A Potts Spin MFT Network Solving Multiple Causal Interactions

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Abstract

In this paper, we propose a Potts spin Mean Field annealed network to address the open, independent and incompatibility classes of causal reasoning (also said abduction, abductive diagnosis). The strong feature of the current work is its characterization of the reasoning task in these classes by an energy/target function. Computation of a scenario (also said explanation) is done by means of Mean Field equations. The application of the model to small and large-scale causal problems reveals its efficacy and robustness in handling varied and multiple causal interactions.

Keywords: Causal Reasoning, Artificial Neural Networks, Mean Field Theory, Potts spin Modeling, Open / Incompatibility / Independent Causal Interactions.

1 Prologue

Causal reasoning (also known as abduction [Thagard, 1989; Ayeb *et al.,* 1998]) is ubiquitous in diagnosis [Peng and Reggia, 1989], scientific reasoning [Thagard, 1989], natural language processing [Charniak, 1987], etc. The intuition behind causal reasoning can be stated as follows ([Goel *et al.*, 1995; Ayeb *et al.*, 1998]). Given an elementary cause c, an observable effect e and the knowledge that *c* causes e, causal reasoning consists in hypothesizing that c occurred. The key feature of causal reasoning is to infer a set of *composite* causes from the elementary ones. These composite causes are said to form a *scenario* (explanation in [Thagard, 1989; Goel *et al.*, 1995]). Unfortunately, computation of a *best* scenario is NP-complete [Bylander *et al.,* 1991]. Hence, the computational time becomes prohibitive, particularly in the presence of incompatibility interactions [Goel *et* al., 1995; Bylander *et al.,* 1991].

Artificial Neural Networks (ANNs) are biologically motivated computational models that turned out to be efficient in solving many NP-complete optimization problems. Mean Field Theory (MFT) [Peterson and Anderson, 1987] is receiving an increasing attention, mainly due to its robustness in solving many large-scale NP-complete optimization problems other methods failed to solve in general; e.g. the "Traveling Salesman" and the "Graph Partitioning" [Peterson and Soderberg, 1989]. MFT is a combination of the Simulated Annealing (SA) of the Boltzmann machine [Kirkpatrick *et* al., 1983] and the Hopfield's network [Hopfield, 1984]. Hopfield's network is mainly characterized by the simplicity of its neurons and its efficient hardware implementation. However, a major drawback of Hopfield's network is its deterministic nature so that the network gets easily trapped into poor local minima as the problem size increases. SA is a stochastic hill-climbing algorithm that uses a gradient descent combined with probabilistic movements. The simulated annealing process consists of first melting the system at high temperatures, then slowly decrease its temperature until the neurons "crystallize" and formulate a solution for the case at hand. However, due to the stochastic and annealing processes, the computational time could be unacceptable. Merging both Hopfield's network and SA, MFT has shown to be efficient in finding optimal solutions for academic and real-world A/'P-complete problems. Another important issue of MFT is its ability to compute optimal (or near optimal) solutions in practical time spans even for large-scale problems.

In the current work, we use Potts spin MFT annealed networks to address the open, independent and incompatibility classes of causal reasoning. The strong feature of the current work is the characterization of the reasoning process in these classes by an energy function. The remainder of this paper is organized as follows. In section 2, we outline the minimal preliminary material and give a real-world causal problem. In Section 3.1, we formulate the energy function for open, independent and incompatibility causal problems. In section 3.2, we use MFT Potts modeling and mean field equations to minimize our energy function. Section 3.3 describes our algorithm and section 3.4 gives illustrations. Finally, the paper is concluded in section 4.

2 Background

The main purpose of this section is to formalize the open, independent and incompatibility classes of causal reasoning and to give an introductory causal problem.

2.1 Preliminaries

A causal problem is entirely described by a set of possible effects $\mathcal{E} = \{e_1, \cdots, e_m\}$, a set of possible causes $\mathcal{C} = \{c_1, \cdots, c_n\}$, and a set of causal interactions $\mathcal{R}$. Several and multiple causal interactions could be modeled in $\mathcal{R}$ and thereby several classes of causal problems could introduced. Formal definitions follow.

Definition 2.1 *We use $\varphi \in \mathcal{R}$ to denote a causal interaction in $\mathcal{R}$.*

- *φ is an **independent** interaction if it is expressed as: $c \xmapsto{w} e$*
- *φ is **incompatibility** interaction if it is expressed as: $c_i \wedge c_k \xmapsto{w'} \perp$*

where w (re: w') is a real number measuring the strength of the causality (resp. incompatibility) between cause c and effect e (resp. causes a and Ck)

In Definition 2.1, $\perp$ denotes the *null* effect, i.e. an effect that can *never* be observed. Now, we express the concept of a manifestation (observation in [Thagard, 1989; Goel *et al,* 1995]) for a causal problem as follows.

Definition 2.2 *Let* $\mathbf{CP}\langle\mathcal{E},\mathcal{C},\mathcal{R}\rangle$ *be a causal problem. A manifestation MM for* CP *is a three-tuple {Ep,EA,Eu), where Ep, EA and EA are the three finite sets of present, absent and unknown effects, respective* ■

According to Definition 2.2, each possible effect in £ is assigned a. state. It is present if it is known to occur, it is absent if it is known not to occur, and it is unknown if we have no "knowledge" about it. In a practical situation, an effect is unknown if its "value" cannot be accessed for some reasons[1]. Now, we are ready to define the independent, open and incompatibility classes of causal problems as follows.

Definition 2.3 *Let* $\mathbf{CP}\langle\mathcal{E},\mathcal{C},\mathcal{R}\rangle$ *be a causal problem. Let* $MM\langle E_P, E_A, E_U\rangle$ *be a manifestation for* CP, *then*

- CP *belongs* **independent** *class if* $\forall\varphi \in \mathcal{R}$, φ *is an independent interaction.*
- CP *belongs* **incompatibility** *class if* $\exists\varphi \in \mathcal{R}$ *such that* φ *is an incompatibility interaction.*
- CP *belongs to the open class* $E_A \neq \emptyset$ ■

[1] Among reasons; let us mention safeness, time.

Now, we need the following notation inspired from [Bylander *et al.,* 1991] to define the concept of a *scenario* (explanation in [Thagard, 1989; Peng and Reggia, 1989)).

Definition 2.4 *Assume that cause c covers effects* e_1 *and* e_2*, i.e.* $c \xmapsto{w_1} e_1$, $c \xmapsto{w_2} e_2$. *We define* COVER, *a function from* $\mathcal{C}$ *to* $\mathcal{E}$, *as:* $\text{COVER}(c) \stackrel{def}{=} \{e_1, e_2\}$. *If C denotes a subset of elementary causes, then* $\text{COVER}(C) \stackrel{def}{=} \bigcup_{c \in C} \text{COVER}(c)$. ■

Definition 2.5 *Let* $\mathbf{CP}\langle\mathcal{E},\mathcal{C},\mathcal{R}\rangle$ *be an independent, incompatibility and/or open causal problem. Let* $MM\langle E_P, E_A, E_U\rangle$ *be a manifestation for* **CP**. *A scenario for CP w.r.t. MM is a subset of elementary causes* $C \in \mathcal{C}$ *such that: (i)* $E_P \subseteq \text{COVER}(C)$*; (ii) C is minimal; (iii)* $\perp \notin \text{COVER}(C)$*; (iv)* $E_A \cap \text{COVER}(C)$ *is minimal. If among all scenarios, C optimizes a given belief function measuring the overall quality of scenarios, then C is called a best scenario.* ■

In Definition 2.5, the minimality criterion is taken w.r.t. set cardinality. One should remark the potential conflict between criteria (i),(u),(m) and *(iv).* Unfortunately, it is not always guaranteed to meet all these criteria in a single scenario. This explains why characterization of a *best* scenario carries certain arbitrariness [Goel *et al*., 1995]. However, a precedence relationship could be adopted. For our concern, we adopt a decreasing precedence relation for criteria *(i)* to *(iv)* with criterion *(i)* assigned the highest precedence.

2.2 A Medical Causal Problem

$$\mathcal{R} = \begin{cases} (\varphi_1) & laryngitis \xmapsto{0.5} inflamed\_throat \\ (\varphi_2) & laryngitis \xmapsto{0.5} expectoration \\ (\varphi_3) & laryngitis \xmapsto{0.5} cough \\ (\varphi_4) & laryngitis \xmapsto{0.45} lost\_voice \\ (\varphi_5) & pneumonia \xmapsto{0.25} fever \\ (\varphi_6) & pneumonia \xmapsto{0.85} cough \\ (\varphi_7) & pneumonia \xmapsto{0.5} headache \\ (\varphi_8) & pneumonia \xmapsto{1.0} dyspnea \\ (\varphi_9) & pneumonia \xmapsto{0.5} expectoration \\ (\varphi_{10}) & pneumonia \xmapsto{1.0} thoracic\_pain \\ (\varphi_{11}) & sinusitis \xmapsto{0.5} headache \\ (\varphi_{12}) & sinusitis \xmapsto{1.0} noise\_pain \\ (\varphi_{13}) & sinusitis \xmapsto{0.5} nasal\_discharge \\ (\varphi_{14}) & tonsillitis \xmapsto{0.25} inflamed\_throat \\ (\varphi_{15}) & tonsillitis \xmapsto{0.25} fever \\ (\varphi_{16}) & sinusitis \wedge pneumonia \xmapsto{1.0} \perp \\ (\varphi_{17}) & laryngitis \wedge tonsillitis \xmapsto{0.85} \perp \end{cases}$$

Figure 1: The Set of Causal Interactions for CPMD

For the sake of illustration, we consider $\mathbf{CP_{MD}}\langle\mathcal{C},\mathcal{E},\mathcal{R}\rangle$, a relatively small causal problem from the medical domain. $\mathcal{E} = \{cough, dyspnea, expectoration, fever, inflamed\_throat, headache, lost\_voice, nasal\_discharge, nose\_pain, thoracic\_pain\}$ are the possible symptoms (effects in our model). $\mathcal{C}$ = *{laryngitis,* *pneumonia, sinusitis, tonsillitis}* are the possible diseases (causes in our model). The set of causal interactions $\mathcal{R}$ is summarized in Figure 1. For instance, the causal interaction φ_8 tells us that *pneumonia* causes *dyspnea* with the maximal causality 1, whereas *(fin* tells us that *laryngitis* and *tonsillitis* are incompatible diseases with a relatively high degree, 0.85. This means that *laryngitis* and *tonsillitis* are incoherent and cannot coexist together. One should remark that the causal interactions in $\mathbf{CP_{MD}}$ (as well as in our modeling) are a matter of degree rather than being either/or (cause/not-cause, compatible/incompatible). This allows flexibility since real-world causal interactions are generally modeled from scratch.

3 Proposal

3.1 Formulation of the Energy Function

Let $MM\langle E_P, E_A, E_U\rangle$ be a manifestation for $\mathbf{CP}\langle\mathcal{E},\mathcal{C},\mathcal{R}\rangle$, the causal problem at hand. We use j and *l* to index all quantities related to effects, while we use i, k and n to index all quantities related to causes. *card(X)* denotes the cardinality of the finite set X. $w_{ij} \in]0,1]$ is the strength of the causality between caus c_i and effect e_j. We denote by $\theta_{ik} \in]0,1]$ the strength of the incompatibility (or incoherence) between cause c_i and c_k. $\Psi = (\psi_1, \cdots, \psi_l)$ measures our belief in the presence (resp. absence) of effects in *Ep* (resp. E_A). Unknown effe (E_U) ave a null certainty degree, i.e. $\psi_j = 0\ \forall e_j \in E_U$. For each *present* effect e_j, we associate a set of discrete variables s_{ij}, defined as follows:

$$s_{ij} = 1 \quad \text{If } e_j \in \textsc{Cover}(c_i) \qquad \text{a)}$$
$$s_{ij} = 0 \qquad \textbf{otherwise} \qquad (2)$$

The variables s_{ij} are called Ising spin variables. S is the vector of variables s_{ij}.

We define the "cover level" of a cause c_i w.r.t. *MM* as follows:

$$E_i(\mathbf{S}) = \sum_{e_j \in E_P} w_{ij}\psi_j s_{ij} = \sum_{e_j \in E_P} c_{ij} s_{ij} \qquad (3)$$

where $c_{ij} = w_{ij}\psi_j\ \forall j$. E_i is null if c_i does not cover any of the present effects. The vector of cover levels is E = $[E_1, E_2, \cdots, E_N]^T$. Hence, the cover level of all possible causes in *C* is computed by:

$$J_1(\mathbf{S}) = \|\mathbf{E}\|^2 = \sum_i E_i^2 \qquad (4)$$

Maximizing *J1* (S) is equivalent to putting "on" all Ising spin variables. Hence, for each effect one or more Ising spin variables, and thereby one or more causes, are active. This kind of competition is referred to in ANNs as *wnner-tokes-others* (WTO), since more than one competing cause are active at the end of the competition. However, a *winner-takes-all* (WTA) competition is necessary to ensure that a cause takes exclusivity in the coverage of a given effect, and thereby meet the minimal cardinality criterion. One possible way is to considerably limit the resources of competition between causes, i.e. to activate the minimal number of Ising spin variables for each present effect. The number of active Ising spin for effect e_j is given by $\Gamma_j = \sum_i s_{ij}$. However, in order to ensure a coverage of e_j, at least one corresponding Ising spin must be active. For this, we define the following constraint:

$$J_2(\mathbf{S}) = \sum_{e_j \in E_P} (\Gamma_j - 1)^2 \qquad (5)$$

Maximizing Ji(S) in (4) under the constraint Ja(S) in (5) will push each cause to maximize its individual cover level and to take the *exclusivity* in covering the present effects. Causes with the highest cover levels are more likely to win the competition than those with the lowest ones.

The open class of causal reasoning is modeled as follows. For each cause c_i, we define a *penalty factor* λ_i by:

$$\lambda_i = \frac{1}{1 + \sum_{ej \in E_A} w_{ij}\psi_j}\ \forall c_i \qquad (6)$$

$\lambda_i = 1$ if c_i does not cover any absent effect, and $\lambda_i \ll 1$ otherwise. We use these factors to weight the cover levels of causes. Thus, we redefine equation (4) to:

$$J_1(\mathbf{S}) = \mathbf{E}^T \mathbf{\Lambda} \mathbf{E} \qquad (7)$$

where $\mathbf{\Lambda} = diag[\lambda_1, \cdots, \lambda_N]$. From equation (7), one can see that causes are strongly penalized if they cover some absent effects. Consequently, these causes enter the competition less effectively, and therefore are more likely to loose it, than other causes than do not cover absent effects.

In computation of a scenario, we should avoid simultaneous activation of *incompatible* causes. For this, we define the following constraint:

$$US) = \mathbf{E}^T \mathbf{\Theta} \mathbf{E} \qquad (8)$$

where $\mathbf{\Theta} = [\theta_{ik}]$ is the matrix of incompatibility interactions. We should note that $\mathbf{\Theta}$ is null-diagonal, i.e. θ_{ii} = 0. In fact, each cause is totally compatible with itself by definition. Minimizing Js(S) would prevent a simultaneous activation of incoherent causes. Particularly, *J3(S)* is null if all active causes are totally coherent.

Using equations (7), (5) and (8), we characterize the reasoning process in the open, independent and incompatibility classes of causal reasoning by the following energy function:

$$J(\mathbf{S}) = -\frac{1}{2}\mathbf{E}^T \mathbf{\Lambda} \mathbf{E} + \frac{\alpha}{2} \sum_{e_j \in E_P} (\Gamma_j - 1)^2 + \frac{\beta}{2}\mathbf{E}^T \mathbf{\Theta} \mathbf{E} \qquad (\bullet)$$

where α and β are Lagrangian multipliers used to weight the distinct constraints in the energy function J(S). Due to the precedence relationship between the coverage (see Definition 2.5), minimal cardinality and coherence criteria, one must have $0 < \alpha < \beta < 1$. Minimization of the energy function J(S) in equation (9) leads to inference of a minimal set of coherent causes covering the present effects and avoiding coverage of the absent ones.

Obviously, minimization of the energy function J(S) in equation (9) is not a trivial matter due to the huge number of possible states. A state is defined by the Ising spin variables *sij*. Naturally, the complexity of the search space is problem-/case-dependent. Hereafter, we derive its expression in the worst-case, i.e. assuming that all effects are present and that each effect is covered by all possible causes. In this case, the dimension of the search space is $2^{N_c \times N_e}$, where N_c = *card(C),* and N_e = $card(\mathcal{E})$. Consequently, the remaining task is to employ a robust method to optimize our energy function: by robustness here, we mean the ability of finding optimal solutions in practical time spans. Evidently, the use of sequential search approaches is inappropriate and indeed impossible for large-scale causal problems. Gradient descent methods which are local in scope are not applicable since they can be easily trapped into poor local minima. Mean Field annealing techniques have been shown to be robust in solving large-scale combinatorial optimization problems as the the "Traveling Salesman" and "Graph Multi-Partitioning" [Peterson and Soderberg, 1989], to cite just a few. For details about MFT methods, we refer the interested reader to the specialized literature, for example [Peterson and Anderson, 1987], [Peterson and Soderberg, 1989]. In the next section, we use MFT Potts spin modeling to derive the mechanics of our model.

3.2 Potts spin MFT Modeling

The Ising spin variables *Sij* are replaced by continuous variable v_{ij} representing their *thermal averages;* i.e. $v_{ij} =< s_j >_T$, where T is said the annealing "temperature" of the system. At a given T, the probability *Vij* obeys a Boltzmann distribution:

$$v_{ij} = \frac{exp(-\phi_{ij}/T)}{\sum_m exp(-\phi_{mj}/T)} \quad (10)$$

where ϕ_{ij} is given by:

$$\phi_{ij} = \frac{\partial J(\mathbf{V})}{\partial v_{ij}} \quad (11)$$

The variables *<f>ij* are called the *mean field variables, vij* are called Potts spin variables and their probabilistic interpretation is evident.

Due to equation (10), the energy of the network J(V) in equation (9) is transformed to:

$$J(\mathbf{V}) = -\frac{1}{2}\sum_i \lambda_i E_i^2 + \frac{\beta}{2}\sum_{k \neq i} \theta_{ik} E_i E_k + \sum_{i,j} v_{ij}(1 - v_{ij}) \quad (12)$$

Let us first remark that due to equation (10), the constraints Γ_j = 1 holds automatically. That's why we have omitted the α**-term** in (12). Due to the continuous nature of v_{ij}, the last term in (12) is added to *for*v_{ij} variables to converge either to 0 or 1 - see [Wang and Ansari, 1997] for a similar approach. Using equations (11) and (12), the mean field variables ϕ_{ij} are given by:

$$\phi_{ij} = c_{ij}\left[-\lambda_i E_i + \beta \sum_{k \neq i} \theta_{ik} E_k\right] + 1 - 2v_{ij} \quad (13)$$

Using equation (10), we can see that $v_{ij} \rightarrow 1/\kappa_j$ when $T \rightarrow +\infty$, wit κ_j eing the number of causes covering e^. As $(T \rightarrow 0)$,, fixed points {0,1} emerge. Potts spin MFT modeling has many advantages for causal reasoning. First, due to formula (10), the dimension of the search space is reduced from $2^{N_c \times N_e}$ to $N_c^{N_e}$. Second, it ensures that a present effect *ej* is *always* covered by some active causes since $\sum_m v_{mj} = 1$ always holds. In the next section, we describe the mechanics of our algorithm.

3.3 The Algorithm

Our algorithm is summarized in Figure 2. In the initialization phase, we compute the λ_i factors due to absent effects. Moreover, we set T^0, ρ and β. We should note that one can use a *progressive* β to emphasize constraint violations at low temperatures, i.e. $\beta \propto 1/T$. The Potts spin variables are initialized to their high-thermal averages and some small random bias is added to "break the symmetry" of the system of neurons (if any). In fact, in

▶ **INITIALIZE**
- Compute λ_i by formula (6) $\forall c_i$;
- T^0; $\beta \in]0,1[$; $\rho \in]0,1]$; $\delta \in [0.8, 0.99]$;
- $t = 0$ ("time-counter");
- $v_{ij} = \frac{1}{\kappa_j} + 0.001 * rand[0,1]$;

▶ **REPEAT**
- $E_i = \sum_{e_j \in E_P} c_{ij} v_{ij}$ $\forall c_i$;
- $F_i = \sum_{k \neq i} \theta_{ik} E_k$ $\forall c_i$;
- $\forall e_j \in E_P$ Do:
  - a) $sum\_spins = 0$; $\forall c_i$ Do:
    - $\phi_{ij}^t = c_{ij}\left[-\lambda_i E_i + \beta F_i\right] + 1 - 2v_{ij}$
    - $v_{ij}^t = exp(-\phi_{ij}^t/T)$;
    - $sum\_spins = sum\_spins + v_{ij}^t$;
  - b) $v_{ij}^t = \rho\left(v_{ij}^t/sum\_spins\right) + (1-\rho)v_{ij}^{t-1}$;
- $T^t = \delta T^{t-1}$; $t = t + 1$;

UNTIL
$(\Sigma = \frac{4}{K}\sum_{e_j \in E_P}\sum_m (v_{mj}^t - 0.5)^2) \geq 0.99$;
or $(\Delta \mathbf{V} = \frac{1}{K}\sum_{ij}(v_{ij}^t - v_{ij}^{t-1})^2) \leq 0.001$;
or $(T \approx 0.001)$;

▶ **RETURN**($\{c_i$ **such that** $E_i > \epsilon\}$);

Figure 2: Solving Multiple Causal Interactions

some cases, more than one equivalent solutions exist for the causal problem at hand, and without an unbiased start the algorithm is unable to choose a direction to follow.

Thereafter, we update *synchronously* mean field and Potts spin variables. In computing Potts spin variables, we use a "time-step" ρ to avoid the "flip-flop" behavior of the synchronous mode [Peterson and Soderberg, 1989]. However, one has to set ρ as close as possible to 1 since this results in a slower convergence by a factor ρ.

The algorithm terminates when all Potts spin variables saturate at 0/1, the rate of change Δ or the temperature of the system are both small enough. In figure $\mathcal{K}$ is the total number of Potts spin variables and is given by $\mathcal{K} = \sum_{c_j \in E_P} \kappa_j$. Regarding the saturation criteria Σ, one can see that:

$$\lim_{v_{ij} \to 0/1} \Sigma = 1$$

The output of our algorithm consists of those causes surviving the competition; i.e. whose cover levels are not null. Hereafter, we illustrate the mechanics of our algorithm on a medical causal problem.

3.4 Illustration

In this section, we run our algorithm on CPMD? the medical problem introduced in section 2.2. In our simulation, we used the following parameters: the cost-term β = 0.5, the initial temperature T(0) = 2.0, the "time-step" ρ = 1.0. Moreover, we have used the annealing scheme $T^{t+1} = 0.92\, T^t$, where t is the "time-counter".

We considered a manifestation *MM(Ep, EA,EU),* where E_P = *{cough, expectoration, inflamedJhroat, headache, lost-voice, nose-pain], EA = {dyspnea, fever}* and E_U = *{nasaLdischarge, thoracic-pain). nose-pain* somewhat occurs. This is modeled by assigning *nose-pain* a degree of ψ_{nose_pain} = 0.4. The rest of the present effects are assumed to occur with

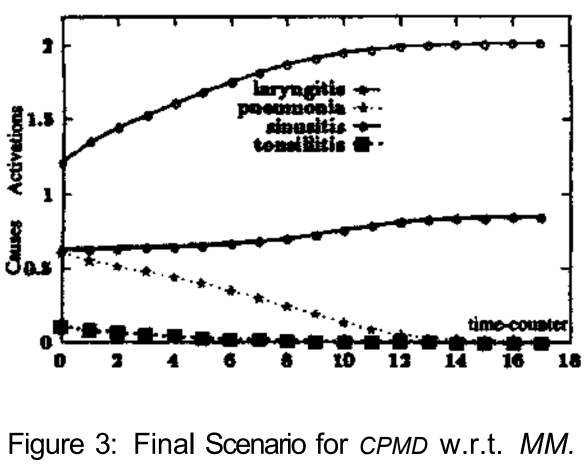

Figure 3: Final Scenario for *CPMD* w.r.t. *MM.*

a relatively high degree of certainty, 0.9. *dyspnea* and *fever* are absent with relatively weak degrees of certainty: $\psi_{dyspnea}$ = 0.2 and ψ_{fever} = 0.3. Remember that unknown effects *{nasaLdischarge* and *thoracic-pain* in this case) are automatically assigned a null degree of certainty by the algorithm. The computed scenario for *MM* by our algorithm is *SCENARIOMD* = **{laryngitis, sinusitis}** since these are the diseases (causes) winning the competition - see Figure 3. *laryngitis* enters the competition with a strong activation since it ensures coverage of the majority of the effects in *Ep.* Tins allowed *laryngitis* to endure the competition more effectively than the rest of the diseases and to be included in *SCENMD.* In contrast, *tonsillitis* starts to compete with a weak activation since it only covers *inflamedJhroat* In addition, *tonsillitis* is penalized since it covers the *absent* effect *fever.* Hence, *tonsillitis* does not survive the competition for long and is dragged down toward 0 around the *1th* iteration - see Figure 3. *sinusitis* and *pneumonia* start the competition with the same activation level. However, *sinusitis* sustain the competition mainly due to its exclusivity in covering the present effect *nose-pain* - see Figure 1. Due to its incompatibility with *sinusitis,* and to its coverage of the absent effects *dyspnea* and /ever, *pneumonia* is slowly "weakened" as the competition continues and "dies" around the *16th* iteration - see Figure 3. One should remark that *SCENMD* (1) ensures coverage of *all* present effects, (2) is composed only of coherent diseases, (3) avoids cover of *all* absent effects, and (4) is minimal w.r.t. set cardinality since removing either *sinusitis* or *laryngitis* from *SCENMD* will lead only to a partial coverage of *Ep.* Remark that *SCENMD* is the *best* scenario that can be proposed. In fact, other scenarios as *{tonsillitis, sinusitis)* or *{tonsillitis, sinusitis, pneumonia}* will respectively lead only to a partial coverage of *Ep* or to a scenario including incoherent diseases.

4 Epilogue

In this paper, we presented a Potts spin MFT annealing model for mechanization of causal reasoning in the open, independent and incompatibility classes. We characterized the reasoning process in these classes by an energy function to be minimized by means of Mean Field equations. The main contributions of this paper can be summarized as follows. To our knowledge, it is the first proposal in the literature that models an explicit energy function for independent, open and incompatibility causal problems; and use MFT Potts spin modeling to encode its dynamics. The use of MFT Potts spin method has many advantages for causal reasoning. First, it automatically ensures a *total* coverage of the present effects. To date, no existing neural model in the literature meets this criterion, particularly in large-scale causal problems. Second, it is able to find, in practical time spans, optimal solutions since it avoids efficiently poor local minima of the energy function. Third, its inherent parallelism raiders the hardware implementation of the model possible and indeed desirable. We have numerically compared our model to a recent abductive model presented in [Ayeb *et cd.,* 1998] on a large-scale medical problem composed of 26 diseases, 56 symptoms and 465 causal interactions; using a huge battery of 1000 cases. Simulation results revealed that our model performs better than the model in [Ayeb *et al.,* 1998] in computing optimal scenarios, in much less CPU time. Due to the space limit, these benchmarks will not be reported herein.

As one should expect, there are some shortcomings in the current work. An important issue is extension of our model to include cancellation and monotonic interactions [Bylander *et al.,* -1991], which have to be addressed in future research. Remember that cancellation interactions involve not only causes, but also effects [Bylander *et al.,* 1991]. Regarding additive interactions, they suggest *cooperation* between causes [Goel *et al.,* 1995]. Unfortunately, additive/cancellation interactions cannot straightforwardly implemented by means of spin (either Potts or Ising) modeling.

Finally, use of MFT methods raises the problem of a so-called "critical temperature" [Peterson and Soderberg, 1989]. During the thermostatic operation of MFT equations, a critical temperature T_c is reached at which each Potts spin begins to move predominantly to 0 or 1. Estimating T_c can save time since annealing at temperatures much higher than T_c is in vain [Peterson and Soderberg, 1989]. Unfortunately, we found no general methodology in the literature for estimating T_c. Most proposals estimate T_c empirically using many assumptions which are valid only for the tackled problem. However, the methodology adopted in [Peterson and Soderberg, 1989] seems to be applicable for many combinatorial optimization problems and could be adapted to our case. In [Peterson and Soderberg, 1989], T_c was estimated by forming a small signal model of the system of spins and finding the eigenvalues of the matrix which expresses the interactions between the linearized neurons. Unfortunately, computing the eigenvalue of the interaction matrix is difficult especially in large-scale causal problems and if the goal is to infer scenarios within reasonable time scales. Most importantly, is that the interaction matrix is case-dependent, i.e. it varies from one manifestation to another given the same causal interactions. Hence, it is unworthy to proceed this way since the computational burden will not be negligible. However, it would be more appropriate to estimate T_c (or even an upper bound of it) from the set of causal strengths (independent and incompatibility) since these do not vary from one case to another. The last point is under investigations.

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