Managing Temporal Uncertainty Through Waypoint Controllability

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Abstract

Simple Temporal Networks have proved useful in applications that involve metric time. However, many applications involve events whose timing is not controlled by the execution agent. A number of properties relating to overall controllability in such cases have been introduced in [Vidal and Ghallab, 1996] and [Vidal and Fargier, 1997], including Weak and Strong Controllability. We derive some new results concerning these properties. In particular, we prove the negation of Weak Controllability is NP-hard, confirming a conjecture in [Vidal and Fargier, 1997]. We also introduce a more general controllability property of which Weak and Strong Controllability are special cases. A propagation algorithm is provided for determining whether the property holds, and we identify tractable cases where the algorithm runs in polynomial time.

1 Introduction

Simple Temporal Networks [Dechter e*t* a/., 1991] have proved useful in Planning and Scheduling applications that involve metric time (e.g. [Bienkowski and Iloebel, 1998; Muscettola *et* a/., 1998a]). However this formalism does not adequately address an important aspect of real execution domains: the occurrence time of some events may not be under the complete control of the execution agent. For example, when a spacecraft commands an instrument or interrogates a sensor, a varying amount of time may intervene before the operation is completed. In cases like this, the execution agent cannot select the precise time delay between events in accord with the timing of previously executed events. Instead, the value is selected by Nature independently of the agent's choices. This can cause inconsistencies at execution time even if the Simple Temporal Network appeared consistent at plan generation time. The problem of control of temporal networks with uncertainty was first addressed formally in [Vidal and Ghallab, 1996 and [Vidal and Fargier, 1997].

In practice, temporal uncertainty is usually eliminated by *padding* each uncertain interval with a flexible *wait* period. For example, a task duration may be modeled as the upper bound of the task's possible executions. In this case, the event at the end of a task does not represent the time at which the task actually ends. Instead, it constitutes a *waypoint,* i.e., a time by which we can guarantee that the task has ended. Synchronization with respect to waypoints can now proceed without uncertainty and Simple Temporal Networks are completely adequate. Although this use of waypoints provides a workable solution to the problem of execution uncontrollability, its indiscriminate application may not be desirable or possible. For example, it may obstruct tight synchronization with respect to follow-on requirements. Moreover, it may not be possible to add a wait period after each uncertain delay if several uncontrollable delays are causally connected.

In this paper we introduce *Waypoint Controllability,* a general framework for a formal analysis of when networks that incorporate uncontrollable delays can be successfully executed. Waypoint controllability generalizes the concepts of *Strong* and *Weak* Controllability (Vidal and Ghallab, 1996; Vidal and Fargier, 1997]. Roughly speaking, Strong Controllability supplies a guarantee of a fixed execution that works irrespective of the outcomes of the uncontrollable delays, while the absence of Weak Controllability means that there are some outcomes for which no execution will work. Subject to restrictions on the type of network, an algorithm is presented in [Vidal and Ghallab, 1996] that determines Strong Controllability in deterministic polynomial time. It is shown in [Vidal and Fargier, 1997] that the negation of Weak Controllability is in $\mathcal{NP}$ and it is conjectured that the problem is $\mathcal{NP}$-Complete.

We prove here that the negation of Weak Controllability is indeed $\mathcal{NP}$-Complete. This complexity result applies also to Waypoint Controllability, of which Weak Controllability is a special case. Sufficient conditions are presented for the tractability of Waypoint Controllability. From these, we are able to derive the tractability of Strong Controllability while relaxing the restrictions in [Vidal and Ghallab, 1996]. We also provide a propagation algorithm for determining Waypoint Controlla-

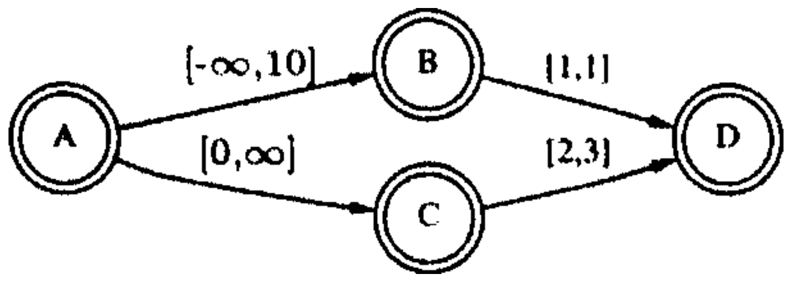

Figure 1: Simple Temporal Network.

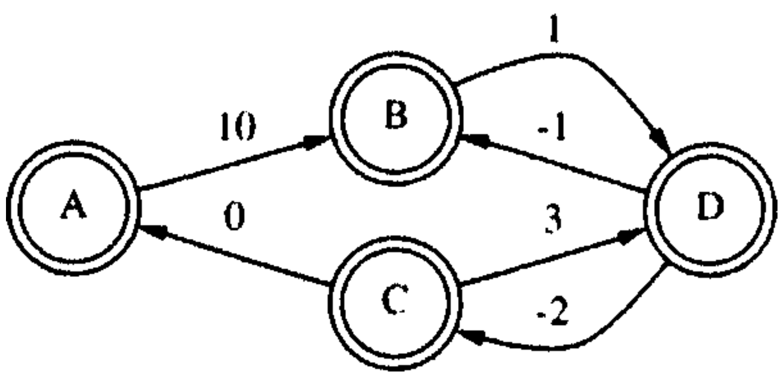

Figure 2: Distance Graph.

bility. Finally we discuss the relationship between Waypoint Controllability and *Dynamic. Controllability,* where knowledge of uncontrollable delays is updated during execution.

2 Preliminaries

We assume the reader is broadly familiar with Simple Temporal Networks [Dechter *et al.,* 1991]. The definition is briefly reviewed here to set the stage for subsequent developments.

A Simple Temporal Network (STN) is a graph in which the edges are labelled with upper and lower numerical bounds. The nodes in the graph represent temporal events or *timepoints,* while the edges correspond to constraints on the durations between the events. Formally, an STN may be described as a 4-tuple $< N, E, l, u >$ where *N* is a set of nodes, *E* is a set of edges, and $l : E \rightarrow \mathbb{R} \cup \{-\infty\}$ and $u : E \rightarrow \mathbb{R} \cup \{+\infty\}$ are functions mapping the edges into extended Real Numbers. Figure 1 shows an example of an STN. Figure 2 shows the corresponding *distance graph* [Dechter *et al.,* 1991], which is an alternate representation useful for mathematical analysis. Note that lower bounds are negated to give the lengths of the reverse edges. Edges of infinite length are omitted. (Given a distance graph, one can also find a corresponding STN, so the representations are interchangeable.) An STN is consistent if and only if the distance graph does not contain a negative cycle.

Controllability problems [Vidal and Ghallab, 1996; Vidal and Fargier, 1997] arise in STNs where the edges, which we will call *links,* are divided into two classes, *causal links* and *requirement links.* The causal links are regarded as representing intervals whose duration is controlled by Nature, subject to the limits imposed by the upper and lower bounds. The timepoints are regarded as variables. The goal is that they should have values that satisfy all the requirement links as well as whatever values are chosen by Nature for the causal links.

Definition 1 *A* Simple Temporal Network with Uncertainty *[Vidal and Fargier, 1997] (STNU) is a 5-tuple* $< N, E, l, u, C >$, *where N,E,l,u are as in a STN, and C is a subset of the edges. The edges in C are called the* causal links, *and the other edges are the* requirement links. *We require* $0 < l(e) < u(e)$ *for each causal link e.*

The requirement that the lower bounds of causal links be positive corresponds to the assumption that causal influences propagate only forward in time. (Only theorem 5 actually makes use of this.) Also notice that if a link *e* is *rigid* (i.e., $l(e) = u(e)$), then there is little point in making e a causal link, since there is no uncertainty in the outcome.

Each set of choices made by Nature for the causal links may be thought of as reducing the STNU to an ordinary STN. Thus, an STNU determines a family of STNs, as in the following definition.

Definition 2 *Suppose* $\Gamma = < N, E, l, u, C >$ *is an STNU. A* projection *[Vidal and Ghallab, 1996] of Y is a Simple Temporal Network derived from F where each requirement link is replaced by an identical STN link, and each causal link c is replaced by an STN link with equal upper and lower bounds* $[b, b]$ *for some b such that* $l(e) \leq b \leq u(e)$.

A boundary *projection [Vidal and Fargier, 1997] is one where, for each causal link e, the above b is chosen so that* $b = l(e)$ *or* $b = u(e)$.

The simplest type of controllability property is concerned with whether some outcomes of the causal choices render it impossible to satisfy the requirements.

Definition 3 *An STNU is* Weakly Controllable [*Vidal and Ghallab, 1996] if every one of its projections is a consistent STN. If an STNU is not Weakly Controllable, we will say it is* Weakly Uncontrollable.

It is shown in [Vidal and Fargier, 1997] that Weak Uncontrollability can be determined in non-deterministic polynomial time, and that in fact the checking may be limited to the boundary projections. (Recall that consistency of a distance graph may be checked in polynomial time by the Bellman-Ford-Moore algorithm [Cormen *et al,* 1990].)

Theorem 1 (Vidal and Fargier, 1997) /f *any projection is inconsistent, then a boundary projection is inconsistent.*

3 Weak Uncontrollability *NP*- Completeness

We now address the issue of showing Weak Uncontrollability is $\mathcal{NP}$-Complete. Since it is known from [Vidal and Fargier, 1997l that the problem is in *AfV,* it remains to show it is $\mathcal{NP}$-Hard.

Theorem 2 *Weak Uncontrollability is AfV-Hard.*

Proof: We show this by reduction of the 3-Coloring Problem, which is known to be AfP-Complete [Cormen *et al,* 1990].

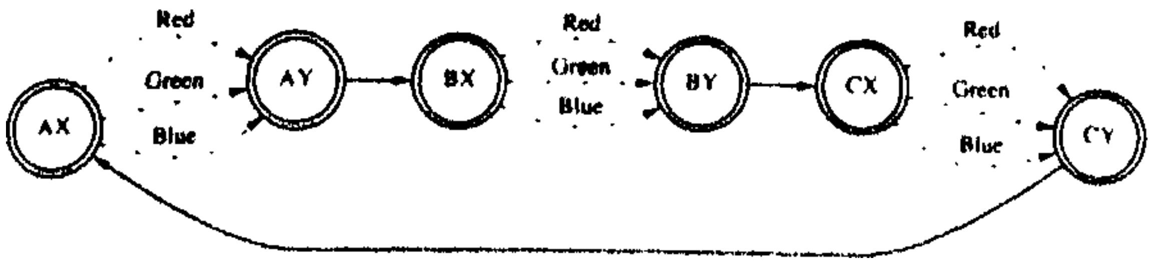

Figure 3: Nodes and Colors Widget.

Our goal will be to construct a mapping from coloring problems to STNU distance graphs such that a solution to the coloring problem maps to a negative cycle in some projection of the STNU, and vice versa. For expository purposes, we adopt the *widget* approach of [Gormen *et* al., 1990].

The first widget describes how nodes and colors from the coloring problem are represented in the STNU distance graph. This is illustrated in figure 3 for a coloring problem with 3 nodes $\{A, B, C\}$. The node *A* from the coloring problem maps to a pair of nodes *AX* and *AY*. The three possible colors of *A* correspond to three concurrent paths from AX to AY as shown. (The labels Red, Green, and Blue do not actually appear in the temporal network; they are merely shown for the convenience of the reader.) A similar mapping is shown for nodes *B* and *C*.

Suppose the coloring problem has *n* nodes $\{A_1, \ldots, A_n\}$ (listed in some arbitrary order). We link all the node widgets to form a cycle by adding temporal edges $A_iY \rightarrow A_{i+1}X$ for $1 \leq i < n$ and $A_nY \rightarrow A_1X$. This is also illustrated in figure 3. Note that each complete set of color choices determines a cycle in the network. Conversely, each cycle selects a single color for each node.

So far, we have not assigned lengths to the distance graph edges. The final edge in the STNU distance graph (e.g., $CY \rightarrow AX$ in figure 3) is given a length of —1. Apart from the mutex widgets (discussed below), all the other edges have zero length. Notice that, so far, all combinations of color choices give rise to slightly negative (value -1) cycles in the distance graph.

Our next task is to represent in the temporal network the mutual exclusion constraints determined by the edges of the coloring problem. (Nodes connected by an edge must have different colors.) We will do this by setting up "mutex" widgets between color paths that correspond to conflicting color choices. Figure 4 shows how to make two color paths mutually exclusive from negative cycles. Here *L* is a positive number large enough so that if a path through the widget has length *L* or greater, then it cannot possibly be on a negative cycle. (Setting $L = +2$ suffices here.) Notice the edges $P \rightarrow Q$ and $Q \rightarrow P$ are each shown with two values for the edge length; these correspond to a causal link from *P* to *Q* with bounds $[2L, 4L]$, and the different lengths are for the different boundary projections.

There are four paths through the mutual exclusion widget that may potentially lie on negative cycles: the two *color* paths $A1 \rightarrow P \rightarrow Q \rightarrow A2$ and $B1 \rightarrow Q \rightarrow P \rightarrow B2$; and the two *short* paths $A1 \rightarrow P \rightarrow B2$ and $B1 \rightarrow Q \rightarrow A2$. Notice that both of the short paths pick up a value of L, so they are definitely not on negative cycles. The two color paths may or may not be on negative cycles.

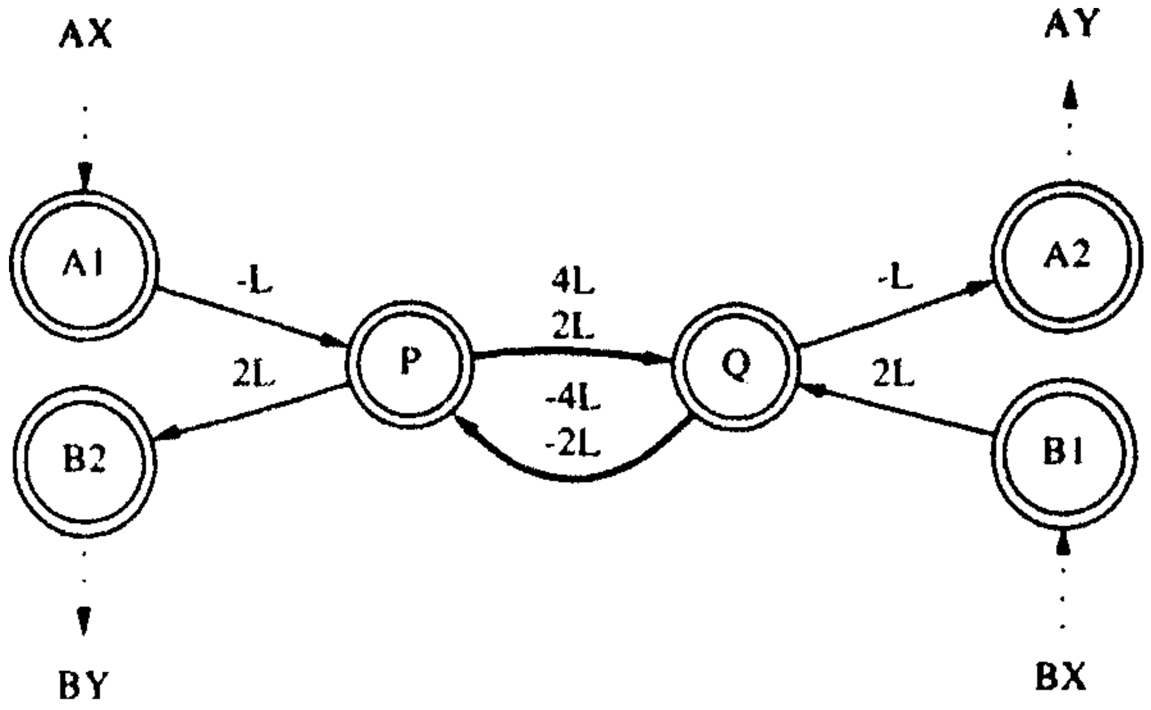

Figure 4: Mutual Exclusion Widget.

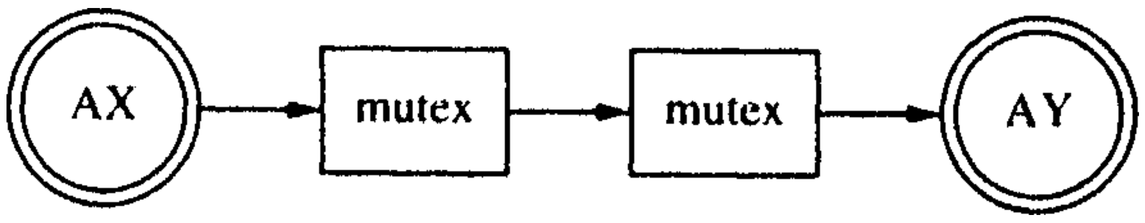

Figure 5: Multiple Constraints.

Recall that to check for Weak Uncontrollability, we need only consider boundary projections, where the *P* —> *Q* causal link has a rigid length of either $2L$ or $4L$. In the distance graph, this corresponds to values of either $+2L$ and $-2L$, respectively, o $+4L$ n d $-4L$, respectively, for the forward (from *P* to *Q)* and backward (from *Q* to *P)* edges. In the first case, the color path from *A* to *A2* has distance 0, while that from $B1$ to $B2$ is $2L$. In the second case, the path from *B* to *B2* has distance 0 while that from .41 to *A2* has distance $2L$, i.e., the values are reversed. Notice that in either case, at most one of the two color paths can be on a negative cycle, i.e., they *mutually exclude* each other from such cycles.

The same node in the color graph may participate in several color constraints. In the temporal network, we represent these by mutual exclusion widgets connected in series, as illustrated in figure 5, where each "mutex" box indicates a mutual exclusion widget between the $AX \rightarrow AY$ color path and some other color path (not shown).

This completes the construction. Now suppose the coloring problem has a solution. We can use the color choices to determine a cyclic path in the distance graph that passes through each mutual exclusion widget at most once, find then traverses the -1 edge. Since there is a projection that sets the chosen passages through the mutual exclusion widgets to zero distance, the cycle will have a total length of - 1 . Thus, the constructed STNU is not Weakly Controllable. Conversely, suppose there is a negative cycle in some projection. This cannot include any of the short paths, since if it did, the cycle would be non-negative. Consequently, the cycle includes only color paths, as illustrated in figure 3. Thus, it selects a single color for each color node. If two of the selections were in conflict, then the cycle would pass through some mutex widget twice, which contradicts the assumption that the cycle is negative. Thus, the color selection provides a solution to the coloring problem.

This reduction of the 3-Coloring Problem shows Weak Uncontrollability is NP-Complete. D

4 Waypoint Controllability

Now we consider a form of controllability in which we distinguish a subset of events, called *waypoints,* whose timing is set independently of the causal outcomes.

Definition 4 *Let* $\Gamma = < N, E, l, u, C >$ *be an STNU. Suppose W is a* won-*empty subset of N. We say* Γ *is* Waypoint Controllable with respect to *W if there is a fixed assignment of time values to the nodes in W that can be extended to a solution in every projection of* Γ*. The nodes in W are called* waypoints.

In [Vidal and Ghallab, 1996], the property of *Strong* Controllability is defined similarly, except a *specific* set of waypoints is used. Their definition corresponds to a waypoint set $C_s \cup (N \setminus C_e)$, where C_s and C_e are the sets of start and end points, respectively, of the causal links. However, Strong Controllability lacks flexibility. For example, if two causal links are *chained* (the end of one is the start of the other), Strong Controllability requires that both the start and end of the earlier causal link be waypoints. However, a fixed assignment to those points cannot be consistent with varying durations of a flexible causal link. Thus, the network is not Strongly Controllable. This limits the usefulness of Strong Controllability in many networks; in the example above, some other set of waypoints, for instance $N \setminus C_e$, may be more realistic.

Note that Weak Controllability is a special case of Waypoint Controllability where the waypoint set is a singleton, since it is always possible to select a single node and give it a fixed value that is extendible to a solution in every consistent projection.

We will shortly prove a useful characterization of Waypoint Controllability. However, some additional definitions are needed. Given an STNU T and a set of waypoints *W*, we can define a Simple Temporal Network S(F, *W)* as follows. The set of nodes of $\mathcal{S}(\Gamma, W)$ is W Between every pair of nodes *x* and *y* in *W*, we place a distance graph edge *e* whose length is given by

$$length(e) = \min\{\delta_p(x,y) \mid p \in P\}$$

where *P* is the set of projections of *T* and $\delta_p(x,y)$ is the shortest-path distance from *x* to *y* in the distance graph of projection p. This leads to the following result, which may be regarded as a generalization of the result in [Vidal and Ghallab, 1996] concerning Strong Controllability in a restricted network.

Theorem 3 *V is Waypoint Controllable with respect to W if and only if it is Weakly Controllable and* $\mathcal{S}(\Gamma, W)$ *is a consistent STN.*

Proof: Suppose T is Waypoint Controllable with respect to *W.* Then there is a fixed assignment $s : W \to \mathbb{R}$ that can be extended to a solution in every projection of T. It follows that every projection has a solution, i.e., F is Weakly Controllable. It also follows that $s(y) - s(x) \leq \delta_p(x,y)$ for every projection p, where δ_p gives the shortest-path distances as before. Hence, $s(y) - s(x) \leq \min\{\delta_p(x,y) \mid p \in P\}$. Thus, *s* gives a solution to $\mathcal{S}(\Gamma, W)$.

Conversely, assuming 5 is a solution to $\mathcal{S}(\Gamma, W)$, the argument can be reversed to deduce that $s(y) - s(x) \leq \delta_p(x,y)$ for every projection p, and every *x* and *y* in *W.* By Weak Controllability, every projection is consistent, so by Decomposability [Dechter *et* al., 1991], *s* can be extended to a solution for every projection. □

It can be shown that in the $\min\{\delta_p(x,y) \mid p \in P\}$ minimizations, the set *P* may be restricted to boundary projections without affecting the minimum value. Nevertheless, the cardinality of *P* is generally exponential in the size of the problem. Indeed, the proof of Theorem 2 can be adapted to show that the minimization problem is $\mathcal{NP}$-hard. However, theorem 3 leads to a propagation algorithm for computing Waypoint Controllability, and there are important special cases where it runs in polynomial time.

5 Propagation Algorithm

The propagation algorithm is an elaboration of the approach used for checking STN consistency [Dechter *et* a*l*., 1991; Cormen *et* al., 1990], and requires that every node be reachable from the initial node, which must be selected to be a waypoint. The basic idea is that we can perform a separate propagation for each projection until we reach a waypoint. At the waypoint, the propagated values for the different projections can be merged, and only the minimum value carried forward. The process continues with new independent propagations to the next waypoint, and so on. Notice that the updates at the waypoints simulate an ordinary STN propagation in S(F, *W).* Both STN inconsistency and Weak Uncontrollability imply a negative cycle, so propagation to quiescence is a reliable indicator of controllability by theorem 3.

Actually, we can do even better if we observe that large segments of the propagation are the same for projections that arc closely related. This suggests sharing the work of propagation between the projections. We do this by associating a tag with each propagated value. The tag carries a description of which causal links have been traversed so far, and which of the boundary values were used in the traversals. We say one tagged value *subsumes* another if its tag is as general (i.e., its causal choices are a subset of the other) and its value is as good (less than or equal to the other). Subsumed tagged values are subject to deletion. Otherwise, the tagged propagations do

```
procedure Propagate-Controllability
begin
   Queue <- {};
   Propagate-Value (0, {}, Initial-Node, Queue);
   while Queue is Non-Empty do
   begin
      <X,V,T> <- Pop (Queue);
      if Bellman-Ford bound is exceeded then
         report Uncontrollability and halt;
      for each causal edge E:X->Y from X do
      begin
         if E/2 does not occur in T then
            Propagate-Value (V+length1(E), T U {E/1}, Y, Queue)
         if E/1 does not occur in T then
            Propagate-Value (V+length2(E), T U {E/2}, Y, Queue)
      end
      for each non-causal edge E:X->Y from X do
         Propagate-Value (V+length(E), T, Y, Queue);
   end
end

procedure Propagate-Value (Val, Tag, X, Queue)
begin
   TV <- (if X is Waypoint then <Val,{}> else <Val,Tag>;
   if TV is not subsumed by an existing tagged value of X then
   begin
      Delete tagged values of X subsumed by TV;
      Add TV to tagged values of X;
      Add <X,val(TV),tag(TV)> to Queue;
   end
end
```

Figure 6: Algorithm for Way point Controllability

not interact until they reach a waypoint, where the tags are reset to empty. The usual Bellman-Ford cutoff terminates the propagation in cases where the network is not controllable. Figure G shows a pseudo-code description of the algorithm. Note that lengthl(E) and length2(E) are the lengths of E corresponding to the two boundary values, and the choice descriptors E/1 and E/2 indicate which of those was used in the propagation.

Similarly to the ordinary STN case, the propagation algorithm can be modified to propagate upper and lower time bounds. Thus, it potentially provides not just a single solution for the waypoints, but a flexible set of acceptable time assignments with the same decomposability property as for an ordinary STN. An execution algorithm can make use of this flexibility [Tsamardinos *et a/.*, 1998] to help manage contingencies that are unanticipated, in addition to the formally represented uncertain durations considered here.

Note that in general the number of possible tags grows exponentially with the number of causal links. Thus, in the worst case, the time and space complexities are exponential. Fortunately, the waypoints can serve to limit the complexity, as seen in the following result.

Theorem 4 *Suppose M is the maximum number of causal links that can occur in a path 'with no interior waypoints. Then the complexity of the Waypoint Controllability propagation algorithm is* $O(M^2 2^{2M}|N||E|)$.

Proof: Since the tags get reset at waypoints, the size of tags is limited to the number of causal links encountered between waypoints. Under the condition of the theorem, at most 2^M different tagged values can reside at any node. The update across an edge then requires at most $O(2^{2M})$ subsumption tests, where each test can be done in $O(M^2)$ time. The Bellman-Ford bound restricts the number of edge updates to $|N|\cdot|E|$ [Cormen *et al.*, 1990]. Thus, the total complexity is $O(M^2 2^{2M}|N||E|)$. □

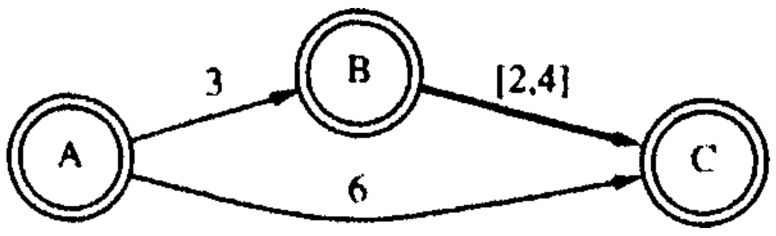

Figure 7: Need for Tags

Corollary 4.1 *The algorithm can determine Strong Controllability in polynomial time.*

Proof: The waypoint set used in Strong Controllability has a waypoint at the start of each causal link. Thus, at most two causal links can occur in a path between waypoints, so *M* is bounded for this class of problems. □

We remark that a "no U-turn" restriction [Muscettola *et* a/., 1998b] is insufficient to avoid interference between projections in computing minimum distance. For example, in figur 7, where $B \to C$ is a causal link, the propagation $A \to B \to C$ would produce a value of 5 at C, which blocks the propagation $A \to C \to B$ that would provide a shorter distance of 2 from *A* to *B.* This kind of interference is avoided by the tag mechanism used here.

6 Dynamic Controllability

Note that Waypoint Controllability requires the setting of times for non-waypoint events to depend on the causal outcomes. However, this is only feasible in practice if information about the causal outcomes is available in time. One scenario is where precise information about causal outcomes is obtained by observation when they occur. This corresponds to a property called *Dynamic Controllability* defined in [Vidal and Fargier, 1997]. Informally, an STNU is Dynamically Controllable if there is a mapping from projections to solutions such that the partial solution up to any time depends only on the durations of the causal outcomes that have completed by that time. We are interested in conditions under which Waypoint Controllability implies Dynamic Controllability.

The following conditions will be relevant. Consider the distance graph of a Waypoint Controllable network. We say a node *x* is *insulated* from another node *y* if every path from *y* to J- passes through a waypoint. (Note that a waypoint is therefore insulated from every node.) A node *x* is *downstream* from a node *y* if, in every projection, there is a path of negative length from *x* to *y.* (We will also say *y* is *upstream,* of *x* in this case.) Note that if *x* is downstream from y, then *x* must occur later than *y* in any solution of any projection. Finally, a node *x* is *protected* from *y* if *x* is either downstream from *y* or is insulated from *y.* This leads to the following result.

Theorem 5 *Let T be an STNU that is Waypoint Controllable with respect to some set of waypoints. Suppose* *every node x is either itself a causal link endpoint, or is protected from every causal link endpoint. Then* Γ *is Dynamically Controllable.*

Proof: From the definition of Waypoint Controllability, there is a fixed assignment of times to the waypoints that is consistent with every projection. Consider any projection *p*. We can obtain a solution of *p* by propagating from the fixed times of the waypoints, and then choosing the latest possible time for each node [Dechter *et* a/., 1991]. (Ensured to be finite by adding a source vertex as in [Cormen *et* a/., 1990, page 541].) This defines a mapping Ψ from projections to solutions.

Let *x* be any node. Suppose pi and *p2* are two projections that agree on causal outcomes that are completed no later than x. Then they must agree on outcomes of causal links whose endpoint is upstream from x\ We wish to show that $\Psi(p1)$ and $\Psi(p2)$ must agree on the timing of *x*. We prove this by induction on the partial ordering imposed by the downstream relation, which allows us to suppose by induction that $\Psi(p1)$ and $\Psi(p2)$ agree on the timing of all nodes upstream of *x*.

First consider the case where *x* is the endpoint of a causal link *e*. Let *x'* be the startpoint of *e*. By the definition of STNU, /(e) > 0. Thus, *x'* is upstream of *x*. It follows that $\Psi(p1)$ and $\Psi(p2)$ agree on the timing of *x'*. Also *pi* and p2 agree on the outcome of *e,* since it is completed no later than *x*. Thus, $\Psi(p1)$ and $\Psi(p2)$ agree on the timing of *x*.

Otherwise, *x* is not the endpoint of a causal link. Let *e* be any causal link, and suppose its endpoint is *y*. By the condition of the theorem, *x* is either downstream of *y* or is insulated from *y*. In the former case, *y* is upstream of *x*, so by the inductive hypothesis $\Psi(p1)$ and $\Psi(p2)$ agree on *y*. Thus, any value propagated from *y* to *x* will be the same for *pi* and *p'2.* If on the other hand *x* is insulated from *y*, then any propagation from *y* to *x* is blocked by the fixed assignments given the waypoints. Thus, .;: will have the same propagated window for both *p* and p2, and so $ will assign *x* the same time value in both cases.

It follows that the value of *x* depends only on causal outcomes that are completed no later than x. Thus, *T* is Dynamically Controllable. □

Corollary 5.1 (Vidal and Fargier, 1997) *Strong Controllability implies Dynamic Controllability.*

Proof: Strong Controllability corresponds to a set of waypoints such that every node is either a, causal link endpoint or a waypoint. In the latter case it is automatically insulated from every node. □

7 Closing Remarks

Waypoint Controllability provides a useful analysis tool that can be used by a planner to decide whether granting the executive access to uncertain events compromises controllability of a flexible plan. We have shown that the cost of controllability analysis can be affordable, provided that the planner judiciously manages the placement of uncontrollable links in the plan topology.

One important consequence of Waypoint Controllability is the possibility of devising more general and principled guidelines on the introduction of wait periods in a plan. The end time point of a suitable wait link can be made a waypoint. Therefore, appropriately located wait periods can partition a plan into compartments that are insulated from each other as far as the effects of uncertain events are concerned.

Previous work on Strong Controllability restricted the plan topologies for which controllability is guaranteed, and severely limited the extent to which an executive can react to uncertain events. For example, Strong Controllability does not allow an executive to have access to an event linking two uncontrollable tasks. Waypoint Controllability shows that such limitations can be lifted provided that small subnetworks of uncontrollable links (with arbitrary topology) are insulated from the rest of the plan through the appropriate use of wait periods.

Acknowledgments

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