

Stochastic processes with applications in supply chain management of electronic industry

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Abstract — A stochastic process which is a sum of a Polya-Aeppli process and a homogeneous Poisson process, called a Non-central Polya-Aeppli process and applied as a counting process in the electronic industry is considered. Some nice properties of the process and some useful applications in electronic process management are given.

Keywords — electronic process management, Non-central Polya-Aeppli process, Polya-Aeppli process, pure birth process, stochastic processes

I. INTRODUCTION

In this paper a Non-central Polya-Aeppli process, introduced by Lazarova and Minkova (2019), [1] is considered and applied in supply chain management of the electronic industry. It is a sum of Polya-Aeppli process, introduced by Minkova (2004), [2] and characterized by Chukova and Minkova (2013), [3] and a homogeneous Poisson process. The first process included in the construction of the Non-central Polya-Aeppli process is the Polya-Aeppli process (PAP) which is a compound Poisson process with geometric compounding distribution and rate $\lambda_1 > 0$.

The notation of this stochastic process is given by $N_1(t) \in PA(\lambda_1 t, \rho)$ where $\rho \in (0,1)$ is an additional parameter is known as correlation coefficient, see [4].

An interesting fact for the Polya-Aeppli process is that it is over-dispersed process related to the standard Poisson process. This property gives a good flexibility of this process especially when it is used for a construction of data with big dispersion. A good indicator for this is the Fisher index of dispersion which is introduced by Fisher in 1934 [5]. It expresses the ratio of the random variable's variance to its mathematical expectation, see [6] and is given by the following formula:

$$(1) \quad \text{FI}(N) = \frac{\text{Var}(N)}{\text{E}(N)}.$$

The Fisher index of dispersion for the Poisson distribution is given by $\text{FI}(N) = 1$. When the Fisher index is $\text{FI}(N) > 1$ then it is an indication that the given distribution is over-dispersed related to the Poisson distribution. When the Fisher index is $\text{FI}(N) < 1$ then the

distribution is under-dispersed related to the Poisson distribution. The motivation is to construct some flexible distributions which are corresponding distributions of stochastic processes which could be applicable in models with count data.

The second process is the homogeneous Poisson process (HPP) with rate $\lambda_2 > 0$ and the notation is given by $N_2(t) \in P_oP(\lambda_2 t)$. The homogeneous Poisson process has exponentially independent identically distributed inter-arrival times and it has an associated counting process which represents the numbers of the events up to time t , see Arnold et al (2020), [7]. The stochastic processes $N_1(t)$ and $N_2(t)$ are independent and the Non-central Polya-Aeppli process is constructed as a sum of these two processes i.e. $N(t) = N_1(t) + N_2(t)$ where the stochastic process's rate is $\lambda_1 + \lambda_2$, see [1]. Often in the literature the expression process's rate is found as an intensity function of the process, see [7].

The paper is organized to show the importance of the chain management in the business of the electronic industry. A new application of using a stochastic counting process in the electronic process and management is given. For the construction of the theoretical profit model in the terms of the stochastic processes in section IV, a Non-central Polya-Aeppli counting process with probability mass function given in section II is used. Section III is devoted to a description of the company's electronic management and the importance of using the supply chain management in business. The significant part of this paper is given in section IV where the application of the electronic process management is shown.

II. THE PROBABILITY GENERATING FUNCTION AND THE PROBABILITY MASS FUNCTION OF THE NON-CENTRAL POLYA-AEPLI PROCESS

The probability generating function and the probability mass function of the Non-central Polya-Aeppli process $N(t)$, see [5] are given by:

$$(2) \quad \psi_{N(t)}(s) = e^{-\lambda_1 t [1 - \psi_1(s)]} \cdot e^{-\lambda_2 t (1-s)},$$

where $\psi_1(s) = \frac{(1-\rho)s}{1-\rho s}$ is the probability generating function of the shifted geometric distribution with success probability $1-\rho < 1$, denoted by $Ge_1(1-\rho)$.

(3) $P(N(t) = i) =$

$$\begin{cases} e^{-(\lambda_1+\lambda_2)t}, & i=0 \\ e^{-(\lambda_1+\lambda_2)t} \left[\frac{(\lambda_2 t)^i}{i!} + \sum_{j=1}^i \frac{(\lambda_2 t)^{i-j}}{(i-j)!} \sum_{k=1}^j \binom{j-1}{k-1} \frac{[\lambda_1(1-\rho)t]^k}{k!} \rho^{j-k} \right] & i=1,2,\dots \end{cases}$$

So this is a counting process which starts at zero and for each $t > 0$ the corresponding distribution of the process is the Non-central Polya-Aeppli distribution, see [8]. Some useful and nice properties of this distribution are presented in [8] where the authors construct some graphics based on the idea of changing the different values of the parameters λ_1 , λ_2 and ρ .

It is shown that the distribution’s sensitivity depends on the two parameters λ_1 and λ_2 in a way that when ρ is fixed and $\lambda_1 < \lambda_2$ then more events happen in the beginning and in this case the distribution forms a right tail. The Non-central Polya-Aeppli distribution is a right tailed distribution and it could be a suitable distribution for data in the industry applicable for such kind of distribution.

A good characteristic for the Non-central Polya-Aeppli is that this stochastic process is a **pure birth** process, [1]

III. ELECTRONIC MANAGEMENT AND INDUSTRY

The description of the company’s management from some certain information, product materials supplying and finally to the company’s production during the supply chain form the component producers and suppliers to the final step - the client’s delivery is called a supply chain management, see [8]. For shortly in the literature the notation SCM is used. As there is a strong connection among all the chain’s members this fact brings the importance of this term in the business because one member of the chain is capable to influence the profit and the wealth of the other business partners. The increasing of the electronic request flow and thus the business growth is dependent on the fact that the company should ensure the electronic requests to its customers on time. Also important for the company policy is to get a feedback on its products and customer service, also to have the ability to determine the future customer’s needs and etc. All these steps are needed and significant for the increasing of the company’s profit. Of course there will be losses too.

In 2004 Van der Vorst, [9] gives a definition of the supply chain as a sequence of decision making processes and information and material flows that aim to meet the final customer requirements and needs. The supply chain

includes the manufacturers, the suppliers, the transporters, the retailers and finally the customers. The retailers and the manufacturers work together to fulfil the customers’ needs. The manufacturers and the supplies work together to supply the final products in such way that there would be a minimal stocks left. Also an important element of the chain is the logistic planning and service and the transport coordination too, [9].

Many authors search and want to create a good chain supply management in the business field. For example in the paper [10] the authors explore the SCM activities performed by electronic manufacturing organizations in Malaysia. They have been conducted on different industrial sectors as grocery, computers, telecommunications etc. And the question that have to be answered is how extensively is the supply chain management adopted in Malaysia? To answer this question 25 SCM activities were presented and pointed out the adoption level of their business. The result is given after one sample T-test for identifying the organizational adoption in practice of the SMC activities is done. The result showed that if the firms would like to improve the revenue growth then it have to deliver its own products on time and directly to the customers, to get a good feedback, to determine the future customers’ needs and also to use a supply chain management specialists. The conclusion that the authors made [10] is that the spread of the idea for using the supply chain management in Malaysian manufacturing industry is not very popular but those firms that decided to took the chance of using the SCM practices had a good benefit in their revenue growth.

The SCM software is an useful tool for delivering goods on time and the benefit of this process is to determine the customers’ added value at minimal cost. Nowadays the companies need to invest in the information and communication technologies (ICT) and the implementation of SCM software is essential because it facilitates the operation of the entire supply chain. The use of such applications provides an accurate information about the processes in the company from the very beginning to the very end of the entire process. Of course each SCM process requires different software but often the IT developers combine and adapt these elements to the needs of each individual client.

The main components of the SCM are as follows:



Fig. 1: Main components of SCM.

The applications that are integrated into the SCM can be divided into three essential groups - transaction management, communication management and relationship management [11-16].

The transactional management includes:

- Enterprise Resource Planning (ERP) which is a software that manages the company's finance resources, the supply chain and operations, commerce, reporting, manufacturing and human resource activities.

- Warehouse Management Systems (WMS) which is a software that allows the organizations to control and administer the warehousing operations from the moment that the goods or materials enter the warehouse until they are exported.

- Transport Management Systems (TMS) which is a software for planning, executing and optimizing the shipment of the goods.

The communication management includes:

- Supply Chain Event Management (SCEM) which is a software that is used to manage events that occur between organizations or partners in the supply chain.

- Radio Frequency Identification (RFID) which is a software that enables close cooperation of the supply chain partners through real-time information visibility.

- Collaborative Planning, Forecasting and Replenishment (CPFR) is an approach that aims to enhance the SCM integration by supporting and assisting joint practices.

The relationship management includes:

- Customer Relationship Management (CRM) which is a software that allows companies to develop their customer's experience using complete real-time information.

- Advanced Planning and Scheduling (APS) which is a software that offers various functions of SCM at the level of strategic, tactical and operational planning.

Nowadays the development of the business and the ICT lead to the popularity of ERP systems consisting of many software components and each of them is the main functional area of the organization [11], [15], [16].

The most common components of ERP are as follows:



Fig.2: ERP components model.

As it is seen in Fig. 2 the main components of the ERP system, see [15-18] are the following:

- Supply Chain Management (SCM);

- Customer Relationship Management (CSM) – this is a strategy for managing interactions with the customers. This component helps the organizations to build the customer relationships i.e. it improves the increasing sales and profitability and also it can improve the customer service;

- Manufacturing Resource Planning (MRP) - this component provides the information about the manufacturing process and methods from which the organization can choose the best for them;

- Financial Management Component (FRM) – this component is the core of the ERP system and here some financial data from different functional departments and generates valuable financial reports are collected;

- HRM (Human Resource Management) - manages human resources and capital and maintains a database with information about employees;

Two years after the beginning of the pandemic Covid-19 and the related restrictions for the business, the cloud ERP systems have already been established as a standard system on the Bulgarian market. The cloud ERP systems are preferred by many managers in our country because of their flexibility which allows the opportunity many companies easily to meet the market changes while maintaining their efficiency.

According to CBN Pannoff, Stoytcheff & Co. prognosis, (<http://web.cbn-bulgaria.com/>) at about 71% of all new ERP projects in Bulgaria for 2021 are cloud solutions. Nowadays the teams are much more mobile and so the business is in a great process of digitalization. This leads to the need of using cloud ERP systems which are much better prepared to answer these challenges. These systems are more preferred in times of crisis which is very important fact.

According to CBN data by the middle of 2021 a total of 27 ERP vendors offer cloud ERP solutions through their partner network or directly in Bulgaria. The number of integrators who have announced that they are implementing such systems for Bulgaria is 56. The data show that in Bulgaria 11 out of 19 industries have installed and working cloud ERP systems. Among them there are some leading industries such as the ICT industry, manufacturing, financial and insurance activities, transport and storage. Thanks to these ERP systems it is much easier to increase or decrease the number of the users at the end of the chain, the functionalities and the parameters according to the needs of each company. Adding some new functionalities and modules is much easier and it is preferred because the customer pays only for what he uses and always works with the latest version of the software.

During the implementation of the ERP cloud system, the company team does not need to take care of the hardware,

the system software and the data storage. Also there is no need to think for the protector hiring and the training of a large number IT professionals.

In the following Table 1 we can see a list of some popular international SCM and ERP software, some of which have been developed by IT specialists in Bulgaria. The ranking of the best suppliers of SCM software for 2022 in the world see [18-21].

Table 1: A list of popular cloud SCM and ERP software, part of which was developed by the Bulgarian IT industry.

Product	Features	Website
Acumatica	It is an intelligent industry which is focused on cloud-based ERP platform for oversees, finances and manufacturing distribution. Also it can perform other roles in the companies.	https://www.acumatica.com
AVAMB LOGICIEL ERP (BG)	It is a cloud system that allows you to track all business processes and it is suitable in the field of online sales, marketing, transport companies and more.	https://avamb-logiciel.com/
bgERP (BG)	It is a free web-based on business management software that can track, manage and automate all important processes in a company.	https://bgerp.com/
DELTEK ERP	It offers a suite of products designed for key work environments and provides a centralized multi-functional management system.	https://www.deltek.com/
Dynamics 365 Business	It is the next generation of business solutions that provides business - ready mobile apps for finance, SCM, Retail and other ERP functionalities.	https://www.isystems.bg/
ECount ERP	It is a web-based ERP system which provides many functions to help the optimization and improvement of the business workflow.	https://www.ecount.com/
E2Open	It is a leading cloud-based, mission-critical, end-to-end supply chain management software.	https://www.e2open.com/
Odoo	It is an open-source solution that features applications, such as inventory management, financial tracking and project organization	https://www.odoo.com/
Oracle NetSuite	It is a cloud based on ERP system and it provides a full set of business solutions in one place: ERP, CRM, BI (Business Intelligence), PSA (Professional Services Automation), SRP (Services Resource Planning), HRMS (Human Resource Management Software) HR, e-commerce, etc.	https://www.balkan-services.com/
Prim (BG)	It is an integrated ERP system with CRM and BI module for sales, purchases, logistics management and financial operations.	https://prim.bg/
Priority Software	Priority is a module-based system offering financial supply chain, customer service and etc.	https://www.priority-software.com/
SAP S4/HANA	The Bulgarian branch of the global software leader SAP SE offers an entire portfolio in the following areas-	https://www.sap.com/

	applications, analysis, mobility, database and technology and the cloud,	
TECHNO CLASS (BG)	It is a cloud-based integrated system designed for process management for companies with different organizational and territorial structure, subject of activity and volume of operations.	https://techno-class.com/
Tonagen ERP (BG)	Module based-integrated software management system with logical division of business processes	https://tonagen.bg
Zeron V/4 (BG)	Zeron V/4 is a software system for business management - ERP, CRM and BI.	https://zeron.bg/

IV. AN APPLICATION IN ELECTRONIC PROCESS MANAGEMENT

In this theoretical model is considered that a business company produces two types of electronic components on the market. The first type of electronic components is a board which contains motion sensors and the second one is a board which contains heat sensors. The flow of the customers' requests comes electronically to the company per unit time. The company processes the requests for a period of three days and after that the ordered components are sent back to the customers. Also the company has a market policy which includes a clause of a company penalty to the client in case of an unprocessed and returned on time request.

The state of a business company at a time t where u is an initial capital can be described by the following stochastic process:

$$P(t) = u + \pi(t) - S_{N(t)}, \quad t \geq 0,$$

where $\pi(t)$ is a non-negative function with finite variation.

The sum $u + \pi(t)$ describes the company's revenue growth and the accumulated sum $S_{N(t)}$ determines the company's expenditures. Then the process $P(t)$ defined on the complete probability space $(\Omega, \mathfrak{F}, P)$ represents the profit of the company's business in the time interval $[0, t]$.

The accumulated sum up to time t of the company's penalties for unprocessed and returned on time requests is given by:

$$S_{N(t)} = \sum_{i=1}^{N(t)} Z_i, \quad t \geq 0$$

The process $\{S_{N(t)}, t \geq 0\}$ is a random sum of random variables with the condition that $S(t) = 0$ when the counting process $N(t) = 0$.

The counting process $N(t) = \sup\{n : \sigma_n \leq t\}$, $t \geq 0$ where $0 \leq \sigma_1 \leq \sigma_2 \leq \dots$ are the times of requests arrivals is the Non-central Polya-Aeppli process, [1].

The interpretation of the counting process is that there are two types of unprocessed and return on time requests to the customers. The first type is counted by the Polya-Aeppli process and the second type is counted by the homogeneous Poisson process. The intensity functions of the two stochastic processes give the processing speed of the two types of the requests that the company offers to the market. They can be measured by the orders flows that the business company couldn't send on time to its customers.

The relations between the times of the requests arrivals and the counting process $\{N(t), t \geq 0\}$ are given by:

$$\{N(t) = n\} = \{\sigma_n \leq t < \sigma_{n+1}\}, \quad n = 0, 1, \dots$$

The sequence $\{Z_n, n = 1, 2, \dots\}$ of independent identically distributed random variables with common distribution function F , $F(0) = 0$ and expectation μ is independent of the counting process $N(t)$.

V CONCLUSION

This paper gives a new approach of using a counting stochastic process in the electronic management and industry. The main aim is to determine the firm's profit and the essential key for constructing a good model with a good application is to use the chain supply management in the business field. In the theoretical part is shown that the company's losses are represented by a sum of the accumulated and not sent on time to the clients requests. The cloud ERP systems have already been discussed in this paper because they are established as a standard system on the Bulgarian market. They are preferred in our country because their flexibility allows the opportunity that the companies could easily meet the market changes.

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