The Power of Beliefs or Translating Default Logic Into Standard Autoepistemic Logic*

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Abstract

Since Konolige's translation of default logic into strongly grounded autoepistemic logic, several other variants of Moore's original autoepistemic logic that embody default logic have been studied. All these logics differ significantly from Moore's autoepistemic logic (stan dard AEL) in that expansions are subject to additional groundedness-coriditioris. Hence, the question naturally arises whether default logic can be translated into standard AEL at all. We show, that a modular translation is not possible. However, we exhibit a a faithful polynomial-time translation from propositional default logic into standard AEL which is non-modular. It follows that the expressive power of standard AEL is strictly greater than that of default logic. Our translation uses as important intermediate step an embedding of Marek's and Truszczyriski's nonmonotonic logic N into standard AEL.

1 Introduction

Reiter's default, logic [16] and Moore's autoepistemic logic [14] are among the most relevant formalizations of nonmonotonic logic. A first investigation into the relationship between default and autoepistemic logic was carried out by Konohge [8].

Konolige [8] encountered some groundedness-problems in Moore's original version of autoepistemic logic (standard AEL), which impeded a straightforward translation of default logic into standard AEL. In particular, the autoepistemic theory *A* that would mast intuitively correspond to a given default theory *T* in many cases admits some additional stable expansions that do not correspond to any default extensions of *T*. These additional expansions are *weakly grounded* in the initial premises in the sense that they contain sentences *p* whose inclusion is merely based on the agent's belief in *p*. Consequently, Konolige has defined a more restrictive version of autoepistemic logic that we will here call *strongly grounded autoepistemic logic (SGAEL)*. In SGIAEL, only specific *strongly grounded* expansions are admissible. Each strongly grounded expansion is a stable expansion in the sense of standard AEL, but not vice-versa. Konolige succeeds in showing that SGAEL exactly corresponds to default logic and exhibits bidirectional translations between these two formalisms [8]

SGAEL differs significantly from standard AEL. In particular, in SGAEL it may be the case that two logically equivalent but syntactically different sets of premises Σ_1 and Σ_2 have a different semantics, i.e., different respective sets of SGAEL-expansions. Further, all strongly grounded extensions of a theory Σ are *stable-set minimal* for Σ, which is not the case in standard AEL.

The second major approach of translating default logic into variants of AEL has been taken by Marek, Schwarz, and Truszczyriski [12; 20; 19] Their approach is based on the concept of nonmonotonic modal logics as introduced by McDermott [13]. They show that default logic is faithfully embeddable in a wide range of different nonmonotonic modal logics. The simplest of these logics is the nonmonotonic counterpart of the *pure logic of necessitation* N the modal logic consisting of propositional calculus augmented by the necessitation rule We will make intensive use of this logic in the present paper. Note that just as SGAEL, nonmonotonic N is considered a stronger logic than standard AEL, since each N-expansion of a set of premises Σ is also a standard AEL-expansion of Σ, but not vice-versa.

In [17] it is shown that standard AEL corresponds to the nonmonotonic version of the modal logic KD45 based on the modal axioms K, 4, 5, and D : $L\phi \rightarrow \neg L \neg\phi$. Unfortunately, the proposed translations schemes which allow the embedding of default logic into a large number of other nonmonotonic modal logics fail to apply to nonmonotonic KD45, hence they are not applicable to standard AEL.

Other methods for translating default logic into formalisms close to AEL have been developed by Nierncla [15J, Lin and Shoham [9], Siegel [18], and Kaminski [7]. Each approach introduces a different version of AEL which captures default logic. Each of these logics is *more restrictive* than standard AEL in the sense that, in general, a set Σ of given premises in these logics admits fewer expansions than in standard AEL.

In summary, all previous methods translate default logic into formalisms that are different from standard

*This is a short version of the full paper [4] available from the author by email/ftp.

AEL- The reason seems to be that standard AEL allows for expansions that are not sufficiently grounded in the premises. Consequently, these approaches are based on more restrictive formalisms that admit less expansions

The question whether default logic can be translated into standard AEL has remained open so far In the present paper, we solve this problem by giving both a negative and a positive[1] answer.

First (in Section 2), we show that there exists no *modular* translation between default logic and AEL This means that after adding a new fact F to the formula set *W* of a default theory (D, *W),* a complete recomputation of the translation becomes necessary. An exception are prerequisite-free default theories, which are modularly translatable to standard AEL.

In Section 3, we show that it is possible to polynomially translate general default theories to standard AEL if one gives up on modularity. Section 4 concludes the paper with a philosophical interpretation of our translation.

In this paper we limit ourselves to consider *propositional* default logic and AEL. Note that the impossibility result for modular translations extends trivially from propositional default logic to the more general first order case. We assume that the reader is familiar with default logic and AEL and do not redefine these concepts.

2 Impossibility of Modular Translation

Let us first define the concept *of faithful* translation.

Definition 2.1 *A* faithful *translation from default logic to AEL is a mapping tr which transforms each default theory D into an autocpistcmic theory tr('P) such that the objective parts of th(autocpistcmic expansions of tr(V) arc identical with the default extensions of D*

The concept of *modular translation* in the context of default logic was introduced by Tomasz Imielinski in [tij. Loosely speaking, a translation scheme is modular if ad ding new facts (not defaults) to a default theory is reflected by the translation through adding these new tacts to the result of the translation, Imielinski [6] considered translations between default logic and circumscription. We adapt his formal definition of modularity to the context where AEL is the target system as follows

Definition 2.2 *A translation tr from default logic to AEL is modular iff for each default set I) and each* $W \subseteq \mathcal{L}$ *it holds that* $tr(\langle D, W\rangle) = tr(\langle D, \emptyset\rangle) \cup W$.

As Imielinski points out, modular translations are highly desirable both from the conceptual an]d the com putational point of view. Indeed, changes to a default theory *(D, W),* due to changes in the underlying "real world", will in most cases affect *W* but not *D,* as the default rules in *D* usually represent time-invariant, proper ties. With a modular translation, when *W* is changed, we do not have to recompute the result of the translation, but we may just add the new elements of W to the old result.

Note that both Konolige's translation from default, logic to SGAEL [8] and Truszczyriskrs translation *T* from default logic to nonmonotonic N are modular. In the rest of this section, we will show that a faithful modular translation from default logic to standard AEL is not possible. In particular, such a translation is not possible even if the defaults are restricted to very simple classes such as normal defaults. We will, however, identify one important class of defaults admitting a faithful modular translation to standard AEL, namely the prerequisite-free defaults.

Theorem 2.1 *There exist no faithful modular translation from default logic into standard AEL. Such a translation does not exist even if the defaults are restricted to be normal.*

PROOF. Consider the normal default theories $\langle D, W_0\rangle$, $\langle D, W_1\rangle$, and $\langle D, W_2\rangle$, where $W_0 = \emptyset$, $W_1 = \{a\}$, $W_2 = \{a \rightarrow b\}$, and

$$D = \{\frac{a \rightarrow b : Ma}{a}, \frac{a : Mb}{b}\}.$$

Each of these three default theories has exactly one extension. $\langle D, W_0\rangle$ has as unique extension $cons(\emptyset)$, while $\langle D, W_1\rangle$ and $\langle D, W_2\rangle$ both have the set $cons(\{a, b\})$ as unique extension. Here $cons(X)$ denotes $\{\phi | X \models \phi\}$.

Assume that there exists a modular faithful translation tr from default logic to standard AEL. We will show that this assumption implies that $cons(\{a, b\})$ is an extension of $\langle D, W_0\rangle$, an obvious contradiction.

Let $tr(\langle D, W_0\rangle) = \Sigma$ Since tr is modular, it must hold that $tr(\langle D, W_1\rangle) = \Sigma \cup \{a\}$. Since tr is faithful, the unique stable AE-expansion Δ of $\Sigma \cup \{a\}$ is $\Delta = E(\{a, b\})$ By the definition of stable expansion we thus have

$$\Delta = cons(\Sigma \cup \{a\} \cup L\Delta \cup \neg L\overline{\Delta}). \tag{1}$$

Since $b \in \Delta$, it follows that

$$\Sigma \cup \{a\} \cup L\Delta \cup \neg L\overline{\Delta} \models b \tag{2}$$

By applying the deduction theorem, we get

$$\Sigma \cup L\Delta \cup \neg L\overline{\Delta} \models a \rightarrow b \tag{3}$$

In other words,

$$a \rightarrow b \in cons(\Sigma \cup L\Delta \cup \neg L\overline{\Delta}). \tag{4}$$

It thus holds that

$$cons(\Sigma \cup L\Delta \cup \neg L\overline{\Delta}) = cons(\Sigma \cup \{a \rightarrow b\} \cup L\Delta \cup \neg L\overline{\Delta}). \tag{5}$$

Now, since tr is modular, it must hold that $tr(\langle D, W_2\rangle) = \Sigma \cup \{a \rightarrow b\}$. Furthermore, since $\langle D, W_1\rangle$ and $\langle D, W_2\rangle$ both have the same unique default extension $cons(\{a, b\})$, their respective translations $tr(\langle D, W_1\rangle)$ and $tr(\langle D, W_2\rangle)$ both have the same unique stable expansion Δ. In particular, since Δ is a stable expansion of $tr(\langle D, W_2\rangle) = \Sigma \cup \{a \rightarrow b\}$, it holds that

$$cons(\Sigma \cup \{a \rightarrow b\} \cup L\Delta \cup \neg L\overline{\Delta}) = \Delta. \tag{6}$$

By Equations (5) and (6), it follows that

$$cons(\Sigma \cup L\Delta \cup \neg L\overline{\Delta}) = \Delta. \tag{7}$$

But this means that Δ is a stable AE-expansion of of Σ. By the faithfulness of tr, it follows that $cons(\{a, b\}) =$

$\Delta \cap \mathcal{L}$ is a default extension of $\langle D, W_0 \rangle$, which is obviously not true. Contradiction. A modular and faithful translation from default logic to standard AEL is thus impossible. □

Note that the above negative result also holds for the class of *justification-free* defaults, i.e., defaults with empty justifications. To see this, just replace $\frac{a \to b \; Ma}{a}$ by $\frac{a \to b}{a}$ and $\frac{a \; Mb}{b}$ by $\frac{a}{b}$ in the above proof. Note that justification-free defaults are ordinary inference rules. Hence, even ordinary rule systems cannot be faithfully translated into autoepistemic logic. This shows where the real problem lies and hints at how we may restrict default theories in order to obtain modular translatability: to prerequisite-free defaults.

For prerequisite-free defaults, we may consider Konolige's most intuitive translation-scheme τ:

$$\tau\left(\frac{: M\beta_1, \ldots, M\beta_n}{\gamma}\right) = \neg L \neg \beta_1 \wedge \ldots \wedge \neg L \neg \beta_n \to \gamma$$

and $$\tau(\langle D, W \rangle) = W \cup \bigcup_{d \in D} \{\tau(d)\}.$$

Theorem 2.2 *r is a faithful, modular, and polynomial translation from the class of prerequisite-jree default theories to standard AEL.*

PROOF. (Sketch) The proof follows directly from previous results (Theorems 4.1.2 and 4.1.4) by Marek and 'IYuszczirisky [11]; the result is also implicitly present in the work of Lin and Shoharn [9]. □

Some authors argue that prerequisite-free defaults are the only natural defaults. Delgrande and Jackson [l], for instance, define *P-Dtfault Logic* based in prerequisite-free defaults. By the above result, P-Default Logic is modularly translatable into standard-AEL.

Note that in Definition 2.2 we defined a rather weak concept of modularity. Stronger types of modularity would require that each single default be *separately* translatable. Since we proved that even weakly modular translations in the sense of Definition 2.2 are impossible, the impossibility of any stronger type of modular translation follows.

3 The Nonmodular Translation

In this Section, we present a faithful polynomial time translation from default logic to standard ALL. By the results of the last section, such a translation must be nonmodular. In fact, our translation is rather involved and is based on sophisticated propositional coding techniques. It heavily exploits the self referential introspective capabilities of AEL. For space limitations, we must omit most of the formal details and proofs and can only present the main ideas. The full development can be found in the extended report [4] which is already available from the author.

We start by giving an informal rationale of the translation.

3.1 Rationale* of the Translation

Marek, Schwarz and Truszczyhski [12; 20; 19] have exhibited different faithful polynomial translations from default logic to nonmonotonic N, a variant of AEL which we will define below. We will use a translation *t* from default logic to nonmonotonic N as an intermediate step in our development. The translation *t* we use is just a slightly modified version of the translation introduced by Truszczyhski in [20], Our modification just makes sure that the translation also works for the inconsistent expansion, which is disregarded in [12; 20; 19]

Since default logic is translatable via *t* to nonmonotonic N, it suffices to establish a faithful translation *h* from nonmonotonic logic N to standard AEL. Then, our desired translation *c* from default logic to standard AEL is obtained by composing / and *h,* i.e., *c — hot.* This is illustrated in Fig. 1.

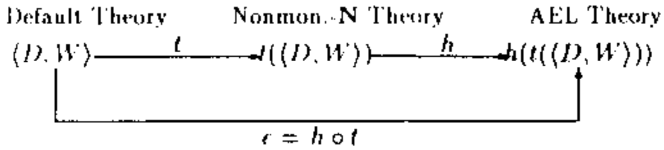

Figure 1: Schema of transformation composition

Nonmonotonic N is stronger than standard AEL in the following sense. Each N-expansion of a set of premises Σ is also a standard-AEL-expansion. On the other hand, Σ may have several additional standard-AEL expansions which are not N-expansions. These additional expansions are in a certain sense weakly grounded and should be eliminated. In order to get rid of these undesired expansions, we define, for each theory Σ a formula $G(\Sigma)$, the so called *grounding formula.* In the context of a particular standard-AEL expansion, $G(\Sigma)$ has the following intuitive meaning:

$$\mathbf{G}(\boldsymbol{\Sigma}) \equiv \text{“I am an N-expansion of } \boldsymbol{\Sigma}\text{”}.$$

In other terms, in the context of each AEL-expansion E of Σ, the formula $G(\Sigma)$ is true if E is a N-expansion, and false if E is not an N-expansion, i.e., if E is a weakly grounded expansion.

It is now easy to cut off the undesired weakly grounded expansions. The trick is to add the formula $LG(\Sigma)$ to Σ. $LG(\Sigma)$ will be a tautology in the context of an N-expansion of Σ and therefore harmless. In the context of a consistent weakly grounded expansion of Σ, however, $LG(\Sigma)$ evaluates to $L\mathit{false}$, which is inconsistent and forces the expansion to vanish. It therefore holds that the N-expansions of Σ exactly correspond to the standard-AEL-expansions of $\Sigma \cup \{LG(\Sigma)\}$. Our transformation h is thus simply obtained by adding $LG(\Sigma)$ to Σ.

We are thus able to enforce groundedness by just adding a single belief $LG(\Sigma)$ to a theory, where $LG(\Sigma)$ informally states "I believe that I am a grounded (i.e., N-) expansion of Σ". This strong power of beliefs has interesting philosophical interpretations that we will discuss in Section 4. In the rest of Section 3, we will outline the formal development of the translation.

3.2 Formal Preliminaries

Let $\mathcal{L}$ be an ordinary propositional language over a countably infinite alphabet of propositional variables, the syntactic operators $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\top$, and $\bot$ (where $\top$ is a constant for truth and $\bot$ is a constant for falsity). The classical propositional logic over $\mathcal{L}$ is denoted by $\mathbf{PL}(\mathcal{L})$. The language $\mathcal{L}_L$ of autoepistemic logic extends $\mathcal{L}$ by a unary modal operator L. The classical propositional logic over the language $\mathcal{L}_L$ is denoted by $\mathbf{PL}(\mathcal{L}_L)$. For $S \subset \mathcal{L}_L$, S^L denotes the set of all subformulas of the form $L\phi$ of each formula of S. If $\phi \in \mathcal{L}_L$ then $\phi^L =_{def} \{\phi\}^L$.

The (monotonic) modal logic $\mathbf{N}$ is defined over the samee language $\mathcal{L}_L$ as standard AEL. $\mathbf{N}$ is defined by the axioms of propositional calculus, modus ponens and the necessitation rule $\phi / L\phi$. $\mathbf{N}$ can be considered as the simplest modal logic. A detailed study of $\mathbf{N}$ is carried-out in [3], where also an appropriate Kripke semantics for this logic is defined. The consequence relation according to $\mathbf{N}$ is denoted by $\models_{\mathbf{N}}$. The consequence set operator $cons_{\mathbf{N}}$ of $\mathbf{N}$ is defined in the usual way: if $S \subseteq \mathcal{L}_L$, then $cons_{\mathbf{N}}(S) = \{\phi \mid S \models_{\mathbf{N}} \phi\}$. Clearly, $cons_{\mathbf{N}}$ is a monotonic operator.

The *nonmonotonic* version of $\mathbf{N}$ (also called *iterative AEL*) is defined from monotonic $\mathbf{N}$ according to McDermott's general definition scheme for nonmonotonic modal logics [13]. The basic concept is the one of an $\mathbf{N}$-expansion. $\Delta \subseteq \mathcal{L}_L$ is an $\mathbf{N}$-expansion (also called iterative expansion) of Σ iff Δ satisfies the fixed point equation $\Delta = cons_{\mathbf{N}}(\Sigma \cup \neg L\overline{\Delta})$. It was shown in [12] that each $\mathbf{N}$-expansion of a set $\Sigma \subseteq \mathcal{L}_L$ is also a stable AEL-expansion of Σ. The converse does not hold.

The *kernel* Λ of a stable expansion Δ of Σ is defined by

$$\Lambda = \Delta \cap \{L\phi, \neg L\phi \mid L\phi \textit{ is subformula of } \Sigma\}.$$

The following proposition shows that $\mathbf{N}$-expansions can be characterized by special properties of their kernels:

Proposition 3.1 ([5; 10]) *Let Δ be a stable expansion of Σ and let Λ be the kernel of Δ. Δ is an $\mathbf{N}$-expansion of Σ iff $\Sigma \cup \Lambda^- \models_{\mathbf{N}} \Lambda^+$, where $\Lambda^- = \{\neg L\phi \mid \neg L\phi \in \Lambda\}$ and $\Lambda^+ = \{L\phi \mid L\phi \in \Lambda\}$.*

For the sake of a simpler notation, we will sometimes implicitly identify *finite sets* and *conjunctions* of propositional formulas.

3.3 Expressing N-inference in PL($\mathcal{L}$)

In this subsection we outline how inference problems in modal logic $\mathbf{N}$ can be polynomially transformed into the problem of tautology-checking in ordinary propositional logic.

Definition 3.1 *Let $S \subset \mathcal{L}_L$ be a finite set formulas and let $\phi \in \mathcal{L}_L$ be a formula. A set $K \subseteq S^L \cup \phi^L$ is called S-ϕ-closed if $S \cup K \not\models \phi$ and for each $L\psi \in (S^L \cup \phi^L) - K$, $S \cup K \not\models \psi$.*

Lemma 3.1 *$S \models_{\mathbf{N}} \phi$ iff there is no S-ϕ-closed set.*

In order to translate $\mathbf{N}$ into classical propositional logic, we proceed in two steps. In the first step we construct a formula $F^+(S,\phi)$ over the language $\mathcal{L}_L$ such that $S \models_{\mathbf{N}} \phi$ iff $F^+(S,\phi)$ is a propositional tautology in $\mathbf{PL}(\mathcal{L}_L)$. In the second step, we transform $F^+(S,\phi)$ by a simple renaming of variables into an ordinary propositional formula $F(S,\phi)$ (over language $\mathcal{L}$) such that $F(S,\phi)$ is a tautology in $\mathbf{PL}(\mathcal{L})$ iff $F^+(S,\phi)$ is a tautology in $\mathbf{PL}(\mathcal{L}_L)$.

Since we will need a number of additional propositional variables for constructing $F^+(S,\phi)$, let us adopt the following convention. If $p \in \mathcal{L}$ is an ordinary propositional variable, then for each formula $\gamma \in \mathcal{L}_L$, p^γ denotes a new distinct propositional variable from $\mathcal{L}$. For $\phi, \gamma \in \mathcal{L}_L$, ϕ^γ denotes the formula obtained from ϕ by uniformly replacing each occurrence (at all levels of L-nesting) of any ordinary propositional variable p in ϕ by p^γ. If $S \subset \mathcal{L}_L$ is a finite set of formulas and $\gamma \in \mathcal{L}_L$, then $S^\gamma = \bigwedge_{\psi \in S} \psi^\gamma$.

Definition 3.2 *Let $S \subset \mathcal{L}_L$ be finite and $\phi \in \mathcal{L}_L$. For each $L\psi \in S^L \cup \phi^L$, let u_ψ denote a new distinct propositional variable. Let z be a propositional variable not occurring in S or ϕ. Then $F^+(S,\phi)$ is defined to be the formula*

$$\left(\left(S^z \wedge \bigwedge_{L\psi \in S^L \cup \phi^L} (u_\psi \rightarrow L\psi^z)\right) \rightarrow \phi^z\right) \vee$$
$$\bigvee_{L\gamma \in S^L \cup \phi^L} \left(\neg u_\gamma \wedge \left(\left(S^\gamma \wedge \bigwedge_{L\psi \in S^L \cup \phi^L} (u_\psi \rightarrow L\psi^\gamma)\right) \rightarrow \gamma^\gamma\right)\right)$$

Intuitively, this formula states that for each truth value assignment to the u_ψ, i.e., for each subset K of $S^L \cup \phi^L$, at least one disjunct must be true, hence K is not S-ϕ closed.

Theorem 3.2 *$S \models_{\mathbf{N}} \phi$ iff $F^+(S,\phi)$ is a tautology in $\mathbf{PL}(\mathcal{L}_L)$.*

Finally, we transform $F^+(S,\phi)$ into a formula $F(S,\phi)$ which does not contain any modal atoms. If $L\psi$ is a modal atom, then $[L\psi]$ denotes a new distinct propositional variable from $\mathcal{L}$. If U is a formula of $\mathbf{PL}(\mathcal{L}_L)$, then $[U]$ is the formula obtained from U by uniformly replacing each occurrence of any subformula of the form $L\psi$ of U not appearing in the scope of some L operator by $[L\psi]$.

Definition 3.3 *$F(S,\phi)$ is defined to be the formula $[F^+(S,\phi)]$.*

Note that $F(S,\phi) \in \mathcal{L}$, since this formula is obtained from $F^+(S,\phi)$ by uniformly replacing all atoms $L\psi$ by ordinary propositional atoms $[L\psi]$.

Theorem 3.3 *$S \models_{\mathbf{N}} \phi$ iff $F(S,\phi)$ is a tautology in $\mathbf{PL}(\mathcal{L})$.*

Remark: The size of $F(S,\phi)$ is at most quadratic in the size of S plus the size of ϕ. Thus, deciding $S \models_{\mathbf{N}} \phi$ is coNP-complete.

3.4 Translating Nonmonotonic N into standard AEL

Our aim is, given a set of premises Σ, to find a formula $G(\Sigma)$ such that the consistent stable AE-expansions of $\Sigma \cup LG(\Sigma)$ exactly coincide with the consistent N-expansions of Σ. The formula $G(\Sigma)$ will enforce any stable AE-expansion to be grounded. More specifically, $G(\Sigma)$ will make sure that for each stable expansion Δ of $\Sigma \cup G(\Sigma)$ with kernel Λ it holds that $\Sigma \cup \Lambda^- \models_{\mathbf{N}} \Lambda^+$, hence, by Proposition 3.1, Δ is an N-expansion.

For any premise set $\Sigma \subseteq \mathcal{L}_L$ we denote by $\mathbf{CEXP}(\Sigma)$ the set of all consistent standard-AE-expansions of Σ. The following theorem – fundamental to our transformation – elucidates the relationship between $\mathbf{CEXP}(\Sigma)$ and $\mathbf{CEXP}(\Sigma \cup \{L\alpha\})$ for autoepistemic formulas α of a certain type. In particular, the theorem shows, how by adding $L\alpha$ to Σ certain expansions of Σ can be suppressed.

Theorem 3.4 (Expansion Cutting Theorem) *Given a set $\Sigma \subseteq \mathcal{L}_L$ and a formula $\alpha \in \mathcal{L}_L$ such that $L\alpha \notin \Sigma^L$ and any atom of α is either a propositional variable not occurring in Σ or an atom $L\psi \in \Sigma^L$, it holds that*

a.) $\mathbf{CEXP}(\Sigma \cup \{L\alpha\}) \subseteq \mathbf{CEXP}(\Sigma)$

b.) *Let $\Delta \in \mathbf{CEXP}(\Sigma)$ be a stable AE-expansion of Σ with kernel Λ. Let α_Λ be the propositional formula obtained from α by replacing each atom $L\psi$ of α by $\top$ if $L\psi \in \Lambda$ and by $\bot$ if $\neg L\psi \in \Lambda$. Then $\Delta \in \mathbf{CEXP}(\Sigma \cup \{L\alpha\})$ iff α_Λ is a propositional tautology.*

We are now ready for defining the grounding formula $G(\Sigma)$.

Definition 3.4 (Grounding Formula) *Let $\Sigma \subset \mathcal{L}_L$ be a finite set of premises. Assume that z is a propositional variable not occurring in Σ. Then $G(\Sigma)$ is the formula displayed in Figure 2.*

In the proof (given in [4]) of the next theorem, the following is shown. In the context of each particular AEL-expansion of Σ with kernel Λ, the grounding formula $G(\Sigma)$ becomes equivalent to the formula $F(\Sigma \cup \Lambda^-, \Lambda^+)$. Thus, by Theorem 3.3, $G(\Sigma)$ is valid iff $\Sigma \cup \Lambda^- \models_{\mathbf{N}} \Lambda^+$, i.e., iff the expansion corresponding to Λ is an N-expansion. The Expansion Cutting Theorem (Thm. 3.4) then guarantees that the theory $\Sigma \cup LG(\Sigma)$ has as standard AEL-expansions exactly the N-expansions of Σ.

Theorem 3.5 *Let $\Sigma \subset \mathcal{L}_L$ be a finite set of premises. Then Δ is a consistent N-expansion of Σ iff Δ is a consistent stable AE-expansion of $\Sigma \cup \{LG(\Sigma)\}$.*

We have thus established a polynomial-time translation from Nonmonotonic N to Standard AEL.

3.5 Translating Default Logic to standard AEL

Our transformation t from default logic to nonmonotonic N is as follows.

Definition 3.5 *Let $\langle D, W\rangle$ be a default theory. Let v be a propositional variable from $\mathcal{L}$ which does not occur in $\langle D, W\rangle$. For each default in $d \in D$ of the form $\alpha : M\beta_1, M\beta_2, \ldots, M\beta_n / \omega$, let $t(d)$ denote the autoepistemic formula $(\neg Lv \wedge L\alpha \wedge L\neg L\neg\beta_1 \wedge \ldots \wedge L\neg L\neg\beta_n) \rightarrow \omega$. Define the translation $t(\langle D, W\rangle)$ by $t(\langle D, W\rangle) = W \cup \{t(d) : d \in D\}$.*

This translation is a slightly modified version of Truszczynski's translation [20]. It handles properly the inconsistent expansion which is disregarded in [20]. By results in [20], it is easy to see that this translation is indeed a faithful embedding of default logic into nonmonotonic N. We now present the definitive translation function e between default logic and standard AEL.

Definition 3.6 *If $\langle D, W\rangle$ is a default theory then $e(\langle D, W\rangle) = t(\langle D, W\rangle) \cup \{LG(t(\langle D, W\rangle))\}$.*

Theorem 3.6 (Faithful Translation) *The extensions of a default theory $\langle D, W\rangle$ are given by the sets $\Delta \cap \mathcal{L}$, i.e., the objective parts of Δ, where Δ is a stable AE-expansion of $e(\langle D, W\rangle)$.*

Theorem 3.6 establishes a faithful polynomial translation from default logic into standard AEL. It is easy to see that a faithful translation in the opposite direction is not possible. Just consider the autoepistemic theory $\Sigma = \{Lp \rightarrow p\}$. Σ has two stable AE-expansions whose objective parts are respectively $O_1 = cons(\emptyset)$ and $O_2 = cons(\{p\})$. Obviously, $O_1 \subset O_2$. However, it is well known (and follows easily from the definition of default-extension [16]) that no extension of a default theory $\langle D, W\rangle$ can be a subset of any other extension of $\langle D, W\rangle$. Thus, the stable AE-expansions of Σ do not correspond to the extensions of any default theory. If the *expressive power* of a nonmonotonic logic stands for its capacity of expressing sets of propositional theories (i.e., sets of alternative epistemic states) through sets of premises, then the expressive power of standard AEL is strictly higher than that of default logic.

4 Philosophical Remarks and Conclusion

Our translatability result admit a somewhat speculative, but nevertheless very appealing and intriguing philosophical interpretation.

Recall that standard AEL is a logic for expressing beliefs, and that a set of premises Σ is conceived as the set of initial beliefs of an ideally rational agent. Hence, the addition of the formula $LG(\Sigma)$ to a theory Σ can be interpreted as the belief of the agent that its own beliefs (w.r.t. Σ) must be sufficiently grounded in Σ. We have shown in this paper that the addition of this formula effectively enforces all beliefs based on Σ to be sufficiently grounded in Σ. Hence, we delivered a mathematical proof for the fact that (polynomially many) beliefs in groundedness-principles about a theory may control the *way* of reasoning on the basis of that theory. In particular, if we agree that more strongly grounded expansions are "better" (i.e., more rational or more scientific) than weakly grounded expansions, then our result may be paraphrased as follows: ***an autoepistemic agent can improve its way of reasoning by believing in better principles of reasoning.***

$$\mathbf{G}(\boldsymbol{\Sigma}) = \Bigg(\Bigg([\Sigma^z] \wedge \bigwedge_{L\psi \in \Sigma^L} (\neg L\psi \rightarrow \neg[L\psi^z]) \wedge \bigwedge_{L\psi \in \Sigma^L} (u_\psi \rightarrow [L\psi^z]) \Bigg) \rightarrow \bigwedge_{L\psi \in \Sigma^L} (L\psi \rightarrow [L\psi^z]) \Bigg) \vee$$

$$\bigvee_{L\gamma \in \Sigma^L} \Bigg(\neg u_\gamma \wedge \Bigg(\Bigg([\Sigma^\gamma] \wedge \bigwedge_{L\psi \in \Sigma^L} (\neg L\psi \rightarrow \neg[L\psi^\gamma]) \wedge \bigwedge_{L\psi \in \Sigma^L} (u_\psi \rightarrow [L\psi^\gamma]) \Bigg) \rightarrow [\gamma^\gamma] \Bigg) \Bigg)$$

Figure 2: The Grounding Formula $\mathbf{G}(\boldsymbol{\Sigma})$

A yet different interpretation or use of our results concerns the problem of *communicating knowledge* to an autoepistemic agent. If we tell a story Σ to such an agent, we must be prepared to the possibility that the agent bases weakly grounded beliefs on Σ and acts according to such beliefs. In order to restrict the agent's phantasy and put its feet back on the ground, it is sufficient to add the formula $LG(\Sigma)$ to the intended information Σ.

When we spoke about *groundedness* in this section, we referred to the type of groundedness inherent in default logic and in the McDermott-style nonmonotonic logics. This kind of groundedness basically limits the "jump" to new nonmonotonic conclusions to such sentences which are obtained by *negative* introspection. In our opinion, this is a very natural type of groundedness. A more complex (and in our opinion slightly less natural) concept of groundedness underlies the definition of *moderately grounded autoepistemic expansion* introduced in [8]. A moderately grounded expansion E of Σ is an AE-expansion of Σ whose objective part $obj(E)$ is minimal in the following sense: there does not exist any stable set containing Σ whose objective part is strictly included in $obj(E)$. Reasoning with moderately grounded expansions is extensively studied in [2] where it is shown that the main reasoning tasks in this formalism are one degree harder in the Polynomial time Hierarchy than the analogous reasoning tasks in AEL or default logic. For example, checking whether a set of premises has a moderately grounded expansion is Σ_3^P-complete [2], while the analogous reasoning tasks in standard AEL and in default logic are Σ_2^P-complete [5]. Thus, unless the Polynomial-time Hierarchy collapses, there cannot exist any polynomial-time embedding (modular or not) of moderately-grounded AEL into standard AEL.

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