With this result we achieve a major milestone towards a complete evaluation of the cross-section at N^{3}LO which will reduce the theoretical uncertainty in the determination of the strengths of the Higgs boson interactions.

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High precision theoretical predictions for the production rate of the Higgs boson are crucial in the study of the recently discovered particle from the ATLAS and CMS collaborations [1] and for inferring the existence of phenomena beyond the Standard Model. With the collection of further data at the upgraded LHC, the theoretical uncertainty for the gluon-fusion cross-section will become soon dominant. It is thus highly timely to improve the theoretical accuracy of the cross-section predictions.

The quest for accurate Higgs boson cross-sections has been long-standing and it is paralleled with major advances in perturbative QCD. State-of-the-art calculations of the gluon-fusion cross-section (for a review, see Ref. [2] and references therein) comprise next-to-leading-order (NLO) QCD corrections in the full Standard-Model theory, next-to-next-to-leading order (NNLO) QCD corrections as an expansion in inverse powers of the top-quark mass $1/m_t$, two-loop electroweak corrections and mixed QCD/electroweak corrections. To improve upon the present accuracy, the most significant correction is expected from the N^{3}LO QCD contribution in the leading order of the $1/m_t$ expansion.

Universal factorization of radiative corrections due to soft emissions, as well as knowledge of the three-loop splitting functions [3], have made possible the derivation of logarithmic contributions to the cross-section beyond NNLO [4]. However, further progress in determining the N^{3}LO correction can only be achieved by direct evaluation of the Feynman diagrams at this order.

Recently, there was rapid progress in this direction. The required three-loop matrix-elements have been computed in Ref. [5]. The partonic cross-sections for the production of a Higgs boson in association with three partons was computed in Ref. [6], while the two-loop matrix-elements for the production of a Higgs boson in association with a single parton and the corresponding two-loop soft current were computed in Ref. [7] and Ref. [8]. Corrections due to one-loop amplitudes for a Higgs boson in association with a single parton were evaluated in Refs. [9], and counter-terms due to ultraviolet [11, 12] and initial-state collinear divergencies were computed in Refs. [10]. The N^{3}LO Wilson coefficient and the renormalization constants of the operator in the effective theory where the top quark is integrated out have been computed in Refs. [11]. Although all these contributions are separately divergent in four dimensions, a finite cross-section can be obtained by combining them with the remaining one-loop matrix elements for the production of the Higgs boson in association with two partons.

The purpose of this Letter is to complete the computation of all matrix-elements integrated over loop momenta and phase-space which are required at N^{3}LO in the limit of Higgs production at threshold. We present the fully analytic result for the first term in the threshold expansion of the gluon-fusion cross-section at N^{3}LO, and we use this result to estimate the impact of N^{3}LO corrections to the inclusive Higgs production cross-section at threshold. Our result is the first calculation of a hadron collider observable at this order in perturbative QCD.

The Higgs production cross-section takes the form

$$\sigma = \sum_{ij} \int dx_1\, dx_2\, f_i(x_1)\, f_j(x_2)\, \hat{\sigma}_{ij}(m_H^2, x_1\, x_2\, s)\,, \quad (1)$$

where $\hat{\sigma}_{ij}$ are the partonic cross-sections for producing a Higgs boson from partons i and j, $f_i(x_1)$ and $f_j(x_2)$ are the corresponding parton distribution functions, and m_H^2 and s denote the mass of the Higgs boson and the hadronic centre-of-mass energy, respectively. We work in an effective theory where the top quark has been integrated out, and the Higgs boson couples directly to the gluons via the effective operator

$$\mathcal{L}_{\rm eff} = -\frac{1}{4v}\, C(\mu^2)\, H\, G^a_{\mu\nu}\, G_a^{\mu\nu}\,, \qquad (2)$$

where $v \simeq 246$ GeV is the vacuum expectation value of the Higgs field and $C(\mu^2)$ is the Wilson coefficient, given as a perturbative expansion in the $\overline{\rm MS}$-renormalized strong coupling constant $\alpha_s \equiv \alpha_s(\mu^2)$ evaluated at the scale μ^2. Up to three loops, we have [11]

$$\begin{aligned} C(\mu^2) = -\frac{\alpha_s}{3\pi}\Bigg\{ & 1 + \frac{11}{4}\frac{\alpha_s}{\pi} \qquad (3)\\ & + \left(\frac{\alpha_s}{\pi}\right)^2 \left[\frac{19}{16}\,L_t + \frac{2777}{288} + N_F\left(\frac{1}{3}\,L_t - \frac{67}{96}\right)\right]\\ & + \left(\frac{\alpha_s}{\pi}\right)^3 \Bigg[\frac{897943}{9216}\,\zeta_3 + \frac{209}{64}\,L_t^2 + \frac{1733}{288}\,L_t - \frac{2892659}{41472}\\ & \quad + N_F\left(-\frac{110779}{13824}\,\zeta_3 + \frac{23}{32}\,L_t^2 + \frac{55}{54}\,L_t + \frac{40291}{20736}\right)\\ & \quad + N_F^2\left(-\frac{1}{18}\,L_t^2 + \frac{77}{1728}\,L_t - \frac{6865}{31104}\right)\Bigg] + \mathcal{O}(\alpha_s^4)\Bigg\}\,, \end{aligned}$$

with $L_t = \log(\mu^2/m_t^2)$ and N_F the number of active light flavours.

The partonic cross-section itself admits the perturbative expansion

$$\hat\sigma_{ij}(m_H^2, \hat s) = \frac{\pi\, C(\mu^2)^2}{v^2\, V^2} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k \eta_{ij}^{(k)}(z)\,, \qquad (4)$$

with $z \equiv m_H^2/\hat s$ and $V = N^2 - 1$, where N denotes the number of colours. The coefficients $\eta_{ij}^{(k)}(z)$ are known explicitly through NNLO in perturbative QCD [13].

If all the partons emitted in the final state are soft, we can approximate the partonic cross-sections by their threshold expansion,

$$\eta_{ij}^{(k)}(z) = \delta_{ig}\,\delta_{jg}\,\hat\eta^{(k)}(z) + \mathcal{O}(1-z)^0\,. \qquad (5)$$

Note that the first term in the threshold expansion, the so-called *soft-virtual* term, only receives contributions from the gluon-gluon initial state. Soft-virtual terms are linear combinations of a δ function and plus-distributions,

$$\int_0^1 dz\,\left[\frac{g(z)}{1-z}\right]_+ f(z) \equiv \int_0^1 dz\,\frac{g(z)}{1-z}\,[f(z) - f(1)]\,. \qquad (6)$$

Through NNLO, we have [13, 14]

$$\hat\eta^{(0)}(z) = \delta(1-z)\,, \qquad (7)$$

$$\hat\eta^{(1)}(z) = 2\,C_A\,\zeta_2\,\delta(1-z) + 4\,C_A\left[\frac{\log(1-z)}{1-z}\right]_+\,, \qquad (8)$$

$$\begin{aligned} \hat\eta^{(2)}(z) = \delta(1-z)\Bigg\{ & C_A^2\left(\frac{67}{18}\,\zeta_2 - \frac{55}{12}\,\zeta_3 - \frac{1}{8}\,\zeta_4 + \frac{93}{16}\right)\\ & + N_F\left[C_F\left(\zeta_3 - \frac{67}{48}\right) - C_A\left(\frac{5}{9}\,\zeta_2 + \frac{1}{6}\,\zeta_3 + \frac{5}{3}\right)\right]\Bigg\}\\ & + \left[\frac{1}{1-z}\right]_+ \Bigg[C_A^2\left(\frac{11}{3}\,\zeta_2 + \frac{39}{2}\,\zeta_3 - \frac{101}{27}\right) \qquad (9)\\ & \quad + N_F\,C_A\left(\frac{14}{27} - \frac{2}{3}\,\zeta_2\right)\Bigg]\\ & + \left[\frac{\log(1-z)}{1-z}\right]_+ \left[C_A^2\left(\frac{67}{9} - 10\,\zeta_2\right) - \frac{10}{9}\,C_A\,N_F\right]\\ & + \left[\frac{\log^2(1-z)}{1-z}\right]_+ \left(\frac{2}{3}\,C_A\,N_F - \frac{11}{3}\,C_A^2\right)\\ & + \left[\frac{\log^3(1-z)}{1-z}\right]_+ 8\,C_A^2\,. \end{aligned}$$

In this expression ζ_n denotes the Riemann zeta function, $C_A = N$ and $C_F = V/(2N)$. For simplicity renormalization and factorisation scales are set equal to the Higgs mass, $\mu_R = \mu_F = m_H$.

The main result of this Letter is the next term in the perturbative expansion, N^{3}LO, of the cross-section for the threshold production of a Higgs boson. All ingredients necessary to compute $\hat\eta^{(3)}(z)$ have recently become available. Each of these contributions is individually divergent. Adding up all the contributions, and including the counter-terms necessary to remove the ultraviolet and infrared divergences, all the poles in the dimensional regulator ϵ cancel, leaving a finite remainder in the Laurent expansion, which, for $\mu_R = \mu_F = m_H$, is given by,

$$\begin{aligned} \hat\eta^{(3)}(z) = \delta(1-z)\Bigg\{ & C_A^3\left(-\frac{2003}{48}\,\zeta_6 + \frac{413}{6}\,\zeta_3^2 - \frac{7579}{144}\,\zeta_5 + \frac{979}{24}\,\zeta_2\,\zeta_3 - \frac{15257}{864}\,\zeta_4 - \frac{819}{16}\,\zeta_3 + \frac{16151}{1296}\,\zeta_2 + \frac{215131}{5184}\right) \qquad (10)\\ & + N_F\Bigg[C_A^2\left(\frac{869}{72}\,\zeta_5 - \frac{125}{12}\,\zeta_3\,\zeta_2 + \frac{2629}{432}\,\zeta_4 + \frac{1231}{216}\,\zeta_3 - \frac{70}{81}\,\zeta_2 - \frac{98059}{5184}\right) \end{aligned}$$

$$
\begin{aligned}
&+ C_A\, C_F\, \left(\frac{5}{2}\,\zeta_5 + 3\zeta_3\zeta_2 + \frac{11}{72}\,\zeta_4 + \frac{13}{2}\,\zeta_3 - \frac{71}{36}\,\zeta_2 - \frac{63991}{5184}\right) + C_F^2\, \left(-5\zeta_5 + \frac{37}{12}\,\zeta_3 + \frac{19}{18}\right)\Bigg] \\
&+ N_F^2 \left[C_A\, \left(-\frac{19}{36}\,\zeta_4 + \frac{43}{108}\,\zeta_3 - \frac{133}{324}\,\zeta_2 + \frac{2515}{1728}\right) + C_F\, \left(-\frac{1}{36}\,\zeta_4 - \frac{7}{6}\,\zeta_3 - \frac{23}{72}\,\zeta_2 + \frac{4481}{2592}\right)\right]\Bigg\} \\
&+ \left[\frac{1}{1-z}\right]_+ \Bigg\{C_A^3\, \left(186\,\zeta_5 - \frac{725}{6}\,\zeta_3\,\zeta_2 + \frac{253}{24}\,\zeta_4 + \frac{8941}{108}\,\zeta_3 + \frac{8563}{324}\,\zeta_2 - \frac{297029}{23328}\right) + N_F^2\, C_A\, \left(\frac{5}{27}\,\zeta_3 + \frac{10}{27}\,\zeta_2 - \frac{58}{729}\right) \\
&+ N_F \left[C_A^2\, \left(-\frac{17}{12}\,\zeta_4 - \frac{475}{36}\,\zeta_3 - \frac{2173}{324}\,\zeta_2 + \frac{31313}{11664}\right) + C_A\, C_F\, \left(-\frac{1}{2}\,\zeta_4 - \frac{19}{18}\,\zeta_3 - \frac{1}{2}\,\zeta_2 + \frac{1711}{864}\right)\right]\Bigg\} \\
&+ \left[\frac{\log(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\, \left(-77\zeta_4 - \frac{352}{3}\,\zeta_3 - \frac{152}{3}\,\zeta_2 + \frac{30569}{648}\right) + N_F^2\, C_A\, \left(-\frac{4}{9}\,\zeta_2 + \frac{25}{81}\right) \\
&+ N_F \left[C_A^2\, \left(\frac{46}{3}\,\zeta_3 + \frac{94}{9}\,\zeta_2 - \frac{4211}{324}\right) + C_A\, C_F\, \left(6\,\zeta_3 - \frac{63}{8}\right)\right]\Bigg\} \\
&+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\, \left(181\,\zeta_3 + \frac{187}{3}\,\zeta_2 - \frac{1051}{27}\right) + N_F \left[C_A^2\, \left(-\frac{34}{3}\,\zeta_2 + \frac{457}{54}\right) + \frac{1}{2}\,C_A\, C_F\right] - \frac{10}{27}\,N_F^2\, C_A\Bigg\} \\
&+ \left[\frac{\log^3(1-z)}{1-z}\right]_+ \Bigg\{C_A^3\, \left(-56\,\zeta_2 + \frac{925}{27}\right) - \frac{164}{27}\,N_F\, C_A^2 + \frac{4}{27}\,N_F^2\, C_A\Bigg\} \\
&+ \left[\frac{\log^4(1-z)}{1-z}\right]_+ \left(\frac{20}{9}\,N_F\, C_A^2 - \frac{110}{9}\,C_A^3\right) + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 8\,C_A^3\,.
\end{aligned}
$$

Equation (10) is the main result of this Letter. While the terms proportional to plus-distributions were previously known [4], we complete the computation of $\hat{\eta}^{(3)}(z)$ by the term proportional to $\delta(1-z)$, which includes in particular all the three-loop virtual corrections.

Before discussing some of the numerical implications of Eq. (10), we have to make a comment about the validity of the threshold approximation. As we will see shortly, the plus-distribution terms show a complicated pattern of strong cancellations at LHC energies; the formally most singular terms cancel against sums of less singular ones. Therefore, exploiting the formal singularity hierarchy of the terms in the partonic cross-section does not guarantee a fast-converging expansion for the hadronic cross-section. Furthermore, the definition of threshold corrections in the integral of Eq. (1) is ambiguous, because the limit of the partonic cross-section at threshold is not affected if we multiply the integrand by a function g such that $\lim_{z\to 1} g(z) = 1$,

$$
\int dx_1\, dx_2\, \left[f_i(x_1)\, f_j(x_2) z g(z)\right] \lim_{z\to 1} \left[\frac{\hat{\sigma}_{ij}(s,z)}{z g(z)}\right]. \qquad (11)
$$

It is obvious that Eq. (11) has the same formal accuracy in the threshold expansion, provided that $\lim_{z\to 1} g(z) = 1$. As we will see in the following, this ambiguity has a substantial numerical implication, and thus presents an obstacle for obtaining precise predictions. We note however that by including in the future further corrections in the threshold expansion, this ambiguity will be reduced.

Bearing this warning in mind, we present some of the numerical implications of our result for $g(z) = 1$. For $N = 3$ and $N_F = 5$, the coefficients of the distributions in Eq. (10) take the numerical values

$$
\begin{aligned}
\hat{\eta}^{(3)}(z) \simeq\ & \delta(1-z)\, 1124.308887\ldots && (\to 5.1\%) \\
&+ \left[\frac{1}{1-z}\right]_+ 1466.478272\ldots && (\to -5.85\%) \\
&- \left[\frac{\log(1-z)}{1-z}\right]_+ 6062.086738\ldots && (\to -22.88\%) \\
&+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ 7116.015302\ldots && (\to -52.45\%) \\
&- \left[\frac{\log^3(1-z)}{1-z}\right]_+ 1824.362531\ldots && (\to -39.90\%) \\
&- \left[\frac{\log^4(1-z)}{1-z}\right]_+ 230 && (\to 20.01\%) \\
&+ \left[\frac{\log^5(1-z)}{1-z}\right]_+ 216\,. && (\to 93.72\%)
\end{aligned}
$$

In parentheses we indicate the correction that each term induces to the hadronic cross-section normalized to the leading order cross-section at a center of mass energy of 14 TeV. The ratio is evaluated with the MSTW NNLO [15] parton densities and α_s at scales $\mu_R = \mu_F = m_H$ in the numerator and denominator. We also fac-

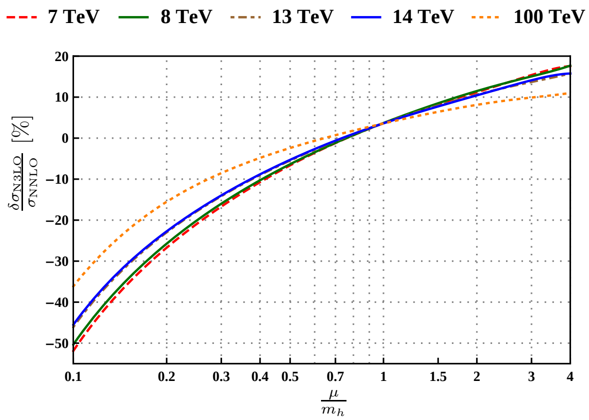

FIG. 1: Percent change from the `ihixs` cross-section at NNLO $\sigma_{\rm NNLO}$ to the N^{3}LO cross-section estimate at threshold for $\sqrt{s} = 7, 8, 13$ and 14 TeV respectively, as a function of the scale $\mu = \mu_R = \mu_F$.

torize the Wilson coefficient at all orders, as in Eq. (4), in both numerator and denominator, and it cancels in the ratio. We find that the pure N^{3}LO threshold correction is approximately -2.27% of the leading order. We observe that the δ-term which we computed for the first time in this publication is as large as the sum of the plus-distribution terms which were already known in the literature and cancels almost completely against them for $\mu_R = \mu_F = m_H$. We note, however, that by choosing a different functional form for the function $g(z)$ in Eq. (11), the conclusion can be substantially different. For example, by choosing $g(z) = 1, z, z^2, 1/z$ we find that the threshold correction to the hadronic cross-section at N^{3}LO normalized to the leading order cross-section is $-2.27\%, 8.19\%, 30.16\%, 7.73\%$ respectively.

In Fig. 1 we present the percentual change of the N^{3}LO threshold corrections to an existing Higgs cross-section estimate based on previously known corrections (NNLO, electroweak, quark-mass effects) in `ihixs` [2] and the settings of Ref. [16]. The new N^{3}LO correction displayed in this plot includes the full logarithmic dependence on the renormalization and factorization scales, as they can be predicted from renormalization group and DGLAP evolution, the Wilson coefficient at N^{3}LO and the threshold limit of Eq. (10). The function $g(z)$ of Eq. (11) is fixed to unity. $\sigma_{\rm NNLO}$ and $\delta\sigma_{\rm N^3LO}$ are defined after expanding the product of the Wilson coefficient and the partonic cross-sections in α_s. We conclude that N^{3}LO corrections are important for a high precision estimation of the Higgs cross-section.

Our result of the N^{3}LO cross-section at threshold demonstrates that it is, in principle, possible to calculate all loop and phase-space integrals required for N^{3}LO QCD corrections for hadron collider processes, albeit in a kinematic limit. With this publication, we open up a new era in precision phenomenology which promises the computation of full N^{3}LO corrections for Higgs production and other processes in the future.

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