The Logic of Time Structures: Temporal and Nonmonotonic Features(†)

Mira Balaban
Dept. of Computer Science
Ben-Gurion University
Beer-Sheva 84105, Israel

Neil V. Murray
Dept. of Computer Science
State Univ. of N.Y. at Albany
Albany, NY 12222

Abstract

We Imbed Into a first order logic a representation language that combines atemporal knowledge with time stamps in a hierarchical fashion. Each time structure contains its own chronology of events: sufficient information for an encoding of a classical temporal logic. By quantifying over time structures, we encode a modal logic of temporal knowledge. In addition, we show how to achieve the effect of nonmonotonic inference, by simulating preferential entailment within a first order framework.

1. Introduction

One approach to the representation of time related knowledge is to embed the time factor in the operational or model-theoretic semantics of the representation. This is the approach in the situation calculus ([McCarthy & Hayes 1969], [Hayes 1971]), in truth maintenance systems, in computation models, in tense logics ([Halpern & Shoham 1986]), and in some computer music systems ([Schottstaedt 1983], Cointe & Rodet 1983]).

Another approach is to make time explicit in the language. Examples are the reified temporal logics of "Allen 1984], [McDermott 1982], Dean & McDermott 1987] , and [Shoham 1987a]; also the Horn-clause logic of [Kowalski & Sergot 1986] and the first order logic of [Haugh 1987].

We imbed into a first order logic a representation language that combines atemporal knowledge with time stamps in a hierarchical fashion. The syntactic unit of the representation language is called a *time structure;* it resides in our logic as a term. A consequence of this is that the implementation of our formalism with logic programming techniques will be relatively straightforward.

Each time structure describes a chronology of events. The time stamps result in an encoding of temporal logic that is similar to that of [Shoham 1987a]. Furthermore, each time structure can play the role of a 'world' in a modal logic, resulting in an unusual encoding of modal logic; unusual because a world is referenced by its explicit representation as a term, rather than by a constant through which certain facts are indexed. Since our 'worlds' are terms, they can be ordered, and we can enforce a preference criterion on that ordering. In this way, we can achieve the effect of nonmonotonic reasoning.

In Section 2 we give our view of temporal domains; The operators (i.e., interpreted symbols) of our logic are introduced in Section 3. Section 4 is a discussion of semantics; in particular, the semantics of the interpreted non-logical symbols. In Section 5 we compare the logic of time structures with Shoham's classical interval temporal logic, and its nonmonotonic version. [Shoham 1987b]. We conclude with an example of reasoning with chronological minimization. Proofs of lemmas and theorems are omitted and can be found in [Balaban & Murray 1988].

2. Domain of Discourse

The temporal world that we describe has no absolute time line. It is built from atemporal objects that, when combined with *time points,* form *histories.* Histories can be combined together to form more complex histories. Each history has its own private time line. The entire domain of discourse consists of atemporal objects, temporal objects (time points), and histories. We now describe each of these types in some detail.

2.1. Atemporal and temporal objects

Atemporal objects are domain elements that, viewed in isolation, are durationless. Of course, in reality 'durationless objects' do not exist, but this is a common abstraction.

We assume a set of *temporal objects* called *time points,* that is totally ordered and that contains an object called Zero.

We distinguish objects, actions, and processes not by means of distinct types, but through the temporal behavior of such entities represented as histories.

2.2. Histories

The building block for histories, called an *elementary combination,* is the association (p, td) of an atemporal object p with a temporal object t_d. The pair (p, t_d) can be thought of as the set of all occurrences of p such that if p starts at some time point t, it clips at time point t + td- We call t_d the *duration* of the elementary combination (p, t_d).

A *history* is a collection of time-stamped histories, or time stamped elementary combinations. A history has its own time line and the histories that occur within it have their own time lines. But the Zero of these time lines is displaced from the Zero of the composite history by their corresponding time stamps. Elementary combinations have no self time lines.

† This research was supported in part under grant CCR-8600848 awarded by the National Science Foundation.

2.3. Formal Definition

Here, a brief formal description of our domain of discourse is given. We have a non-empty domain *D* of atemporal objects, and a domain *T* of time points that is totally ordered and that contains Zero. Time points and objects can be combined to form the set *H* of histories as follows:

(a) The empty set 0 is a history called the *empty history.*

(b) For $i \in \Gamma$ an arbitrary indices set, $h_i \in H \cup EC$, and $t_i \in T$: $h = \{(h_i, t_i) \mid i \in \Gamma\} \in H$;

h is the *history* in which the time line of sub-history *hi* is displaced by t_i.

The entire domain of discourse *U* consists of $(D \cup T \cup EC \cup H)$. We assume a set *F* of total functions $U^k \longrightarrow U$. No a priori restrictions are placed on these functions, except that they are total and have a signature indicating the sub-domains corresponding to their arguments and to their range. In other words, the functions of *F* must be well-typed with respect to D, 7\ EC, and *H.*

The need for 'inter-typed' functions arises from the non-homogeneous nature of *U.* Elementary combinations are obtained by a function whose signature is: $D \times T \rightarrow EC$; histories require the signature $\{ (H_i \cup EC_i) \times D_i)\}_{i \in \Gamma} \rightarrow H$. A duration function would have the signature $H \longrightarrow T$; atemporal functions like "angry" would be of type $D \longrightarrow D$.

3. Time Structures

In correspondence to the four types in *U,* there are four kinds of terms: *primitive terms* that denote atemporal objects, *time terms* that denote time points, *elementary pairs* that denote elementary combinations, and *time structures* that denote histories. All are terms of first order logic.

Given:

C = a set of constant symbols.

S = {d,t,ec,h} – sort symbols, that correspond to the three types *D, T, EC, H.*

0 ∈ C – denoting the Zero object of T.

NIL ∈ C – denoting the empty history ∅ in H.

V = a set of variables.

FS = a set of function symbols.

[ ] ∈ FS – describing elementary histories.

● ∈ FS – describing histories. ● is called the ***temporal concatenation*** operator.

sgn = a mapping as follows:

$$\begin{aligned}&\mathrm{sgn}: C \rightarrow S\\ &\mathrm{sgn}: V \rightarrow S\\ &\mathrm{sgn}: FS^{(n)} \rightarrow \times_{i=1}^{n+1} S\\ &\mathrm{sgn}(0) = t\\ &\mathrm{sgn}(NIL) = h\\ &\mathrm{sgn}([\,]) = (d,t,ec)\\ &\mathrm{sgn}(\bullet) = (v,t,h,h) \quad \text{where } v \in \{h, ec\}\end{aligned}$$

Terms: Defined as usual, with sort restrictions taken into account as follows:

The sort of constants and variables is their signature; the sort of $f(t_1, \ldots, t_n)$ is the last element of the signature of f, where the i^{th} element of sig(f) must agree with the sort of tj, l<i<n.

We discuss the terms of sorts d, t, ec, and h below. We express functions of arity one or greater as lower case identifiers; variables are also lower case but are italicized; constants are functions of zero arity and may be upper or lower case; the time structure operators appear in boldface.

3.1. Primitive Terms and Time Terms

Primitive terms are terms of sort *d.* They denote atemporal objects. Time terms are terms with sort *t.* One distinguished time term is the constant 0; it denotes the time point Zero.

3.2. Elementary Pairs

An *elementary pair* is a sort ec term of the form [p, tj ; it associates a non-negative time term t_d with a primitive term p. It denotes the elementary combination (p', td'), where p', td' are the denotations of p, td, respectively. For example, the elementary pair [FIDO-BARKS,35] can denote the elementary combination of fido barking, that clips at time point t + 35, if it starts at time point t.

3.3. Time Structures

Time structures are terms of sort h that denote histories; they include (not exclusively) NIL and all terms generated by •. Some examples are:

NIL denotes the empty history.

•([p,d],t,NIL) denotes the history {((p',d'),t')}, where p', d', and t' are the denotations of p, d, and t, respectively.

$\bullet([p, d_1], t, \bullet([q, d_2], 0, NIL))$ denotes the history $\{((p', d_1'), t'), ((q', d_2'), 0)\}$

$\bullet(\bullet([p,d], t_1, NIL), t_2, NIL)$ denotes a single element history: $\{ (\{ ((p', d'), t_1'), \}, t_2') \}$

$\bullet([p,d], t, ts)$ denotes: $\{((p',d'),t')\} \cup h$, where h is any history (possibly unbounded or infinite).

$\bullet(ts, 2{*}t, \bullet(ts, t, NIL))$ denotes histories of the form: $\{ (h, 2{*}t'), (h, t') \}$, i.e. a pattern of histories.

The following axioms characterize the basic properties of the ● operator.

Axiom ([ ]): The denotation of every sort ec term is that of some [ ] term:

$$\forall ep \; (\mathrm{sgn}(ep) = ec \;\rightarrow\; \exists\, p,d \;\; ep = [p,d]) \qquad ([])$$

Axiom (●NIL): NIL is a unit element of the ● operator:

$$\forall t,ts \quad \bullet(NIL, t, ts) = ts \qquad (\bullet NIL)$$

Axiom (●h): The denotation of every non NIL sort h term is that of some ● term:

$$\forall ts \;\; ((\mathrm{sgn}(ts){=}h \wedge ts \neq NIL) \;\rightarrow\; \exists\, ts_1, ts_2, t \;\; (ts = \bullet(ts_1, t, ts_2))) \qquad (\bullet h)$$

Note that this ● term is unique up to repetitions and commutativity.

Idempotence:

$$\forall ts_1, t_1, ts \qquad (\bullet idp)$$

$$\bullet(ts_1, t_1, \bullet(ts_1, t_1, ts)) = \bullet(ts_1, t_1, ts)$$

Commutativity:

$\forall ts_1, t_1, ts_2, t_2, ts_3$ (•cmt)

$$\bullet(ts_1, t_1, \bullet(ts_2, t_2, ts_3)) = \bullet(ts_2, t_2, \bullet(ts_1, t_1, ts_3))$$

We define the intended meaning of NIL, [ ], and • terras in Section 4. The intended meaning of all other h terms is defined in terms of these via equality axioms.

3.4. Operators

By an "operator", we mean a symbol that has a well-defined signature and whose definition is fixed. An operator plays the same role as any function symbol except that it is *interpreted.* We have already introduced • and [ ]; beyond those, we discuss only the *time operators* here; many others are introduced in [Balaban & Murray 1988].

The *self-clip time* of a time structure *ts* is written *clip self(ts);* intuitively it is the latest point on the time line of *ts* at which one of its constituents clips. The *self-start time* of *ts,* written $start_{self}(ts)$ is similarly defined; the *duration,* written duration(ts), is their difference.

The operators $clip(ts_1, ts_2)$ and $start(ts_1, ts_2)$, compute the more useful *relative* clip and start times for a time structure or an elementary pair *ts1* that occurs within ts_2. Note that each occurrence of ts_1 in ts_2 has a relative start and clip time. Therefore, these operators yield a *list* of time points.

The interval operator computes a list of pairs of time points that describe intervals during which a given time structure is "playing" within another time structure. Interval is definable from start and clip (see [Balaban & Murray 1988]).

4. Semantics of Time Structures

We assume a standard first order logic syntax including the usual logical symbols: $\forall$, $\exists$, $\wedge$, $\vee$, $\neg$, $\Rightarrow$, and $\Leftrightarrow$. Predicates begin with an upper case character, and we use the notation from Section 2 otherwise.

An equality and total ordering predicate are characterized by appropriate axioms. We avoid introducing a sort symbol b, and a type B of boolean values; the signature of a predicate is taken as just the tuple of its argument sorts, and the sort is understood as boolean. We extend our syntax as follows:

P is a set of predicates that includes < and =.

We extend sgn as follows: sgn: $P^{(n)} \rightarrow \times_{i=1}^{n} S$, where $sgn(\leq) = (t,t)$, and $sgn(=)$ is (d,d), (t,t), (ec,ec), or (h,h). (we use "=" in the obvious polymorphic way.)

Our principle axiomatic temporal relation is the *completion* of a time structure over a given interval within a context time structure. The notion of completion is similar to the TRUE notation of [Shoham 1987a]. It is, later on, used to define additional temporal relations, and to classify temporal behaviors of time structures. The notion of completion is expressed by a predicate "C", where $C(t1, t_2, ts_1, ts_2)$ means ts_1 occurs as a sub-time structure or sub-elementary pair within ts_2, and it starts and clips at t_1 and t_2, respectively. The signature of C is (t,t,h,h).

The C predicate reflects the time structure operator interval and is defined as follows:

$\forall t_1, t_2, ts_1, ts_2$

$$(C(t_1, t_2, ts_1, ts_2) \Leftrightarrow (t_1, t_2) \in \mathbf{interval}(ts_1, ts_2))$$

$\forall t_1, t_2, p_1, p_2, d, ts \ (C(t_1, t_2, [p_1 c p_2, d], ts) \Rightarrow$ (C2)

$$C(t_1, t_2, [p_1, d], ts) \ C_L \ C(t_1, t_2, [p_2, d], ts))$$

where "c" denotes a *term level connective:* &, v, ⊃, ~, and CL denotes the corresponding logic level connective (in the case of ~, there is no p,, and C^ is unary). Axiom (C2) defines the term level connectives to be merely abbreviations for non-atomic formulas involving the C predicate. But we might add the condition $c \neq \sim$ to the premise of (C2); then inconsistent information could be represented within a time structure and yet allow the logical system to remain consistent. Note that keeping the implication uni-directional, the strong law of the excluded middle does not apply to completion of primitive terms: It is not the case that for every primitive term and every interval, either the primitive term or its negation completes over a time structure (otherwise, NIL, for example, cannot be a legitimate time structure).

As a 1-st order theory, our logic admits the regular 1-st order semantics, within the type restrictions. We now define the notions of *intended* (standard) *interpretation,* and *model.*

An intended interpretation is a domain and denotation function (U,T), where

U = D ∪ T ∪ EC ∪ H, and D, T, EC, H are mutually disjoint.

D = is the Herbrand Universe defined by all constant symbols having a signature in $\times_{i=1}^{n} S \times \{d\}$, for all $n \geq 0$.

T = a non-empty set of time points.

Zero ∈ T

EC = a set of elementary combinations over D and T (section 2.4).

H = a set of histories over EC and T (section 2.4).

Υ = a denotation function as follows:

Maps constants of signature t to elements of T.

Maps constants whose signature is d to themselves.

Maps 0 to Zero.

Maps ≤ to Time-order.

Maps = to the equality relation on U.

Maps NIL to ∅ (the empty history).

Maps | | to a function: D × T → EC that maps d ∈ D and t ∈ T to the pair (d,t).

Maps • to a function: (H ∪ EC) × T × H → H, defined by:

$$(h_1, t, h_2) \rightarrow \{(h_1, t)\} \cup h_2$$

$$\text{or:} \quad \bullet(h_1,t,h_2)^T = \{ (h_1^T, t^T) \} \cup h_2^T .$$

The time structure operators like start, clip, interval, etc., are assigned their history counter-parts.

A variable assignment is partitioned as follows:

A *simple-variable assignment* is an assignment to the variables of signature d, t and ec.

An *h-variable assignment* is an assignment to the variables of signature h.

An interpretation I and a h-variable assignment V form a *model* to a theory in the logic of time structures, if they satisfy:

1. Axioms (eNIL) (tcmt), (eh) and (•idp); the equality axioms for =, the total ordering axioms for <, and axioms defining the time structure operators.
2. Axioms for the arithmetic time functions, and the list functions; see [Balaban & Murray 1988]).
3. Axioms (CI), (C2).
4. The proper formulas of the theory for every simple-variable assignment.

The notions of satisfiability, validity, logical consequence are defined as usual.

4.1. Characterization of Temporal Behaviors

Taking C as our main predicate reflects the interval based nature of our calculus. However, we believe that there is a need also for point-wise characterization of temporal behavior. For that purpose we introduce a predicate, called P (PLAYS), where $P(t, ts_1, ts_2)$ stands for "the time structure tS_1 plays at time point t in time structure ts_2."

$$\forall t, ts_1, ts_2 \; (\; P(t, ts_1, ts_2) \iff (\exists \; t_1, t_2) \; t_1 \leq t \leq t_2 \wedge C(t_1, t_2, ts_1, ts_2) \;) \qquad \text{(P1)}$$

Note that the subject of the point-wise truth is a time structure tsi, and a context time structure ts_2. This reflects the view underlying our formalism, that a "fact" (i.e., some atemporal object) cannot "hold" or be "true", unless it is part of some time structure. Hence, it is, always, a *time structure* about which we wish to state some point-wise truth.

The C and P predicates have the following properties:

1) A time structure is playing at every time along an interval on which it completes:

$$\forall t_1, t_2, t, ts_1, ts_2, \; (\; (C(t_1, t_2, ts_1, ts_2) \wedge t_1 \leq t \leq t_2) \Rightarrow P(t, ts_1, ts_2) \;)$$

2) A time structure does not complete on any proper subinterval of an interval on which it completes:

$$\forall t_1, t_2, t_3, t_4, ts_1, ts_2, \; (\; (C(t_1, t_2, ts_1, ts_2) \wedge t_1 \leq t_3 \leq t_4 \leq t_2 \wedge (t_1 \neq t_2 \vee t_4 \neq t_2) \wedge (t_1 \neq t_4 \wedge t_3 \neq t_2)) \Rightarrow \neg C(t_3, t_4, ts_1, ts_2) \;)$$

Note that by the last property, point-wise completion is still different from playing (which is always point-wise): On every point of the interval $[t_1, t_2]$, ts_1 plays but does not complete!

The issue of whether to analyze the truth of some entity over time intervals or over time points has been discussed at some length in the literature of temporal logics in AI [Allen 1984], [McDermott 1982], and [Shoham 1987a]. Using the C and P predicates, we are able to handle both.

5. Related Work

The most well known treatments of temporal information in AI are the works of Allen, McDermott, Shoham, Kowalski, and Haugh. Shoham's logics seem to subsume the other above mentioned works. Therefore, we compare our logic to his.

First we show that every formula of Shoham's first order interval temporal logic can be simulated by a formula in the logic of time structures that has the context time structure as a free variable. This simulation can be further augmented to apply to Shoham's monotonic modal logic of Temporal Knowledge - but this is an option that we do not pursue. Instead, we show that the preference criterion on models of the modal logic of Temporal Knowledge, called *chronologically more ignorant,* can be formulated as a first order formula, involving comparison of time structures. A major property of this simulation is that it avoids problems associated with the chronological minimization of models of Shoham's classical interval logic; problems that caused Shoham to switch to the more sophisticated modal logic of *chronological ignorance.* We conclude with an example that demonstrates the simulation, in the logic of time structures, of nonmonotonic reasoning in Shoham's modal logic of chronological ignorance.

5.1. Shoham's Classical Interval Temporal Logic

Shoham's logic contains, variables, constant symbols, predicates, logical connectives and quantifiers, that are analogous to those of the logic of time structures, with minor modifications. His logic makes a syntactic distinction between variables and terms denoting time points versus those denoting domain objects, whereas we made only a semantic one. Also, in the classical version, Shoham assumes only a partial ordering on the set of time points, not a total ordering as we did. Our comparison applies only to totally ordered sets of time points. Note, that in the modal version, Shoham assumes the structure of the integers, i.e., a total ordering.

A special construct of Shoham's logic is $TRUE(t_1, t_2, p)$ where t1 and t_2 are time terms, and p is a predication. This is just notation for the interval-predication pair $<<t_1, t_2>, p>$. The semantics of the logic will assign, under a given interpretation and variable assignment, a set of time intervals to the predication p. If $<t_1, t_2>$ is in this set, then TRUE^, t_2, p) is true under this interpretation and assignment. Notice that in this interval logic, there is one world containing a single time line, and all formulas are related to this one time line. Therefore, in this comparison, we will restrict formulas to refer to a single time structure as the fourth argument of the C predicate.

Corresponding to a set of wffs Φ of Shoham's logic, we have the wff Φ_t *(ts)* where Φ_t of our logic is the following set of translated wffs:

1. **If Φ is $t_1 = t_2$, or $t_1 \lesssim t_2$, for temporal terms t_1, t_2, then Φ_t is Φ.**
2. **For a predication p (without negation), if Φ is $TRUE(t_1, t_2, p)$, then $\Phi_t(ts)$ is $C(t_1, t_2, [p, t_2 - t_1], ts)$.**
3. **If Φ^1 and Φ^2 are wffs, then $(\Phi^1 \wedge \Phi^2)_t(ts)$ is $\Phi^1_t(ts) \wedge \Phi^2_t(ts)$.**
4. **The cases for disjunction, negation, and quantifiers are similar.**

We want now to prove that the translation preserves satisfiability, and logical implication. For that purpose we first relate the notions of interpretations in the two logics. We look only at interpretations in the logic of time structures that satisfy all the apriori requirements for being a model: The assignments to the special symbols 0, NIL, [ ], •, <, =, C, to the arithmetic time functions, to list functions, and to the time structure operators, must satisfy their axioms and "definitions".

Such interpretations are *regular* interpretations.

Definition: Given I, a regular time structure interpretation + variable assignment (to all variables), *ts,* a variable of signature h, and J, an interval logic interpretation + variable assignment. We say that (I, *ts) corresponds* to J if:

1. I and J have the same set of time points, with the same ordering; assignments in I to constants and variables of signature t is the same as the assignments in J to time point symbols and to temporal variables.
2. The assignment to *ts* in I satisfies: For all time terms t_1, t_2, and predication p in the interval logic:

$$J \models \mathrm{TRUE}(t_1, t_2, p) \quad \text{iff}$$
$$I \models (t_1, t_2) \in \mathbf{interval}([p, t_2 - t_1], ts).$$

In the following lemma we restrict the time structures to include only the symbols appearing in a given set Ψ of wffs. The notion of correspondence between interpretations turns into Ψ-correspondence.

Lemma 1. Given Ψ, a set of time structure wffs, we assume that the language of time structures includes only the symbols in Ψ. Then, for every regular time structure interpretation + variable assignment I, and a signature h variable *ts,* there exists a Ψ-corresponding interval logic interpretation + variable assignment J, and vice-versa.

Comment: The restriction on the language of time structures is needed because we have to split the function symbols of signature $\times_{i=1}^{n} S \times \{d\}$ in the logic of time structures, into predicates and function symbols in the interval logic.

Lemma 2. Every pair of Φ and $\Phi_t(ts)$, a set of wffs in the interval logic, and its translation into the time structure logic, is cqui-satisfiable.

Corollary: If J and (I, *ts)* are Φ_t-corresponding interpretations, then

$$J \models \Phi \quad \text{iff} \quad I \models \Phi_t(ts).$$

Theorem 1. $\Phi \models \phi, \quad \text{iff} \quad \Phi_t(ts) \models \phi_t(ts).$

5.2. Chronological Ignorance In Time Structures

Shoham introduces two versions of *chronological minimization.* First, for a given set S of primitive propositions, he defines a preference criterion called *chronologically smaller in S,* on models of the classical interval logic. Intuitively, a model M_2 is chronologically smaller in S than a model M1 if, for all propositions in S, they 'agree' up to a certain time point to, and at t_0 M_1 has information about a proposition in S, that M_2 does not. The problem with this preference criterion is its dependency on the set S, whose selection seems to depend on the desired conclusions. To solve this problem, Shoham introduces a logic called CI (Chronological Ignorance), which is a nonmonotonic version of a modal logic of temporal knowledge. The nonmonotonicity of CI results from a preference criterion on Kripke structures, called *chronologically more ignorant.* The idea behind this criterion is similar to the previous one, but now the minimization involves all known propositions. This idea could not be applied directly to the classical interval logic (i.e., take S as the set of all propositions), since in classical logic for every proposition p, either p or its negation is true over any given interval (the strong law of excluded middle), thereby turning every model into a chronologically minimal one.

In this section we try to incorporate the chronologically more ignorant criterion described above, into the (first order) logic of time structures. The idea is that history H_2 is *chronologically more ignorant* than history H_1, if they 'agree* up to a certain time point to, and H_1 has information at t_0 that H_2 does not. This relation between histories is captured by the predicate >, called *chronologically more ignorant.* The predicate is defined by the following first order formula:

Definition - The chronologically more ignorant predicate:

$$ts_2 \gtrsim ts_1 \leftrightarrow \exists\, t_0$$
$$[\ (\forall p, d, t \le t_0, t' \le t \quad (C(t', t, [p,d], ts_2) \rightarrow C(t', t, [p,d], ts_1))) \quad \wedge$$
$$(\exists\, p, d, t \le t_0 \quad (C(t, t_0, [p,d], ts_1) \wedge \neg C(t, t_0, [p,d], ts_2)))\ ]$$

Note that, as with Shoham's latter chronologically more ignorant criterion, our chronological preference predicate is defined with respect to all primitive terms, not just terms within a given set of propositional primitive terms. The reason we can do that, within the scope of first order logic, is that the strong law of excluded middle does not apply to time structures: it is not the case that for any primitive proposition, either it completes over a given interval, or its (term level) negation completes over that interval.

A *chronologically maximally ignorant* history with respect to a given theory is characterized by the CMI predicate as follows:

Definition: Given $\Psi(ts)$, a formula in the logic of time structures. The CMI predicate, with respect to Ψ is defined by:

$$\mathrm{CMI}_\Psi(ts) \leftrightarrow (\Psi(ts) \wedge \forall ts' (\Psi(ts') \Rightarrow \neg(ts' \gtrsim ts)))$$

The following three claims summarize the relationship between the nonmonotonic version of Shoham's classical logic, defined by the *chronologically smaller in S* preference criterion, to the logic of time structures. All claims refer to a formula Φ of the interval logic, and to Φ_t, its translation into the logic of time structures. The set S is fixed as the set of all primitive terms in Φ_t.

Lemma 3. Let J_i and (I, *tSi),* for i=1,2, be Φ_t-corresponding interpretations and variable assignments. Then,

J_2 is chronologically smaller in S than J_1, (denoted $J_1 \subset_S J_2$ iff $I \models ts_1 \lesssim ts_2$

That is, the correspondence between interpretations preserves the chronologically smaller relationship.

Lemma 4. If J and (I, *ts)* are Φ_t-corresponding interpretations and variable assignments, then

J is a chronologically smallest in S model of Φ iff $I \models \mathrm{CMI}_{\Phi_t}(ts)$.

Theorem 2. $\Phi \models_{\subset_S} \phi$ iff $\Phi_t(ts) \wedge$ {definitions of $\gtrsim$, CMI_{Φ_t} } $\wedge\ \mathrm{CMI}_{\Phi_t}(ts) \models \phi_t(ts)$

We have shown that the notion of chronological ignorance can be implemented in the logic of time structures, using first order tools alone. Below, we present an example of chronological ignorance based reasoning in our logic.

6. An Example

Shoham characterizes two major problems in reasoning about change; the solutions he suggests are within his modal logic of chronological ignorance [Shoham 1987b].

The *qualification problem* is "the problem of having to specify too many conditions in order to make even a single prediction about the future". The *extended prediction* problem is "the difficulty of predicting things about extended periods of time in the future". Shoham claims that the extended prediction problem subsumes the famous *frame* or *persistence* problem.

We consider the Yale shooting example, which is now firmly established in the AI folklore. In [Balaban & Murray 1988], we provide two formulations of this example in our logic that address, respectively, qualification and extended prediction. Here we present the latter formulation which is a generalization of the former. The set of time points is assumed to have the structure of the integers.

Point-wise facts that are expected to persist (like C(l,l,[loaded,0],ts)) are interval-wise statements, with unspecified upper time points. The CMI criterion forces these statements to be clipped at the latest possible time, i.e., to persist as long as possible. The frame axiom is, of course, dropped. This formulation is similar to Shohan's potential histories formulation.

There are *six* axioms:

1. $\exists$ v, d C(1, v, [loaded,d], ts)
2. C(5, 5, [pull-trigger,0], ts)
3. $\forall t, d$
   a. C(t, t, [pull-trigger,0], ts) $\Rightarrow$ $\neg$P(t+1, [loaded,d], ts)
   b. C(t, t, [emptied-manually,0], ts) $\rightarrow$ $\neg$P(t+1, [loaded,d], ts)
4. $\forall t, d$
   ([P(t, [loaded,d], ts) $\wedge$ C(t, t, [pull-trigger,0], ts) $\wedge$ Φ] $\rightarrow$ C(t+1, t+1, [noise,0], ts))

   where Φ is the conjunction of denials of 'abnormalities' like $\neg$P(t, [vaccum, d],ts).
5. Definition of $\gtrsim$
6. Definition of CMI, with Ψ taken as the conjunction of formulas 1 to 4 above.

Under this formulation, the "expected results" are obtained. Although we have no knowledge about persistence of propositions over time, we can conclude that loading persists up to time point 5, and causes the noise at time 6.

7. Conclusion

We have shown that we can formulate Shoham's notion (in his classical interval logic) of chronological ignorance. Clearly, we can formulate his preference criterion on Kripke structures by a preference criterion on models of the logic of time structures. But then we end up with a nonmonotonic logic built on a first order one, rather than on a modal logic. We prefer capturing these notions with purely first order formulas involving time structures.

The main point here is not the imitation of Shoham's work, but that we have implemented some kind of model preference mechanism through axioms in a first order logic. Should we desire an entirely different preference criterion, we might only require a new axiom set; we need not build a new logic, define its semantics, and design its inference or model-building engine from scratch. Furthermore, because of our particular encoding of temporal knowledge, the strong law of the excluded middle does not hold in any of our "worlds." Thus we have not been forced away from first order and toward modal logic.

References

[Allen 1984] J.F. Allen, "Towards a General Theory of Action and Time," *Artificial Intelligence,* vol. 23(2), pp. 123-154, 1984.

[Balaban & Murray 1988] M. Balaban and N. Murray, "The Logic of Time Structures: Temporal, Modal, and Nonmonotonic Features," Technical Report 88-32, SUNY at Albany, 1988.

[Cointe & Rodet 1983] P. Cointe and X. Rodet, "FORMES: A New Object-Language for Managing of Hierarchy of Events," Internal Report, Institut de Recherche et de Coordination Acoustique-Musique, Paris, 1983.

[Dean & McDermott 1987] T.L. Dean and D.V. McDermott, "Temporal Data Base Management," *Artificial Intelligence,* vol. 32(1), pp. 1-55, 1987.

[Halpern & Shoham 1986] J.Y. Halpern and Y. Shoham, "A propositional Modal Logic of Time Intervals," *Proc. Symp. on Logic in Computer Science, IEEE,* Boston, Ma, 1986.

[Haugh 1987] B. Haugh, "Non-Standard Semantics for The Method of Temporal Arguments," *Proceedings, 1JCA1-87,* pp. 449-455, 1987.

[Hayes 1971] P.J. Hayes, "A Logic of Actions," in *Machine Intelligence 6,* ed. B. Meltzer & O. Michie, pp. 495-520, Edinburgh University Press, Edinburgh, 1971.

[Kowalski & Sergot 1986] R. Kowalski and M. Sergot, "A Logic-based Calculus of Events," *New Generation Computing,* vol. 4, pp. 67-95, 1986.

[McCarthy & Hayes 1969] J. McCarthy and P.J. Hayes, "Some Philosophical Problems from the Standpoint of Artificial Intelligence," in *Machine Intelligence,* ed. B. Meltzer and D. Michie, vol. 4, pp. 463-502, 1969.

[McDermott 1982] D. McDermott, "A Temporal Logic for Reasoning About Processes and Plans," *Cognitive Science,* vol. 6, pp. 101-155, 1982.

[Schottstaedt 1983] B. Schottstaedt, "PLA: A Composer's Idea of a Language," *Computer Music Journal,* vol. 7(1), pp. 11-20, 1983.

[Shoham 1987b] Y. Shoham, *Reasoning About Change: Time and Causation from the Standpoint of Artificial Intelligence,* MIT Press, 1987.

[Shoham 1987a] Y. Shoham, "Temporal Logics in AI: Semantical and Ontological Considerations," *Artificial Intelligence,* vol. 33 (1), pp. 89-104, 1987.