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THE MOMENTUM DISTRIBUTION OF THE DECELERATED DRIVE BEAM IN CLIC AND THE TWO-BEAM TEST STAND AT CTF3

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Abstract

We present analytical calculations of the momentum spectrum of the drive beam in (CLIC and) CTF3 after part of its kinetic energy is converted to microwaves used to accelerate the main beam. The resulting expressions can be used to determine parameters of the power conversion process in the PETS structures installed in the Two-beam test stand in CTF3.

INTRODUCTION

The drive beam in CLIC [1] or CTF3 looses a significant amount of energy in the power extraction and transfer structures (PETS), which is converted to microwaves that are subsequently used to accelerate a second beam. The efficiency of the process depends on many parameters and in this report we attempt to understand how they affect the energy spectrum of the decelerated drive beam in order to optimize the power production process. The rather violent energy exchange from beam to radio frequency power can be visualized as the longitudinal distribution of a drive beam bunch 'wrapped' over the decelerating longitudinal field. The particles lose energy according to their longitudinal position or phase which will determine the energy distribution of the beam after the PETS. This energy distribution is what we calculate in this report, both for beams with vanishing and with finite initial energy distribution. The energy distribution is normally measured with a transverse beam size monitor in a spectrometer. Therefore, we calculate the expected image including the effect of smearing due to finite emittance. Finally, we discuss surprising results from measurements recently performed in the two-beam test stand [2] of CTF3 with the goal of comparing the calculations to measurements.

MODEL

We describe the distribution in longitudinal phase space $\psi(E,\phi)$ of the beam upstream of the PETS by the product of a Gaussian in phase ϕ and a general (normalized) energy distribution Ψ_0

$$\psi(E,\phi) = \frac{1}{\sqrt{2\pi}\sigma_t} e^{-(\phi-\phi_0)^2/2\sigma_t^2} \Psi_0(E-E_0) \tag{1}$$

where E is the energy in MeV, E_0 is the average energy of the beam and ϕ_0 the phase, that might be different from zero, which we define to be on crest of the RF.

In the PETS the beam looses energy by working against the already present longitudinal electric field, which thereby increases in amplitude. The energy E_1 after the interaction for each particle with energy E is therefore given by

$$E_1 = E - A\cos\phi \tag{2}$$

where A is the amplitude of the field. The distribution of energy after the PETS can be calculated by summing up all particles with energy E that end up with energy E_1, provided Eq. 2 is true, which is

$$\begin{aligned}\Phi(E_1) &= \int\int \frac{1}{\sqrt{2\pi}\sigma_t} e^{-(\phi-\phi_0)^2/2\sigma_t^2} \Psi_0(E-E_0) \\ &\quad \times\delta(E_1 - E + A\cos\phi)\,dE\,d\phi \end{aligned} \tag{3}$$

and upon utilizing the delta-function to evaluate the integral over the energy E we find

$$\begin{aligned}\Phi(E_1) &= \frac{1}{\sqrt{2\pi}\sigma_t} \int e^{-(\phi-\phi_0)^2/2\sigma_t^2} \\ &\quad \times\Psi_0(E_1 - E_0 + A\cos\phi)\,d\phi \end{aligned} \tag{4}$$

which needs to be evaluated numerically for a general initial distribution $\Psi_0(E-E_0)$.

ZERO ENERGY SPREAD

In the limit of vanishing initial momentum spread, or $\Psi_0(E-E_0) = \delta(E-E_0)$ the integral over the phase ϕ is reduced to

$$\Phi(E_1) = \frac{1}{\sqrt{2\pi}\sigma_t} \int e^{-(\phi-\phi_0)^2/2\sigma_t^2} \delta(E_1 - E_0 + A\cos\phi)\,d\phi \tag{5}$$

whence the remaining delta function can be rewritten in terms of delta functions, where the phase ϕ appears linearly. For this we use the relation

$$\delta(g(x)) = \sum_{zeros} \frac{\delta(x-x_0)}{|g'(x_0)|} \tag{6}$$

where x_0 is given by $g(x)$ or $g(x_0) = 0$ and $g'(x) = dg(x)/dx$. In our case we have $g(\phi) = E_1 - E_0 + A\cos\phi$ and $\phi = \pm\arccos((E_0 - E_1)/A)$. For the derivative we obtain

$$\begin{aligned}\left|\frac{d(E_1 - E_0 + A\cos\phi)}{d\phi}\right| &= |A\sin\phi| \\ &= \left|A\sqrt{1-(E_0-E_1)^2/A^2}\right| \end{aligned} \tag{7}$$

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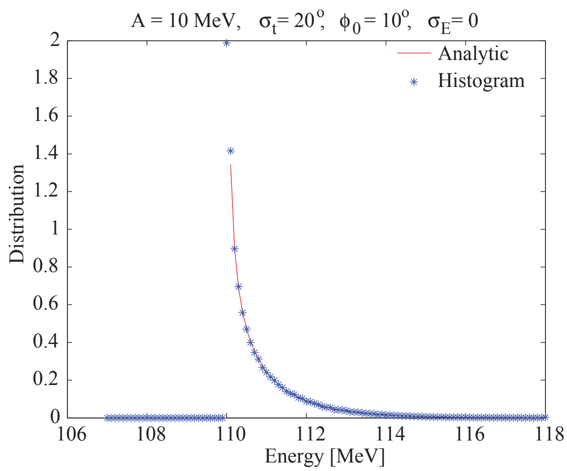

Figure 1: Energy distribution after the PETS for a beam with initial energy $E_0 = 120$ MeV and a maximum loss of $A = 10$ MeV. The blue dots are generated in a simulation with 100000 particles and the red line is the result from Eq. 8.

where we use $\sin(\arccos(x)) = \sqrt{1-x^2}$. Inserting into Eq. 5 we finally get

$$\Phi(E_1) = \frac{1}{\sqrt{2\pi}\sigma_t}\frac{1}{\sqrt{A^2-(E_1-E_0)^2}} \times\left\{\exp\left[-\frac{(\arccos((E_0-E_1)/A)-\phi_0)^2}{2\sigma_t^2}\right] + \exp\left[-\frac{(\arccos((E_0-E_1)/A)+\phi_0)^2}{2\sigma_t^2}\right]\right\} \tag{8}$$

Note that we used only two zeros of Eq. 2 close to the maximum of the beam distribution near the phase ϕ_0, because the assumed Gaussian bunch distribution falls off rapidly. Basically, we assume that the entire bunch is localized near the crest of the RF and we can safely neglect zeros of Eq. 2 that are 2π or further away.

In Eq. 8 we have a closed form expression for the energy distribution after the PETS, where the maximum energy loss is given by A. In Fig. 1 we display the final energy distribution for a beam with initial energy $E_0 = 120$ MeV, energy loss of $A = 10$ MeV, an initial phase of $\phi_0 = 10°$ off-crest and an rms bunch length of 20 degree. The analytic result from Eq. 8 is shown as a red line. A numerical simulation with 10^5 particles distributed to achieve an initial distribution that has energy E_0 and Gaussian distributed phase ϕ, each subjected to the energy loss in Eq. 2 and binned in a histogram, we show as blue asterisks.

FINITE ENERGY SPREAD

If the initial energy distribution can be approximated by a Gaussian

$$\Psi_0(E-E_0) = \frac{1}{\sqrt{2\pi}\sigma_E}e^{-(E-E_0)^2/2\sigma_E^2} \tag{9}$$

with rms width σ_E, the energy distribution after the PETS can be written as

$$\Phi(E_1) = \int\int\frac{1}{2\pi\sigma_t\sigma_E}e^{-(\phi-\phi_0)^2/2\sigma_t^2}e^{-(E-E_0)^2/2\sigma_E^2} \times\delta(E_1-E+A\cos\phi)dEd\phi \tag{10}$$

which can be simplified by evaluating the integral over the energy E and exploiting the delta function

$$\Phi(E_1) = \frac{1}{2\pi\sigma_t\sigma_E}\int e^{-(\phi-\phi_0)^2/2\sigma_t^2} \times e^{-(E_1-E_0+A\cos\phi)^2/2\sigma_E^2}d\phi \tag{11}$$

which needs to be evaluated numerically. This integrand, however, is rather well-behaved to integrate numerically and in Fig. 2 we show the distribution for the same parameters as before, but initial momentum spread $\sigma_E = 0.5$ MeV. The red line is from the evaluation of the integral in Eq. 11 and the blue asterisks come from a numerical simulation with initial Gaussian momentum and phase distributions. We observe that the peak is reduced and the distribution is wider, reflecting the initial energy spread.

EMITTANCE

In the previous sections we calculated the energy distribution after the deceleration process in the PETS, but what we actually observe is the position of particles on a screen in a dispersive section. To simplify the analysis we assume that the momentum spread is Gaussian and that the displacement of a particle with energy offset is linear in the energy. This approximation is justified in case the energy spread is small. We therefore start from Eq. 11 and transform the en-

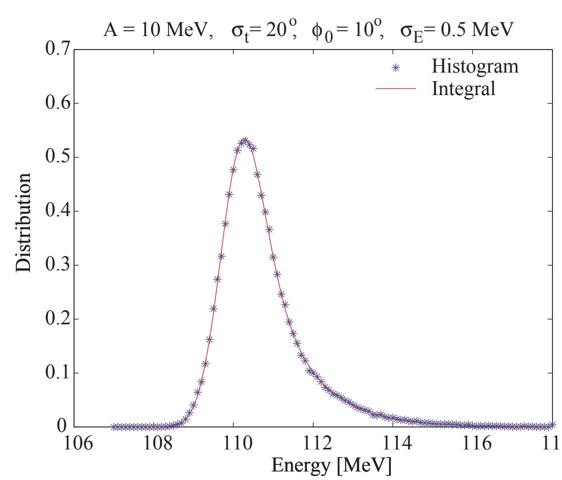

Figure 2: Energy distribution after the PETS for a beam with initial energy $E_0 = 120$ MeV and maximum loss of $A = 10$ MeV. Here the initial rms energy spread was taken to be 0.5 MeV. The blue dots are generated in a simulation the red line is the result of Eq. 11.

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ergy variable E_1 to position on the screen x using

$$x = D\frac{E_1 - E_c}{E_c} \quad \text{or} \quad E_1 = E_c\left(1 + \frac{x}{D}\right) \tag{12}$$

where we introduced the dispersion D and the energy of the center of the beam E_c after deceleration, which is a somewhat arbitrary normalization constant. The distribution function $\Xi(x)$ can then be calculated from

$$\Xi(x) = \int \Phi(E_1)\delta\left(x - D(E_1 - E_c)/E_c\right) dE_1 \tag{13}$$

where $\Phi(E_1)$ is given in Eq. 11. Evaluating the integral using standard methods yields

$$\Xi(x) = \frac{1}{2\pi\sigma_t D\sigma_\delta}\int d\phi e^{-(\phi-\phi_0)^2/2\sigma_t^2} \times \exp\left[-\frac{(x - D(E_0 - E_c)/E_c - D(A/E_c)\cos\phi)^2}{2D^2\sigma_\delta^2}\right] \tag{14}$$

with $\sigma_\delta = \sigma_E/E_c$. If we now assume that the beam has a finite emittance that results in a beam size σ_x on the screen, we need to convolute the previous expression with a Gaussian window function $W(x - X)$ of width σ_x

$$W(x - X) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-(x-X)^2/2\sigma_x^2} . \tag{15}$$

We therefore have to evaluate the integral

$$\Upsilon(X) = \int W(x - X)\Xi(x)dx \tag{16}$$

and observe that Ξ is Gaussian in the variable x and the convolution integral is just the convolution of two Gaussians, one with width σ_x and the other with width $D\sigma_\delta$. It is well-known that the convolution of two normalized Gaussians yields another Gaussian with width equal to the sum of squares of the original widths

$$\Upsilon(X) = \frac{1}{2\pi\sigma_t\sqrt{\sigma_x^2 + D^2\sigma_\delta^2}}\int e^{-(\phi-\phi_0)^2/2\sigma_t^2} \times \exp\left[-\frac{\left(X - D\left[\frac{E_0-E_c}{E_c} - \frac{A}{E_c}\cos\phi\right]\right)^2}{2\left(\sigma_x^2 + D^2\sigma_\delta^2\right)}\right] d\phi \tag{17}$$

and we find that it has the same structure as Eq. 11, just some of the constants are a little different. Again, the integral over the phase ϕ needs to be evaluated numerically.

SURPRISING MEASUREMENTS

After calculating spectra, as described in earlier sections of this report, we intended to compare the calculations with measurements from the two-beam test stand. Consequently, during a recent beam time we recorded spectra of the drive beam after deceleration under conditions where the PETS are turned ON or OFF [3] which is accomplished by two phase shifters that shifts the phase of the RF inside the PETS with respect to the beam. We had expected distributions similar to those calculated, but found the spectra shown in Fig. 3 where the decelerated beam (PETS ON, blue) is located to the left and is much narrower than the beam that is not decelerated (PETS OFF, green). This behavior can be understood by realizing that manipulating the phase using the PETS-OFF phase shifter generates a fluctuating interference pattern inside the PETS that causes different parts of the bunch train to experience different amplitudes and different phases and therefore experience an apparently random variation of their energy. On the spectrometer screen the jumping bunches can not be individually resolved and appear as a widened energy spread in PETS OFF conditions.

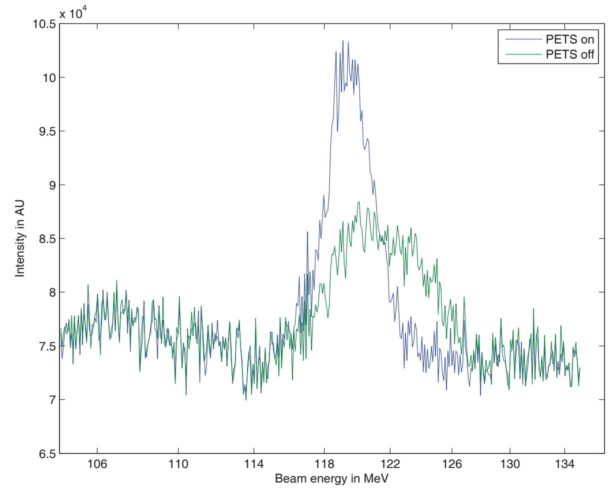

Figure 3: The energy spread of the beam on the spectrometer screen with PETS ON (blue) and OFF (green). Note that the beam energy is higher but the profile is wider in the case of PETS OFF.

We conclude that the way in which the PETS is turned off affects the drive beam by increasing its projected energy spread. We will investigate, both theoretically and in experiments, the importance of this effect on stable and reliable operation of the CLIC decelerator with a large number of PETS in series in the future.

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