Global Analysis of a SEIQV Epidemic Model for Scanning Worms with Quarantine Strategy

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Abstract

Active scanning worms have drawn a significant attention due to their enormous threats to the Internet infrastructure and services. In order to effectively defend against them, this paper proposes a novel epidemic SEIQV model with quarantine strategy. Using this SEIQV model, we obtain the basic reproduction number for determining whether the worm dies out completely. The global stabilities of worm-free equilibrium and endemic equilibrium are proved, and determined by the basic reproduction number. The impact of different parameters of this model is studied. Simulation results show that the number of susceptible, infected and vaccinated hosts are consistent with theoretical analysis. The model provides a theoretical foundation for controlling and forecasting for active scanning worms.

Keywords: Basic reproduction number, network security, quarantine strategy, stability analysis, worm propagation model

1 Introduction

Active scanning worms are malicious codes which can replicate themselves and actively infected other hosts with certain vulnerability via Internet. With the ever increasing number of Internet applications and the emergence of new technologies, worms have become a great threat to our work and daily life, caused tremendous economic losses. Especially, the advent of the Internet of things would make the threat increasingly serious. How to combat Internet worms effectively is an urgent issue confronted with defenders. Therefore, it is necessary to comprehend the long-term behavior of worms and to propose effective strategies to defend against worms.

Based on the infectivity between a worm and a biological virus, some epidemic models representing worm propagations were presented to depict the propagation of worms, e.g., SIR model [15], $SIRS$ model [9, 14], $SIQV$ model [24], $SIDQV$ model [25], which all assume that susceptible hosts can immediately translate into infected ones. This assumption is unreasonable. Actually, it will take a certain time to send worm copies to susceptible hosts. To overcome previous drawbacks, some researchers added a state (E), namely the exposed state, and then proposed some propagation models, e.g., $SEIR$ model [10], $SEIRS$ model [11, 13, 17], $SEIQV$ model [18], $SEIQRS$ model [8], which assume that exposed hosts can not infect other ones. Actually, an infected host which is in latency can infect other hosts by means of some methods, e.g., vulnerability seeking. All the previous models do not take this passive infectivity into consideration. Recently, Yang et al. [20, 21, 22, 23] proposed some models, by taking into account the fact that a host immediately possesses infectivity once it is infected. These model, however, all make an assumption that exposed hosts and infected hosts have the same infectivity. This is not consistent with the reality. Although an exposed host also sends scanning packets to find susceptive hosts with certain vulnerabilities, the scanning packets sent by an exposed host are less than an infected one. Usually, the infection rate of exposed hosts is less than that of infected ones. Therefore, they should have different infection rates.

Recently, more attention has been paid to the combina-

tion of worm propagation model and countermeasures to study the prevalence of worms, e.g., quarantine [24, 25] and vaccination [4]. The implementation of quarantine strategy relies on the intrusion detection systems (IDSes). Intrusion detection systems can be classified into two categories: misuse and anomaly intrusion detection systems. The former is mainly based on a database with the feature of known attack behaviors, which fails to detect new ones. The latter can detect both novel and known worms, but false positive rate is high. In summary, both classes of intrusion detection systems have defects that affect their performances.

SEIQV model [18] takes quarantine and vaccination into consideration as the two main recovery countermeasures, and analyzes the global stability of its worm-free equilibrium. Inspired by *SEIQV* model [18], we propose a new extended model, referred to as e-SEIQV (susceptible - exposed - infected - quarantined - vaccinated) model. In comparison with *SEIQV* model [18], the model proposed takes two infection rates into account. Some susceptible hosts can be directly vaccinated. Using the basic reproduction number, we derive global stabilities of a worm-free equilibrium and a unique endemic equilibrium by a Lyapunov function and a geometric approach. Based on these results and further analysis, some effective methods for controlling worms are recommended.

The rest of this paper is organized as follows. Section 2 formulates the new model and obtains its basic reproduction number. Section 3 proves the local and global stabilities of the worm-free equilibrium. Section 4 examines the local and global stabilities of the endemic equilibrium. Section 5 covers the numerical analysis and the simulations. Section 6 summarizes the paper with some future directions.

2 Mathematical Model Formulation

The total host population N is partitioned into five groups and any hosts can potential be in any of these groups at any time t: the susceptible, exposed, infected, quarantined, vaccinated, with sizes denoted by S, E, I, Q, V, respectively. The total number of population N at time t is given by $N(t) = S(t) + E(t) + I(t) + Q(t) + V(t)$. The dynamical transfer of hosts is depicted in Figure 1.

In the model, susceptible hosts can be infected by worms with efficient infection rates β_1, β_2 and become into exposed ones and infected ones, or patched into the vaccinated state with rate ρ. ω is the transfer rate between the exposed and the infected. Some exposed and infected ones can be detected by a misuse detection system and then constantly quarantined at rates α_1, α_2, respectively. α_1, α_2 are determined by the misuse detection system, which will become larger if the detection system is set to be sensitive to worms' activities. A high performance detection system has higher detection rate and lower false alarm rate. For example, the detection system should
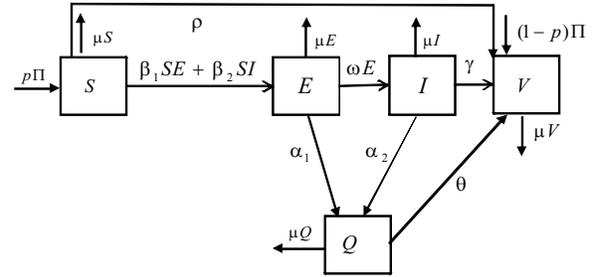

Figure 1: State transition diagram with quarantine

have larger α_1, α_2. Some hosts in the quarantined state become vaccinated ones by repairing and then patching at rate θ. Some infected hosts can be detected and then manually patched at rate γ. The positive parameter μ is the death rate in each of the five states. Some hosts enter the network at the number Π, a fraction $1 - p$ of which is patched into the vaccinated state directly at "birth".

Based on the compartment model presented Figure 1, our model having infected force in the exposed, infected period is described by the following system of differential equations:

$$\begin{cases} S^{'}(t) = p\Pi - \beta_1 SE - \beta_2 SI - (\rho + \mu)S, \\ E^{'}(t) = \beta_1 SE + \beta_2 SI - (\omega + \alpha_1 + \mu)E, \\ I^{'}(t) = \omega E - (\gamma + \alpha_2 + \mu)I, \\ Q^{'}(t) = \alpha_1 E + \alpha_2 I - (\theta + \mu)Q, \\ V^{'}(t) = \rho S + \gamma I + \theta Q + (1 - p)\Pi - \mu V. \end{cases} \quad (1)$$

Summing the equations of System (1), we obtain

$$N(t)^{'} = \Pi - \mu(S + E + I + Q + V). \quad (2)$$

Therefore, the total population size $N(t)$ converges to the equilibrium Π/μ. It follows from Equation (2) that $\liminf_{t \to \infty} N(t) \le \Pi/\mu$. We thus study our System (1), in the following feasible region:

$$\Omega = \{(S, E, I, Q, V) \in \mathbb{R}^5_+ : S + E + I + Q + V \le \Pi/\mu\},$$

which is a positively invariant set of Model (1). We next consider the dynamic behavior of Model (1) on Ω. It is easy to see that Model (1) always has a worm-free equilibrium, $P_0 = (p\Pi/(\rho + \mu), 0, 0, 0, \frac{\Pi}{\mu}(1 - \frac{p}{\rho + \mu}))$.

Let $x = (E, I, Q, V, S)^T$, then Model (1) can be written as

$$\frac{dx}{dt} = \mathcal{F}(x) - \mathcal{V}(x),$$

where

$$\mathcal{F}(x) = \begin{pmatrix} \beta_1 SE + \beta_2 SI \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

$$\mathcal{V}(x) = \begin{pmatrix} (\omega + \alpha_1 + \mu)E \\ (\gamma + \alpha_2 + \mu)I - \omega E \\ (\theta + \mu)Q - \alpha_1 E - \alpha_2 I \\ \mu V - \rho S - \gamma I - \theta Q - (1-p)\Pi \\ \beta_1 SE + \beta_2 SI + (\rho + \mu)S - p\Pi \end{pmatrix}.$$

Differentiating $\mathcal{F}(x)$ and $\mathcal{V}(x)$ with respect to E, I, Q, V, S and evaluating at the worm-free equilibrium $P_0 = (p\Pi/(\rho + \mu), 0, 0, 0, \frac{\Pi}{\mu}(1 - \frac{p}{\rho + \mu}))$, respectively, we have

$$D\mathcal{F}(P_0) = \begin{pmatrix} F_{2\times 2} & 0_{2\times 3} \\ 0_{3\times 2} & 0_{3\times 3} \end{pmatrix},$$

$$D\mathcal{V}(P_0) = \begin{pmatrix} Y_{2\times 2} & 0 & 0 & 0 \\ & 0 & 0 & 0 \\ Y'_{3\times 2} & \theta + \mu & 0 & 0 \\ & -\theta & \mu & -\rho \\ & 0 & 0 & \rho + \mu \end{pmatrix},$$

where

$$F_{2\times 2} = \begin{pmatrix} \beta_1 p\Pi/(\rho + \mu) & \beta_2 p\Pi/(\rho + \mu) \\ 0 & 0 \end{pmatrix},$$

$$Y'_{3\times 2} = \begin{pmatrix} -\alpha_1 & -\alpha_2 \\ 0 & -\gamma \\ \beta_1 p\Pi/(\rho + \mu) & \beta_2 p\Pi/(\rho + \mu) \end{pmatrix},$$

and

$$Y_{2\times 2} = \begin{pmatrix} \omega + \alpha_1 + \mu & 0 \\ -\omega & \gamma + \alpha_2 + \mu \end{pmatrix}.$$

$\mathcal{F}\mathcal{V}^{-1}$ is the next generation matrix for Model (1). Thus, the spectral radius of the next generation matrix $\mathcal{F}\mathcal{V}^{-1}$ can be obtained as,

$$\begin{aligned} \rho(\mathcal{F}\mathcal{V}^{-1}) &= \rho(F_{2\times 2}Y_{2\times 2}^{-1}) \\ &= \frac{p\Pi(\beta_1(\gamma + \alpha_2 + \mu) + \beta_2\omega)}{(\rho + \mu)(\omega + \alpha_1 + \mu)(\gamma + \alpha_2 + \mu)}. \end{aligned}$$

According to Theorem 2 in [2], the basic reproduction number of Model (1) is

$$R_0 = \frac{p\Pi(\beta_1(\gamma + \alpha_2 + \mu) + \beta_2\omega)}{(\rho + \mu)(\omega + \alpha_1 + \mu)(\gamma + \alpha_2 + \mu)}. \tag{3}$$

For the concision of notation, let $m = \omega + \alpha_1 + \mu$ and $n = \gamma + \alpha_2 + \mu$. Thus $R_0 = \frac{p\Pi(\beta_1 n + \beta_2\omega)}{(\rho + \mu)mn}$.

The first four equations in System (1) have no dependence on the fifth one. Therefore, the fifth equation can be omitted. Thus, System (1) can be rewritten as the following four-dimensional system:

$$\begin{cases} S'(t) = p\Pi - \beta_1 SE - \beta_2 SI - (\rho + \mu)S, \\ E'(t) = \beta_1 SE + \beta_2 SI - (\omega + \alpha_1 + \mu)E, \\ I'(t) = \omega E - (\gamma + \alpha_2 + \mu)I, \\ Q'(t) = \alpha_1 E + \alpha_2 I - (\theta + \mu)Q. \end{cases} \tag{4}$$

Next, we will study the stabilities of worm-free equilibrium and endemic equilibrium on System (4).

3 Stability of Worm-free Equilibrium

It is easily obtained that System (4) has a worm-free equilibrium given by $P_0 = (p\Pi/(\rho + \mu), 0, 0, 0)$.

Lemma 1. *When $R_0 < 1$, the worm-free equilibrium P_0 is locally asymptotically stable in Ω. When $R_0 > 1$, the worm-free equilibrium P_0 is an unstable saddle point.*

Proof. The Jacobian matrix of Model (4) at P_0 is

$$J(P_0) = \begin{pmatrix} -(\rho + \mu) & -\frac{\beta_1 p\Pi}{(\rho + \mu)} & -\frac{\beta_2 p\Pi}{(\rho + \mu)} & 0 \\ 0 & \frac{\beta_1 p\Pi}{(\rho + \mu)} - m & \frac{\beta_2 p\Pi}{(\rho + \mu)} & 0 \\ 0 & \omega & -n & 0 \\ 0 & \alpha_1 & \alpha_2 & -(\theta + \mu) \end{pmatrix}$$

It is easily obtained that $J(P_0)$ has two negative eigenvalues $\lambda_1 = -(\rho + \mu)$, and $\lambda_2 = -(\theta + \mu)$, the other eigenvalues of $J(P_0)$ are determined by the following equation

$$\lambda^2 + (m + n - \frac{\beta_1 p\Pi}{(\rho + \mu)})\lambda + mn - \frac{(n\beta_1 + \omega\beta_2)p\Pi}{(\rho + \mu)} = 0. \tag{5}$$

When $R_0 < 1$, then $mn > (n\beta_1 + \omega\beta_2)p\Pi/(\rho + \mu)$.

For $mn > (n\beta_1 + \omega\beta_2)p\Pi/(\rho + \mu)$, we can obtain $m + n > n + \beta_1 p\Pi/(\rho + \mu) + p\Pi\beta_2\omega/(\rho + \mu)$, thus $m + n - \beta_1 p\Pi/(\rho + \mu) > n + \beta_2 p\Pi/(\rho + \mu) > 0$, which means the Equation (5) has two negative roots. Therefore, the worm-free equilibrium P_0 is locally asymptotically stable.

When $R_0 > 1$, then $mn - (n\beta_1 + \omega\beta_2)p\Pi/(\rho + \mu) < 0$, which means the Equation (5) has a positive root and a negative root. Therefore, the worm-free equilibrium P_0 is unstable saddle point. □

Lemma 2. *When $R_0 \leq 1$, the worm-free equilibrium P_0 is globally asymptotically stable in Ω. When $R_0 > 1$, all solutions starting in Ω and sufficiently close to P_0 move away from $\{P_0\}$.*

Proof. Consider the Lyapunov function

$$L = \frac{\beta_1 n + \beta_2\omega}{mn}E + \frac{\beta_2}{n}I.$$

Its derivative along the solutions to Model (4) is

$$\begin{aligned} L' &= \frac{\beta_1 n + \beta_2\omega}{mn}(\beta_1 SE + \beta_2 SI - mE) + \frac{\beta_2}{n}(\omega E - nI) \\ &= \frac{\beta_1 n + \beta_2\omega}{mn}(\beta_1 SE + \beta_2 SI) - (\beta_1 E + \beta_2 I) \\ &= (\beta_1 E + \beta_2 I)(\frac{\beta_1 n + \beta_2\omega}{mn}S - 1) \\ &\leq (\beta_1 E + \beta_2 I)(\frac{p\Pi(\beta_1 n + \beta_2\omega)}{mn(\rho + \mu)} - 1) \\ &= (\beta_1 E + \beta_2 I)(R_0 - 1) \\ &\leq 0. \end{aligned}$$

Furthermore, $L' = 0$ if and only if $E = I = 0$ or $R_0 = 1$. Thus, the largest compact invariant set in $\{(S, E, I, Q)|L' = 0\}$ is the singleton $\{P_0\}$. When $R_0 \leq 1$, the global stability of P_0 follows from LaSalle's invariance principle [5]. LaSalle's invariance principle [5] implies that P_0 is globally asymptotically stable in Ω. When $R_0 > 1$, it follows from the fact $L' > 0$ if $E > 0$ and $I > 0$. This completes the proof. □

4 Stability of Endemic Equilibrium

The endemic equilibrium $P^*(S^*, E^*, I^*, Q^*)$ of Model (4) is determined by equations

$$
\begin{cases}
p\Pi - \beta_1 SE - \beta_2 SI - (\rho + \mu)S = 0, \\
\beta_1 SE + \beta_2 SI - (\omega + \alpha_1 + \mu)E = 0, \\
\omega E - (\gamma + \alpha_2 + \mu)I = 0, \\
\alpha_1 E + \alpha_2 I - (\theta + \mu)Q = 0.
\end{cases}
\tag{6}
$$

By some simple computation, we obtain

$$
\begin{cases}
S^* = \frac{p\Pi}{R_0(\rho+\mu)}, \\
E^* = \frac{(\gamma+\alpha_2+\mu)I}{\omega} = \frac{nI}{\omega}, \\
I^* = \frac{p\Pi(1-1/R_0)\omega}{(\omega+\alpha_1+\mu)(\gamma+\alpha_2+\mu)} = \frac{p\Pi(1-1/R_0)\omega}{mn}, \\
Q^* = \frac{(\alpha_1(\gamma+\alpha_2+\mu)+\alpha_2\omega)I}{\omega(\theta+\mu)} = \frac{(\alpha_1 n+\alpha_2\omega)I}{\omega(\theta+\mu)}.
\end{cases}
\tag{7}
$$

Now we investigate the local stability of the endemic equilibrium $P^*(S^*, E^*, I^*, Q^*)$. The Jacobian matrix of Equation (4) at the endemic equilibrium P^* is

$$
J(P^*) = \begin{pmatrix}
B_1 & -\beta_1 S & -\beta_2 S & 0 \\
\beta_1 E + \beta_2 I & \beta_1 S - m & \beta_2 S & 0 \\
0 & \omega & -n & 0 \\
0 & \alpha_1 & \alpha_2 & -\theta - \mu
\end{pmatrix}
\tag{8}
$$

where, $B_1 = -\beta_1 E - \beta_2 I - \rho - \mu$. Its characteristic equation is $det(\lambda I - J(P^*)) = 0$, where I is the unit matrix. Therefore,

$$
det(\lambda I - J(P^*)) = (\lambda_1 + \theta + \mu)(\lambda^3 + A\lambda^2 + B\lambda + C) = 0, \tag{9}
$$

where

$$
A = n + \rho + \mu + \beta_1 E + \beta_2 I + \frac{\beta_2 \omega m}{\beta_1 n + \beta_2 \omega} > 0,
$$

$$
\begin{aligned}
B &= \frac{\beta_2 m\omega}{\beta_1 n + \beta_2 \omega}(\rho + \mu + \beta_1 E + \beta_2 I) \\
&+ \beta_1 S(\beta_1 E + \beta_2 I) + n(\rho + \mu + \beta_1 E + \beta_2 I) \\
&> 0,
\end{aligned}
$$

$$
C = mn(\beta_1 E + \beta_2 I) > 0.
$$

By a direct calculation, we obtain that $AB - C > 0$. According to the theorem of Routh-Hurwitz [1], it follows that all the roots of Equation (9) have negative real parts. Therefore, the endemic equilibrium P^* is locally asymptotically stable.

From the above discussion, we can summarize the following conclusion.

Lemma 3. *When $R_0 > 1$, the endemic equilibrium P^* is locally asymptotically stable in Ω.*

Next, we apply the geometrical approach [7] to investigate the global stability of the endemic equilibrium P^* in the region Ω.

Theorem 1. [7] *Consider the following systems:*
$x' = f(x),\ x \in \Omega.$
If the following conditions are satisfied:

1) *The system (∗) exists a compact absorbing set $K \subset \Omega$ and has a unique equilibrium P^* in Ω;*

2) *P^* is locally asymptotically stable;*

3) *The system (∗) satisfies a Poincaré-Bendixson criterion;*

4) *A periodic orbit of the system (∗) is asymptotically orbitally stable, then the only equilibrium P^* is the globally asymptotically stable in Ω.*

Lemma 4. *If $R_0 > 1$, the unique positive equilibrium P^* of Model (4) is globally asymptotically stable in Ω.*

Proof. We only need to prove that all assumptions of Theorem 1 hold.

If $R_0 > 1$, then the worm-free equilibrium is unstable according to Lemma 1. Moreover, the behavior of the local dynamics near the region P_0 described in Lemma 1 implies that Model (4) is uniformly persistent in the region Ω. That is, there exists a constant $c > 0$, such that any solution $(S(t), E(t), I(t), Q(t))$ of Model (4) with initial value $(S(0), E(0), I(0), Q(0))$ in Ω satisfies

$$
min\{\liminf_{t\to\infty} S(t), \liminf_{t\to\infty} E(t), \liminf_{t\to\infty} I(t), \liminf_{t\to\infty} Q(t)\} \geq c.
$$

This can be proved by applying a uniform persistent result in [3] and by the use of a similar argument as in the proof in [6]. The uniform persistence of System (4) in the bounded set Ω is equivalent to the existence of a compact $K \in \Omega$ that is absorbing for System (4). During the process of obtaining the endemic equilibrium P^*, we can know that P^* is the unique equilibrium in the interval $(0, \Pi/\mu)$. Assumption (1) holds.

According to Lemma 3, we know that the endemic equilibrium P^* is locally asymptotically stable in the region Ω. Assumption (2) holds.

The Jacobian matrix of Model (4) is denoted by Equation (8). Choosing the matrix H as $H = diag(-1, 1, -1, -1)$, it is easy to prove that HJH has non-positive off-diagonal elements, thus we can obtain that System (4) is competitive. This verifies the Assumption (3).

The second compound matrix $J^{[2]}(P^*)$ of $J(P^*)$ can be calculated as follows:

$$
J^{[2]}(P^*) = \begin{pmatrix}
A1 & \beta_2 S & 0 & \beta_2 S & 0 & 0 \\
\omega & A2 & 0 & -\beta_1 S & 0 & 0 \\
\alpha_1 & \alpha_2 & A3 & 0 & -\beta_1 S & -\beta_2 S \\
0 & b & 0 & A4 & 0 & 0 \\
0 & 0 & b & \alpha_2 & A5 & \beta_2 S \\
0 & 0 & 0 & -\alpha_1 & \omega & A6
\end{pmatrix}
\tag{10}
$$

where,
$$
A1 = -(\beta_1 E + \beta_2 I + \rho + \mu + m - \beta_1 S),
$$

$A2 = -(\beta_1 E + \beta_2 I + \rho + \mu + n),$
$A3 = -(\beta_1 E + \beta_2 I + \rho + 2\mu + \theta),$
$A4 = -(m + n - \beta_1 S),$
$A5 = -(m + \mu + \theta - \beta_1 S),$
$A6 = -(n + \theta + \mu),$
$b = \beta_1 E + \beta_2 I.$

The second compound system of Model (4) in a periodic solution can be represented by the following differential equations:

$$
\begin{cases}
X'(t) = A1X + \beta_2 SY + \beta_2 SL, \\
Y'(t) = \omega X + A2Y - \beta_1 SL, \\
Z'(t) = \alpha_1 X + \alpha_2 Y + A3Z - \beta_1 SM - \beta_2 SU, \\
L'(t) = bY + (\beta_1 S - m - n)L, \\
M'(t) = bZ + \alpha_2 L + A5M + \beta_2 SU, \\
U'(t) = -\alpha_1 L + \omega M - (n + \mu + \theta)U.
\end{cases}
\quad (11)
$$

In order to prove that System (11) is asymptotically stable, we consider the following Lyapunov function:

$$
V(X, Y, Z, L, M, U; S, E, I, Q)
$$
$$
= \sup\{|X| + |L| + |M|, \frac{E}{I}(|Y| + |Z| + |U|)\}.
$$

By the use of the uniform persistence, we obtain that the orbit of $P(t) = (S(t), E(t), I(t), Q(t))$ remains a positive distance from the boundary of Ω, thus, we know that there exists a constant c satisfying

$$
V(X, Y, Z, L, M, U; S, E, I, Q)
$$
$$
\geq c \sup\{|X|, |Y|, |Z|, |L|, |M|, |U|\},
$$

for all $(X, Y, Z, L, M, U) \in \mathbb{R}^6$ and $(S, E, I, Q) \in P(t)$.

For the differential equations in Equation (11), we can obtain the following differential inequalities by direct calculations:

$$
\begin{aligned}
[D_+(|X| + |Y| + |Z|) &\leq -(2\mu + \omega + \alpha_1)(|X| + |L| \\
&\quad + |M|) + \frac{E}{I}(\beta_1 S + \beta_2 S \frac{I}{E}) \\
&\quad (|Y| + |Z| + |U|), \\
D_+(|L| + |M| + |U|) &\leq \omega(|X| + |L| + |M|) - (2\mu \\
&\quad + \alpha_2 + \gamma)(|Y| + |Z| + |U|).
\end{aligned}
$$

Then,

$$
\begin{aligned}
D_+ \frac{E}{I}(|Y| + |Z| + |U|) &\leq \omega \frac{E}{I}(|X| + |L| + |M|) \\
&\quad + (\frac{E'}{E} - \frac{I'}{I} - (2\mu + \alpha_2 \\
&\quad + \gamma)) \frac{E}{I}(|Y| + |Z| + |U|).
\end{aligned}
$$

From the pervious formula, we can obtain

$$
D_+ |V(t)| \leq \max\{g_1(t), g_2(t)\} V(t),
$$

where,

$$
\begin{aligned}
g_1(t) &= -(2\mu + \delta_1 + \omega) + (\beta_1 S + \beta_2 S \frac{I}{E}), \\
g_2(t) &= \omega \frac{E}{I} + \frac{E'}{E} - \frac{I'}{I} - (2\mu + \alpha + \delta_2 + p).
\end{aligned}
$$

From Model (4), we can obtain

$$
\begin{aligned}
\frac{E'}{E} &= \beta_1 S + \beta_2 S \frac{I}{E} - (\omega + \alpha_1 + \mu), \\
\frac{I'}{I} &= \omega \frac{E}{I} - (\gamma + \alpha_2 + \mu).
\end{aligned}
$$

Therefore,

$$
g_1(t) = \frac{E'}{E} - \mu, g_2(t) = \frac{E'}{E} - \mu.
$$

Then,

$$
\int_0^\zeta \sup\{g_1(t), g_2(t)\} dt \leq \ln E(t)|_0^\zeta - \mu\zeta = -\mu\zeta < 0,
$$

which implies that $(X(t), Y(t), Z(t), L(t), M(t), U(t)) \to 0$, as $t \to \infty$. Thus, the second compound System (11) is asymptotically stable. This verifies the Assumption (4).

We verify all the assumptions of Theorem 1. Therefore, P^* is globally asymptotically stable in Ω. $\qquad \square$

5 Numerical Simulations

In this experiment, we choose the Slammer as basic behavior of a worm. To obtain the spread of worms in a large-scale network, 1,000,000 hosts are selected as the population size. According to the real conditions of the Slammer worm, the worm's average scan rate is $s = 4000$ per second [12]. Slammer worm's infection rate can then be computed as $\beta_2 = s/2^{32} = 0.00000093$, $\beta_1 = 0.0000009$. At the beginning, the number of susceptible, exposed, infected, quarantined and vaccinated hosts are $S(0) = 999,990$, $E(0) = 0$, $I(0) = 10$, $Q(0) = 0$ and $V(0) = 0$, respectively. The quarantined rates of exposed hosts and infected hosts are $\alpha_1 = 0.0001$, $\alpha_2 = 0.004$ per minute, respectively.

Other parameters in these simulations are given as follows: $\mu = 0.00001$, $\rho = 0.00002$, $\theta = 0.005$, $\omega = 0.05$, $p = 0.1$, $\gamma = 0.001$, where $R_0 = 0.677 < 1$. The worm will gradually disappear according to Theory 2. Figure 2 illustrates the number of susceptible, exposed and infected hosts when R_0 is 0.677. From Figure 2, we can clearly see that the tendency of the worm propagation is depressive, which is consistent with Lemma 2. Finally, all infected hosts vanish, and become into vaccinated state. In order to effectively defend against such worms, we must adopt some feasible methods to decrease the infection rate [16, 19] or increase the following parameters (e.g., the transfer rates between the exposed and the recovered, between the exposed and the infected) to guarantee the basic reproduction number $R_0 < 1$.
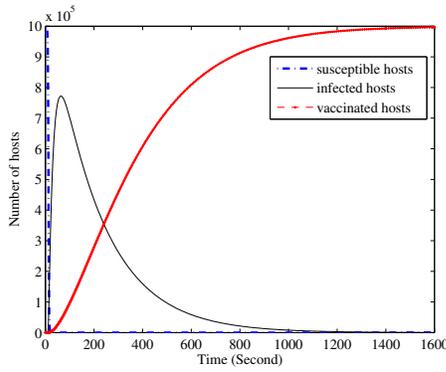
Figure 2: Globally asymptotically stable worm-free equilibrium

Figure 4: Effect of quarantined rate α_1

When $\alpha_1 = 0.02$, $\alpha_2 = 0.06$, $\theta = 0.009$, we can obtain $R_0 = 2.654 > 1$. Other parameters do not vary. We can see the results in Figure 3. As can be seen from Figure 3, the number of susceptible and infected hosts eventually become positive values between 0 and Π/μ. $S(t)$, $I(t)$, $V(t)$ all approach their steady state, and the worm persists. This is fully consistent with the conclusions of Lemma 4.
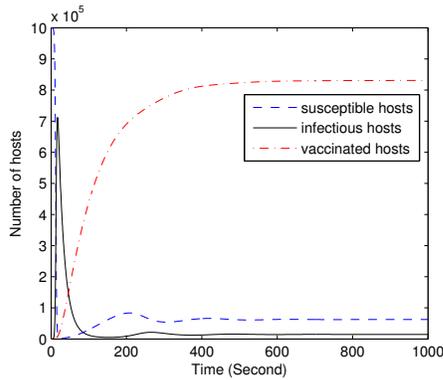
number of infected hosts are. The quarantined rate plays an important role in containing the infected hosts. The quarantined rate relies mainly on the accuracy of intrusion detection systems. The detection rate depends mainly on the quarantined rate α_2 of infected hosts. That is, a more effective detection rate will generate a larger quarantined rate. We can improve the efficiency and decrease the false positive of intrusion detection systems to obtain a larger quarantined rate.

Figure 3: Globally asymptotically stable endemic equilibrium

Figure 5: Effect of quarantined rate α_2

In our model, the main defending method is the use of quarantine strategy. There are two quarantined rates in the proposed model. Intuitively, both of them all play an important role in decreasing the number of infected hosts. Next, we study the effect of the quarantined rates. when α_1 is equal to 0.0001, 0.001 and 0.01, respectively, we can see the result in Figure 4. From Figure 4, we can see that the quarantined rate α_1 plays a minor role in decreasing the number of hosts infected by worms. On the other hand, a larger α_1 might cause a higher false alarm rate of the detection system, and block some users' normal activities.

When we change the values of the quarantined rate α_2, e.g., 0.002, 0.004, 0.006 and 0.008, we obtain the result in Figure 5. From Figure 5, it can be seen that the quarantined rate α_2 has an obvious significant effect on defending worms. The larger the quarantined rate is, the less the
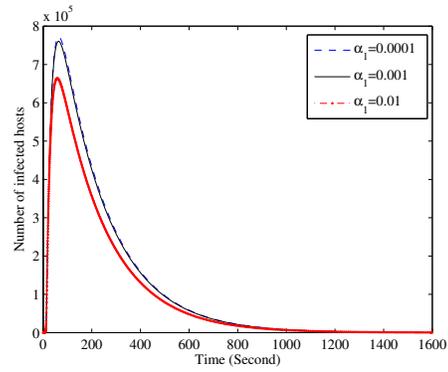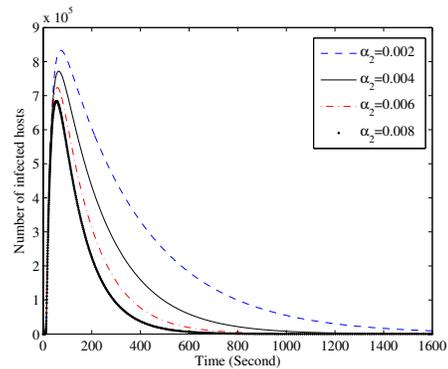
6 Conclusions

This paper proposed an epidemic model to defend the propagation of active scanning worms, which takes the quarantine strategy into account. Firstly, we obtain the basic reproduction number using the next generation matrix. Next, with the help of the reproduction number, we prove the stabilities of worm-free equilibrium and endemic equilibrium. When the reproduction number is less than or equal to one, our model has only a worm-free equilibrium which is globally stable, which implies the worm dies out eventually; when the reproduction number is larger than one, our model has a unique endemic equilibrium which is globally stable, it implies that the worm persists in the whole population and tends to a steady state. Finally, simulation results are given to verify our conclu-

sions. Our future work will expand this model which can characterize more features of Internet worms, e.g., taking delay or impulse into consideration.

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