

# An Asymmetric Cryptographic Key Assignment Scheme for Access Control in Tree Structural Hierarchies

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## Abstract

In a hierarchical structure, a user in a security class has access to information items of another class if and only if the former class is a predecessor of latter. Based upon cryptographic techniques, several schemes have been proposed for solving the problem of access control in hierarchical structures. In this paper, we propose a new scheme for an access control in tree structural hierarchies based on asymmetric cryptographic key assignment scheme. Further, our encryption and decryption procedures are based on asymmetric cryptographic technique. We show that proposed scheme requires less amount of storage space to store public parameters and also retains the same security level compared to the previous published schemes. Furthermore, our scheme achieves better generality compared to the Hwang's scheme.

*Keywords:* Access control, authentication, cryptography, data security

## 1 Introduction

In real life, hierarchical structures are used in many applications organizations like the military, government organizations, school systems, college systems, private corporations, computer network systems [16, 17, 19, 20], operating systems [10] and database management systems [5, 7, 8, 9], etc.

We consider an organizational structure in which the users and their own information items (e.g., a message, data, etc.) are divided into a number of disjoint set of security classes, say  $C_1, C_2, \dots, C_n$ . We can define a binary relation  $\leq$ , which partially orders the set  $C = \{C_1, C_2, \dots, C_n\}$ . In the partially ordered,  $(C, \leq)$ ,  $C_i \leq C_j$  means that  $C_i$  has security clearance lower than or equal to  $C_j$ . In other words, the users in  $C_j$  can access the encrypted information items held by the users in  $C_i$ . However, the converse is not permitted. Figure 1 shows

an example of four-level hierarchical structure. Top level class possesses the greatest authority, and authority decreases with the increase in level. Thus, users in bottom level classes have the least authority. For the partially ordered set structure,  $C_i \leq C_j$ ,  $C_i$  is called a successor of  $C_j$ , where as  $C_j$  is called a predecessor of  $C_i$ . If there does not exist  $C_k$  such that  $C_i \leq C_k \leq C_j$ ,  $C_i$  is called an immediate successor of  $C_j$ , and  $C_j$  is called an immediate predecessor of  $C_i$ . If there does not exist  $C_i$  such that  $C_i \leq C_j$ ,  $C_j$  is called a leaf security class. Without loss of generality, we identify the classes in a hierarchical system as follows. Let  $G$  be a set consists of integers  $1, 2, \dots, g$ , i.e.,  $G = \{1, 2, \dots, g\}$ , where  $g$  is the degree of a tree structure. Let  $C_i$  be a class. Then, the immediate successors of  $C_i$  are represented by  $C_{i_j}$ , where  $i_j = i \cdot g + j - 2$  for some  $j \in G$  and  $i$  is the identity of the class  $C_i$ . Let us consider an example as follows. Assume that the user in the security class  $C_{15}$  in Figure 1 encrypts a message (information items) with her own encryption key  $e_{15}$ . Because of access control in a hierarchical structure, only the users in the security class  $C_{15}$  and her predecessors classes (i.e.,  $C_5, C_2, C_1$ ) can decrypt this encrypted message, whereas nobody else can decrypt this encrypted message.

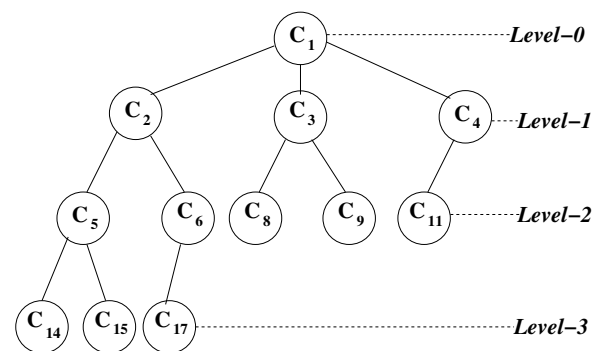


Figure 1: An example of a tree hierarchical structure

A straightforward access control scheme for poset hierarchy is to assign each security class with a key, and each class has the keys of all its successors. The information items belonging to a class is encrypted with the key assigned to that class. As a result, if a class encrypts the information items, its predecessors can only decrypt the encrypted information items. The drawback of such scheme is to store the keys in higher hierarchical classes. Several methods have been proposed in order to solve such type of problems based on the concept of the master key [4]. In 1983, Akl and Taylor [2] proposed a scheme based on symmetric key cryptosystem. Each security class  $C_i$  is assigned with a public parameter,  $PB_i$  and a secret key  $K_i = K_0^{PB_i} \bmod N$ , where  $N$  is widely separated secret pair of primes and  $K_0$  is kept secret by the central authority (CA, for short). If  $C_i \leq C_j$ ,  $PB_i/PB_j$  is an integer,  $C_j$  can derive the secret key,  $K_i$  as  $K_i = K_0^{PB_i} = K_0^{PB_j \cdot (PB_i/PB_j)} = K_j^{(PB_i/PB_j)} \bmod N$  of the class  $C_i$ .

In 1985, Mackinnon et al. [18] proposed an improved algorithm for the Akl-Taylor scheme based on top-down approach of poset hierarchy for reducing the value of public parameters. In 1988, Sandhu [24] introduced a cryptographic implementation of a tree hierarchy for access control based on one-way function. In 1990, Harn and Lin [11] proposed a scheme which is similar to the scheme of Akl-Taylor, but, it is based on bottom-up approach for key generation. In 1992 and 1993, both Chang et al. [3] and Liaw et al. [13, 14] proposed the scheme based on Newton's interpolation method and one-way function. The key generation procedures of the above schemes in such a way that higher level security class can derive the secret key of lower level security class using her secret key and the public parameters. In 2000, Hwang [12] proposed an access control scheme for a totally ordered hierarchy based on asymmetric cryptosystem. Recently, many related schemes have been proposed [15, 26]. In 2003, Lin-Hwang-Chang [15] proposed a scheme for access control, where each security class contains a secret key  $SK_i$  and derivation key  $DK_i$  which are kept secret by the class  $C_i$ . If  $C_i \leq C_j$ , the class  $C_j$  can derive the secret key of the class  $C_i$  using the derivation key  $DK_j$  and public parameters. In this scheme requires only small amount of storage space to store public parameters compared to the Akl-Taylor's [2]. In this paper, we propose a new scheme for access control in tree structural hierarchy based on asymmetric cryptosystem which is the generalization of Hwang's [12] proposed scheme. Besides, our scheme requires less amount of storage space to store public parameters. Moreover, our encryption and decryption techniques are based on asymmetric cryptographic technique.

In a multilevel access control scheme based on asymmetric cryptosystem, each security class  $C_i$  has a distinct encryption key  $e_i$  and a distinct decryption key  $d_i$  for encryption and decryption respectively. A user can encrypt the information items (message) in  $C_i$  with  $e_i$ . The only user in the same security class  $C_i$  can decrypt using

the decryption key  $d_i$  and public parameters, whereas the users in the higher security classes can decrypt that encrypted information items using their own decryption key and public parameters. But, no one else can decrypt that encrypted information items.

The remainder of this paper is organized as follows. Section 2 gives a brief review of the Hwang's scheme. In Section 3, we describe our proposed scheme for access control in tree structural hierarchies. Section 4 shows the space and time complexity of our scheme. In Section 5, we discuss the security analysis. Section 6 shows the advantages of our scheme. In section 7, our scheme is compared with previous published schemes. Finally, Section 8 concludes the paper.

## 2 Review of the Hwang's Scheme

In this section, we now review briefly the Hwang's scheme [12].

In the key generation phase, for  $n$  security classes  $C_1, C_2, \dots, C_n$  in totally-ordered hierarchy, CA performs the following techniques to generate and distribute keys. At first CA chooses a large number  $N$  so that  $N$  is product of two large primes. Then CA chooses  $e_i$  so that  $e_i$  and  $\phi(N)$  are relatively prime and computes  $s_i = e_i^{-1} \bmod \phi(N)$ , where  $\phi(\cdot)$  represents the usual Euler's totient function. After that CA selects parameters  $\beta$ ,  $2 \leq \beta \leq \phi(\phi(N)) - 1$  and  $t$ ,  $2 \leq t \leq \phi(N) - 1$  such that  $\gcd(\beta, \phi(\phi(N))) = 1$ . CA computes  $\alpha = \beta^{-1} \bmod \phi(\phi(N))$ . Then, CA also computes  $p_i, d_i, w_i$ , where

$$\begin{cases} p_1 = \alpha^t \bmod \phi(\phi(N)), \\ d_1 = \beta^t \bmod \phi(\phi(N)), \\ w_1 = s_1^{p_1} \bmod \phi(N). \end{cases}$$

and

$$\begin{cases} p_i = p_{i-1}^2 \bmod \phi(\phi(N)), \\ d_i = d_{i-1}^2 \bmod \phi(\phi(N)), \\ w_i = s_i^{p_i} \bmod \phi(N), \end{cases}$$

for  $i = 2, 3, \dots, n$ . After that CA sends securely encryption key  $(e_i, N)$ , decryption key  $d_i$  to the security class  $C_i$ , where  $(e_i, N)$  is public and  $d_i$  is kept secret by  $C_i$ . CA keeps  $w_i$  as public.

### 2.1 Encryption Technique

The encryption technique of this scheme is as follows. Let  $M$  be the message to be encrypted. Encrypted message  $T$  of  $M (< N)$  for a user in a security class  $C_i$  is defined as

$$T = M^{e_i} \bmod N.$$

### 2.2 Decryption Technique

Let us assume that  $C_i \leq C_j$ . If a user in security class  $C_j$  wants to decrypt the message which is encrypted by a

user in security class  $C_i$ , the following is the technique of this scheme:

$$M = T^{w_i^{d_i^{2^{(L_i-L_j)}}}} \pmod{N},$$

where  $L_i$  and  $L_j$  are the level of security classes  $C_i$  and  $C_j$  respectively, and  $M$  is the decrypted message.

### 3 Our Scheme

In this section, we present a new key assignment scheme for access control in a tree structural hierarchy based on asymmetric cryptosystem. We assume that there is a trusted CA in the system. The main purpose of CA is to generate keys and distribute them securely to the classes. We use the following notations for describing key generation procedure.

- $C_i$ : A class with identity  $i$ .
- $C_{i_j}$ : A successor class of a class  $C_i$ , where  $i_j = i \cdot g + j - 2, j \in \{1, 2, \dots, g\}$  and  $g$  is the degree of a tree.  $i_j$  is the identity of the class  $C_{i_j}$ .
- $E_i, D_i, p_i, w_i, s_i$ : These parameters are assigned for the class  $C_i$ , which are kept secret by CA.
- $e_i, d_i$ : Encryption key and decryption key for the class  $C_i$  respectively.
- $E_{i_j}, D_{i_j}, p_{i_j}, w_{i_j}, s_{i_j}$ : These parameters are assigned for the class  $C_{i_j}$ , which is a successor class of  $C_i$ . These parameters are kept secret by CA.
- $e_{i_j}, d_{i_j}$ : Encryption key and decryption key of the class  $C_{i_j}$ .
- $SI_{ij}$ :  $SI_{ij}$  (secret information) is computed and kept secret by CA in between the classes  $C_i$  and  $C_j$  with  $C_i \leq C_j$ .
- $SRI_{ij}$ :  $SRI_{ij}$  (secret relational information) is computed by CA in between the classes  $C_i$  and  $C_j$  with  $C_i \leq C_j$  and is kept secret by  $C_j$ .

#### 3.1 Key Generation Procedure

In this subsection, we discuss the procedure to generate keys for all classes.

**Step 1:** CA chooses a large number  $N$ , so that  $N$  is a product of two large primes which are widely separated and  $\phi(\phi(N))$  has at least two large prime factors.  $\phi(\cdot)$  is Euler's totient function. CA keeps  $N$  as public parameter.

**Step 2:** CA chooses a prime  $h$  ( $h > 2$ ) and another number  $h_i$  distinct from  $h$  so that  $2 \leq h \cdot h_i \leq \phi(N) - 1$  and  $\gcd(hh_i, \phi(N)) = 1$ . Then CA computes the distinct encryption key  $e_i = hh_i$  and secret key  $s_i$  such

that  $e_i s_i = 1 \pmod{\phi(N)}$  for the security class  $C_i$ . Although  $e_i$  and  $s_i$  use the same common modular, it is not possible to derive  $s_i$  by common modular attack [21] because of the fact that  $s_i$  are kept secret only by CA.

**Step 3:** CA chooses a prime  $E_i$  so that  $2 \leq E_i \leq \phi(\phi(N)) - 1$  and  $\gcd(E_i, \phi(\phi(N))) = 1$ . Then CA calculates the multiplicative inverse,  $D_i$ , for each  $E_i$ , where  $D_i E_i = 1 \pmod{\phi(\phi(N))}$ .  $E_i$  and  $D_i$  are kept secret by CA.

**Step 4:** CA chooses a secret parameter  $t$ ,  $2 \leq t \leq \phi(\phi(\phi(N))) - 1$ .

**Step 5:** CA computes a secret parameters  $p_1, w_1$ , and decryption key  $d_1$  of the security class  $C_1$  as follows:

$$\begin{cases} p_1 = D_1^t \pmod{\phi(\phi(N))}, \\ d_1 = E_1^t \pmod{\phi(\phi(N))}, \\ w_1 = s_1^{p_1} \pmod{\phi(N)}. \end{cases}$$

Let us consider CA has computed the secret parameters  $p_i, w_i$  and decryption key  $d_i$  for the class  $C_i$ . Let  $C_{i_j}$  be an immediate successor of the class  $C_i$ . The secret parameters  $p_{i_j}, w_{i_j}$  and decryption key  $d_{i_j}$  of the class  $C_{i_j}$  as follows:

$$\begin{aligned} p_{i_j} &= (D_{i_j} p_{\lceil \frac{i_j-1}{g} \rceil})^2 \pmod{\phi(\phi(N))} \\ &= (D_{i_j} p_i)^2 \pmod{\phi(\phi(N))}, \\ d_{i_j} &= (E_{i_j} d_{\lceil \frac{i_j-1}{g} \rceil})^2 \pmod{\phi(\phi(N))} \\ &= (E_{i_j} d_i)^2 \pmod{\phi(\phi(N))}, \\ w_{i_j} &= s_{i_j}^{p_{i_j}} \pmod{\phi(N)}, \end{aligned}$$

where  $g$  is the degree of the tree structure.

**Step 6:** CA computes secret information  $SI_{ij}$  in between two same or different classes  $C_i$  and  $C_j$  with  $C_i \leq C_j$  using the following algorithm:

**SI\_Function( $i, j$ )**

**Step 6.1:** Set  $k := 1, SI := 1$ ;

**Step 6.2:** If ( $i == j$ ) then  
Goto **Step 6.5**;

**Step 6.3:** If ( $i > j$ )  
 $SI := SI \cdot E_i^{2^k} \pmod{\phi(\phi(N))}$ ;  
 $i = \lceil \frac{i-1}{g} \rceil$ ;  
 $k := k + 1$ ;

**Step 6.4:** Go to **Step 6.2**;

**Step 6.5:** Return  $SI$ ;

All  $SI_{ij}$  are kept secret by CA.

**Step 7:** Then CA computes secret relation information  $SRI_{ij}$  in between two same or different classes  $C_i$  and  $C_j$  with  $C_i \leq C_j$  as follows:

$$SRI_{ij} = w_i^{SI_{ij}} \pmod{\phi(N)}.$$

Then, CA transmits securely the decryption key  $d_i$  and encryption key  $(e_i, N)$ , and secret relational information  $SRI_{ki}$  for all  $k$  with  $C_k \leq C_i$  to the class  $C_i$  in the system. The class  $C_i$  keeps secret  $d_i$  and  $SRI_{ki}$  for all  $k$  with  $C_k \leq C_i$ . The encryption key  $(e_i, N)$  of each security classes  $C_i$  are published by CA.  $E_i, D_i, s_i, p_i, w_i$  and  $SI_{ki}$  for all  $k$  with  $C_k \leq C_i$  are kept secret by CA.

### 3.2 Encryption Technique

We define our encryption technique as follows. Let  $M$  be the information items or message to be encrypted. Encrypted message  $T$  of  $M$  for a user in a security class  $C_i$  is defined as

$$T = M^{e_i} \text{ mod } N.$$

### 3.3 Decryption Technique

Let us assume that  $C_i \leq C_j$ . If a user in security class  $C_j$  wants to decrypt this encrypted message which is encrypted by a user in security class  $C_i$ , we use the following technique.

$$M = T^{SRI_{ij}^{d_j^{2(L_i-L_j)}}} \text{ mod } N,$$

where  $L_i = \lceil \log_g((g-1) \cdot (\text{identity of the class } C_i + 1)) - 1 \rceil$  and  $L_i$  and  $L_j$  are the level of security classes  $C_i$  and  $C_j$  respectively,  $M$  is the decrypted message, and  $SRI_{ij}$  are secret relational information of the class  $C_j$ .

### 3.4 Correctness

In this subsection, we prove that plaintext  $M$  can be derived using our decryption technique. We have already shown that encrypted message by the security class  $C_{15}$  is decrypted by the security class  $C_2$  because of the fact that the security class  $C_2$  is in the higher level security class than  $C_{15}$ . The proof as follows:

Let  $T = M^{e_{15}} \text{ mod } N$ , where  $T$  is the ciphertext encrypted by a user in the security class  $C_{15}$ . Then

$$M = T^{SRI_{ij}^{d_j^{2(L_i-L_j)}}} \text{ mod } N,$$

where

$$\begin{aligned} L_{15} &= \lceil \log_3(2 \cdot 15 + 1) - 1 \rceil \\ &= 3, \\ L_2 &= \lceil \log_3(2 \cdot 2 + 1) - 1 \rceil \\ &= 1, \\ d_2 &= E_2^2 \cdot E_1^{2t} \text{ mod } \phi(\phi(N)), \\ SI_{15,2} &= E_{15}^2 \cdot E_5^4 \text{ mod } \phi(\phi(N)). \end{aligned}$$

Now,

$$\begin{aligned} &SRI_{15,2}^{d_2^{2(L_{15}-L_2)}} \\ &= w_{15}^{(d_2^{2(L_{15}-L_2)})SI_{15,2}} \\ &= s_{15}^{p_{15} \cdot (d_2^{2(L_{15}-L_2)})SI_{15,2}} \\ &= s_{15}^{(D_{15}^2 \cdot D_5^4 \cdot D_2^8 \cdot D_1^{8t}) \cdot (E_2^2 \cdot E_1^{2t})^4 \cdot (E_{15}^2 \cdot E_5^4)} \\ &= s_{15} \text{ mod } \phi(N), \end{aligned}$$

where  $p_{15} = D_{15}^2 \cdot D_5^4 \cdot D_2^8 \cdot D_1^{8t} \text{ mod } \phi(\phi(N))$ .

Therefore,

$$\begin{aligned} &T^{SRI_{15,2}^{d_2^{2(L_{15}-L_2)}}} \text{ mod } N \\ &= M^{e_{15} \cdot s_{15}} \text{ mod } N \\ &= M. \end{aligned}$$

## 4 Storage Requirement and Computational Complexity

### Storage Requirement:

Let us consider  $k$  be the number of successors of the class  $C_i$ . Then from the key generation procedure, the class  $C_i$  has to store  $k + 1$  secret relational information, where each secret relational information lies between 1 and  $\phi(N)$  ( $< N$ ) and the decryption key  $d_i$  lies between 1 and  $\phi(\phi(N))$  ( $< N$ ). Therefore, the storage requirement for storing the secret information (parameters) is the sum of storing  $k + 1$  secret relational information and one decryption key. Thus, the required storage for storing the secret information is  $(k + 2)\lceil \log_2 N \rceil$  bits for the class  $C_i$ . Let us assume that there are  $u$  classes in the hierarchical systems. So, the total number of public parameters,  $\{e_i | i \in \{\text{identity of the classes in the hierarchy}\}\}$  and  $N$  is  $u + 1$ . Also, each  $e_i$  lies between 1 and  $\phi(N)$  ( $< N$ ). Therefore, the total amount of space required for storing the public parameters is  $(u + 1)\lceil \log_2 N \rceil$  bits.

### Time Complexity:

The time requirement for encryption and decryption techniques in our scheme using the repeated square and multiply algorithm are described as follows.

### Time Requirement for Encryption:

Since the encryption key lies between 1 and  $\phi(N)$  ( $\leq N$ ), the time requirement to encrypt a message is  $O(\log_2^3(N))$  in terms of bit operations.

### Time Requirement for Decryption:

Suppose  $C_i$  encrypts a message  $M$ , where encrypted message is  $T$  and  $C_j$  plans to decrypt this encrypted message. The total time required for decryption can be attributed to three basic stages.

- 1) Computation of  $d_j^{2^{(L_i-L_j)}}$ : The number of bit operations required is  $O(2^{L_i-L_j} \log_2^2(N))$ .
- 2) Computation of  $SRI_{ij}^{d_j^{2^{(L_i-L_j)}}$ : The number of bit operations required is  $O(N^{2^{L_i-L_j}} \log_2^2(N))$ .
- 3) Computation of  $T^{SRI_{ij}^{d_j^{2^{(L_i-L_j)}}} \bmod N$ : The number of bit operations required is  $O(N^{2^{L_i-L_j}} \log_2^3(N))$ .

Thus, in our scheme, computational time is  $O(N^{2^L} \log_2^3(N))$  in terms of bit operations, where  $L = L_i - L_j$ . So, computational time required for encryption and decryption of our scheme are same as the Hwang's scheme.

## 5 Security Analysis

In our proposed scheme, the decryption key  $d_i$  of a security class  $C_i$  is equal to the square of multiplication of the parameter  $E_i$ , which is kept secret by CA and its immediate predecessor's decryption key  $d_{\lceil \frac{i-1}{g} \rceil}$ . So, a user in a lower level security class  $C_i$  can derive its predecessor's decryption key unless that lower level class is able to compute the square root mod  $\phi(\phi(N))$  of her decryption key as well as to compute the  $D_i$  which is the inverse of  $E_i$ . Since  $N$  is product of two large primes. So, it is difficult to compute  $\phi(N)$  from  $N$ . Hence, it is also difficult to compute  $\phi(\phi(N))$  from  $N$ . Also, it is known that the problem to compute  $n$ -th root of  $x^n \bmod m$  for any integer  $n \geq 2$  is as difficult as factoring  $m$  [23], where  $m$  is product of two large primes and this has been proven in [22] for the case of  $n = 2$ . Again,  $\phi(\phi(N))$  has at least two large prime factors. As a result, it is hard to compute square root mod  $\phi(\phi(N))$ . Further,  $E_i$  and  $D_i$  are kept by CA. So, it is hard to compute  $D_i$  or  $D_i^2$  from secret relational information of  $C_i$ . Therefore, in our scheme, it is difficult to compute the decryption key of upper level class by a class of lower level class is as difficult as factoring the product of two large primes.

### Collaboration Attacks:

Collaboration attack is the case when two or more security classes at the lower level in the hierarchy wish to derive the decryption key of their predecessor class. Let  $C_i$  and  $C_j$  be the immediate successors of the class  $C_k$ . The decryption keys of  $C_i, C_j$ , and  $C_k$  are  $d_i (= (E_i d_k)^2 \bmod \phi(\phi(N)))$ ,  $d_j (= (E_j d_k)^2 \bmod \phi(\phi(N)))$  and  $d_k$  respectively. Let us assume that  $C_i$  and  $C_j$  compromise their  $d_i, d_j$  and their secret relational information  $SRI$ . Because of the factorization problem, it is hard to compute  $\phi(\phi(N))$  from  $N$ . Again, it is difficult to compute  $D_i^2$  (inverse of  $E_i^2$ ) or  $D_j^2$  (inverse of  $E_j^2$ ) from  $SRI$  as well as it is also difficult to compute the square root of  $d_k^2 \bmod \phi(\phi(N))$ . Thus, it is hard to compute  $d_k$  from  $d_i, d_j$  and  $SRI$ . Hence our scheme is secure against

such type of attacks.

### Common Subordinate Attacks:

This is the case when the subordinate class  $C_k$  is accessible by two or more predecessor classes  $C_i$  and  $C_j$ . Let us consider  $C_k \leq C_i \leq C_j$ , where  $C_k$  and  $C_j$  are the immediate successor and predecessor of the class  $C_i$  respectively. Let us assume that  $C_i$  and  $C_k$  compromise their decryption keys and their secret relational information  $SRI$ .  $d_i (= (E_i d_j)^2 \bmod \phi(\phi(N)))$  and  $d_k (= (E_k d_j)^2 = E_k^2 E_i^4 d_j^4 \bmod \phi(\phi(N)))$  are the decryption keys of the classes  $C_i$  and  $C_k$  respectively. But, in our proposed scheme, it is difficult to compute  $d_j$  using  $d_i, d_k$  and secret relational information  $SRI$  of the classes  $C_i$  and  $C_k$  because it is difficult to compute  $n$ -th root ( $n = 4$ ) of  $d_j^n \bmod \phi(\phi(N))$  for any integer  $n \geq 2$ . Further, it is difficult to compute  $D_i^2$  as well as  $D_k^2$  by a user in  $C_i$  is same as in collaboration attacks. Therefore, in our scheme, it is difficult to compute  $d_j$  using  $d_i, d_k$  and  $SRI$  by a user in  $C_i$ . As a result, proposed scheme is secure against such type of attacks.

### Common Modulus Attacks:

There are two types of common modulus attacks.

- 1) The first type of common modulus attacks uses the same message  $M$  and same modulus  $N$  for two different encryption keys (public keys)  $e_1$  and  $e_2$ . Then,  $T_1 = M^{e_1} \bmod N$  and  $T_2 = M^{e_2} \bmod N$ . If  $e_1$  and  $e_2$  are relatively prime, there exist integers  $x$  and  $y$  such that  $xe_1 + ye_2 = 1$ . In this case, message  $M$  can be retrieved by the following technique [21, 25]:

$$\begin{aligned} T_1^x (T_2^y) \bmod N &= (M^{e_1})^x (M^{e_2})^y \\ &= M^{xe_1 + ye_2} \\ &= M. \end{aligned}$$

But, in our proposed scheme, adversary can calculate  $\gcd(e_1, e_2) = h$ , where  $h > 2$ . As a result, it is difficult to compute  $h$ -th root of  $M^h \bmod N$  without factoring to  $N$ . So, our scheme is secure against this type of attacks.

- 2) The second type of common modulus attacks [6, 21] is that a user in a class can use her own encryption key (public key)  $e_2$  and decryption key  $d_2$  together to retrieve the decryption key  $d_1$  of another user of a class  $C_1$  using encryption key  $e_1$ . The user first finds the  $\gcd$  of  $e_1$  and  $e_2 d_2 - 1$  by the Euclidean algorithm. Let  $u = \gcd(e_1, e_2 d_2 - 1)$ . Then, the user finds  $v$  such that  $v = (e_2 d_2 - 1)/u$ . Since  $u$  divides  $e_1$  and  $\gcd(e_1, \phi(m)) = 1$ ,  $u$  must be relatively prime to  $\phi(m)$ . As  $e_2 d_2 - 1 = 0 \bmod \phi(m)$  and  $uv = e_2 d_2 - 1$ ,  $uv$  is a multiple of  $\phi(m)$ . As a result,  $v$  must be a multiple of  $\phi(m)$ . Since  $v$  is relatively prime to  $e_1$ , there exist integers  $x$  and  $y$  such that the relation  $xv + ye_1 = 1$  holds. Therefore,  $ye_1 = 1 \bmod \phi(m)$ . Since  $v$  is multiple of  $\phi(m)$ . Thus,  $y = d_1$ . Hence,

the user in a security class can derive the decryption key  $d_1$  of another user.

Let us consider our scheme. Assume that a user in a security class  $C_i$  wants to retrieve decryption key  $d_j$  of another user in a class  $C_j$ . A user in a class knows encryption key  $e_i$ , decryption key  $d_i$  of itself and encryption key  $e_j$  of  $C_j$ . But, encryption and decryption keys are on different modulus. So,  $C_i$  have to compute  $p_i$  from  $d_i$ , where  $p_i$  is the inverse of  $d_i$ . As it is hard to compute  $\phi(\phi(N))$  from  $N$ , it is difficult to compute  $p_i$  from  $d_i$  by the class  $C_i$ . Hence, a user in  $C_i$  cannot derive  $d_j$  from  $e_i$ ,  $d_i$  and  $e_j$ . As a result, our scheme is secure against this type common modulus attacks.

## 6 Advantages

In this section, we discuss the various kind of advantages achieved from our proposed scheme. The following advantages are as follows:

- Encryption and decryption techniques are based on asymmetric cryptosystem.
- The security is equivalent as RSA cryptosystem.
- Key generation procedure is based on asymmetric cryptosystem which is done by CA. So, it supports authentication.
- This scheme is secure against all possible attacks.
- No class can derive the decryption key of other class. An encrypted message of a class can be decrypted by the class itself and its predecessor classes. But, the reverse is not true.

## 7 Comparison

In this section, we compare our method with previous published schemes.

- 1) Our proposed scheme provides a hierarchical access control based on asymmetric key assignment in tree (non linear) hierarchical structure, whereas the M.S. Hwang's scheme provides totally order (linear order) hierarchical structure for access control although both schemes provide encryption and decryption techniques based on asymmetric cryptosystem.
- 2) In our scheme, the size of public parameters only depends on the magnitude of  $N$  and does not depend on the number of security classes in the hierarchical system. As a result, our scheme may be applicable even if the number of security classes is more. On the other hand, if the number of security classes is large, the Akl and Taylor [2], Mackinnon et al. [18] and Harn and Lin [11] scheme cannot be applicable.

## 8 Conclusion

In this paper, we have proposed a new scheme for solving the multilevel key generation technique. Our scheme is based on asymmetric cryptosystem for access control of information items in an organization. In fact, this scheme does not require large amount of storage space to store public parameters. Our scheme also provides encryption and decryption techniques using asymmetric cryptosystem. Furthermore, our proposed scheme retains the same security level compared to the schemes previously published.

Hence, we conclude that our scheme is a novel scheme to be used as asymmetric cryptosystem in tree structured access control hierarchies for key generation as well as for encryption and decryption.

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