

THE INFLUENCE OF NONUNIFORMITY IN THE MODELS OF AUTOMATA
COLLECTIVE BEHAVIOUR

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A number of specific model examples show that the introduction of nonuniformity of any kind into a collective of automata implementing a common task may result in improving the behaviour of the entire collective. Ways of introducing such purposeful nonuniformity are discussed.

Introduction

In mid-fifties I.M. Gel'fand and M.L. Tsetlin have formulated a hypothesis that complex forms of behaviour observed may be explained by the integrated activity of a set of subsystems, each being local by its nature and having limited information on the task of the whole system. This hypothesis has served as a core for a number of collective behaviour models in which the role of local subsystems was played by deterministic or nonprobabilistic finite automata. The choice of such subsystems was dictated by the results obtained by M.L. Tsetlin and his school. They have shown that in a stationary random array such automata provide purposeful behaviour and that under unrestricted increase of their memory depth the behaviour of each subsystem becomes asymptotically optimal [1]. In many cases of nonstationary random array the probabilistic automata with reorganising structure also provide good behaviour with a priori ignorance of the array properties [2]. In [1] and [2] designs of numerous automata providing purposeful behaviour and models of col-

lective behaviour of a uniform collective of such automata are described.

The uniformity of automata collective was useful for two reasons. Firstly, these models have indicated a rather natural approach to their implementation with the help of uniform arrays. Secondly, the uniformity ensured analytical description of the whole set of automata rather than of a single one and that essentially facilitated their analytical study. Besides, most practical models used by the experts in collective automata behaviour were technical and biological models for which the assumption of collective uniformity looked very natural.

A different situation arose when in practical applications models of an individual's behaviour were used in which he faced both his local tasks and those of the whole of his collective. Ignoring the individual differences in such collectives led to no important conclusions from simulation results.

In this paper two kinds of collective nonuniformity will be described. The nonuniformity of a first kind results from the personal experience of each member of the collective. It is the nature of this experience that influences his decision-making. The nonuniformity of a second kind is caused by the fact that certain members of the collective possess more information than the others. The main conclusion is that the introduction of such nonuniformities improves the behaviour of the collective in terms of its

overall task.

"Fishing" model

N devoted fishing fans live in a town. Each Sunday they take their fishing-tackle and go out fishing. The nearest environs of the town have M fishing places but to all fishermen's regret M is much less than N. Therefore they can never count on loneliness. This makes them hold on to a certain "code of honour": have n people gathered at one place, all of their catch is divided equally among them. The local task of each fisherman is evident, to maximise the amount of his catch in a certain time interval T (his summer vacation season, for example). The town has its own task which is to maximise the total amount of all fishermen's catch in a season. In this case the town populations needs for fresh fish are met especially well.

Fishing places are not all of the same value and characterised by some objective and time invariant parameter P_i . This parameter can be treated as the probability of fish being found in the i-th fishing place. But the values of P_i are unknown a priori to fishermen. During the season all fishermen accumulate information on the fishing places. This personal individual experience of each of them leads to two individual estimates π_i^j and ν_i^j where i is the number of a fishing place and j the ordinal number of a fisherman. The estimate or π_i^j serves for approximation of the unknown parameter P_i and represents the mean frequency of fish to be found in the i-th fishing place. ν_i^j approximates the mathematical expectation of the number of fishermen at the i-th place. ν_i^j is practically calculated as the mean number of fishermen during j-th fisherman's visits to the i-th place.

To make a decision on the choice of the fishing place the fishermen are apt to use the estimates of their estimates rather than the estimates themselves. One can consider, for instance, that the four numbers a_1^j, a_2^j, b_1^j and b_2^j are given such that with $\pi_i^j > a_2^j$ the j-th fisherman takes the value of π_i^j for favourable, with $\pi_i^j < a_1^j$ takes π_i^j for unfavourable and with $a_1^j \leq \pi_i^j \leq a_2^j$ takes it for uncertain. (For $m \neq j$ it would be natural to consider the other four threshold numbers). b_1^j and b_2^j are used to estimate ν_i^j . With $\nu_i^j < b_1^j$ the value of ν_i^j is taken for favourable, with $\nu_i^j > b_2^j$ for unfavourable and with $b_1^j \leq \nu_i^j \leq b_2^j$ for uncertain. If the value of 1 is assigned to the favourable estimate, 0, to unfavourable and 1/2 - to uncertain, then each fishing place can be estimated by the j-th fisherman as a set of two components $\langle \xi_i^j, \eta_i^j \rangle$ where ξ_i^j and η_i^j can take their values from the set $L_3 = \{0, 1/2, 1\}$. Time and experience can change this set of estimates for a certain fishing place.

To make his decision a fisherman should choose among the fishing places with a given set of estimates for all fishing places in the form or $\langle \xi_i^j, \eta_i^j \rangle$. It is natural to suppose that all entire fishing places are estimated by the fishermen on a three-stage scale as well: a good place, a poor place and an uncertain place. Presenting these estimates in the form of L_3 a set elements the fisherman can estimate the i-th fishing place from the mapping $L_3^2 \rightarrow L_3$. On obtaining the estimates for all the fishing places the choice of the next place to go fishing can take into account all the places having the highest estimate with equal probabilities. The function which implements the mapping of L_3^2 into L_3 is a usual ternary logical function.

It is difficult to imagine that with a larger graduation of estimates for π_i^j and ν_i^j or for the fishing places nothing

principally changes but the mapping $O1$ L_3^2 onto L_k which is implemented by a K-ary logical function. With the increase of the number of parameters for fishing place estimate (for instance the distance to a fishing place from the town) the logical function will only have a different number of arguments because with q estimating parameters it will be necessary to organise the mapping of L_k^q onto L_k .

To describe the individual differences of the fishermen one can use the threshold values a_1^j, a_2^j, b_1^j and b_2^j and the form of the function implementing the mapping of L_3^2 onto L_3 . It can be shown that; varying these thresholds is in a certain sense equivalent to varying the form of the mapping function. Therefore we shall further assume that the thresholds do not depend on a fisherman but reflect their consensus on "what's good and what's bad". Their individual differences will therefore be determined only by the form of the mapping function.

For a "FISHING" model it is natural to consider the mapping function to be of conjunctive nature since the fishing place estimate depends simultaneously on the estimate of π_i^j and that of ν_i^j . The following table shows several functions which can be used as the estimating ones.

The function ψ corresponds to the usual ternary conjunction. Such function is characteristic of an objective fisherman who estimates fishing places with no emotions or assumptions. The functions ψ_0^1 and ψ_0^2 can be called optimistic. The fishermen using them are apt to consider that "the world is good". With lack of information on something and thus no chance to estimate it the optimists consider such estimate to be a favourable one. The function ψ_0^1 is characteristic of a careful optimist inclined to change just one value of 1/2 for 1. Under complete uncertainty considers that "every thing will be alright". On the contrary, the functions ψ_p^1 and ψ_p^2 are characteristic of the pessimistic fishermen. They always think that the "world does not love them" and "what good they would see from it"?

A first-rank pessimist, or a careful one with one uncertain estimate in the set still dare not substitute it for unfavourable but he does it for the set $(\frac{1}{2}, \frac{1}{2})$ without hesitation. Whereas a frantic pessimist replaces all estimates of 1/2 by 0.

In computer simulation of the fishermen behaviour a uniform collective was first considered with the common estimating functions for all the members of the collective. Under these conditions the "FISHING" problem turned into an insigni-

ξ_i	η_i	ψ	ψ_0^1	ψ_0^2	ψ_p^1	ψ_p^2	ξ_i	η_i	ψ	ψ_0^1	ψ_0^2	ψ_p^1	ψ_p^2
0	0	0	0	0	0	0	$\frac{1}{2}$	1	$\frac{1}{2}$	1	1	1/2	0
0	$\frac{1}{2}$	0	0	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$\frac{1}{2}$	0
$\frac{1}{2}$	0	0	0	0	0	0	1	1	1	1	1	1	1
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0							

ficant modification of the well known problem of distribution of automata among feeding-racks* As in the classical approach the process of redistribution of automata among the feeding-racks (fishing places) was established but the rate of reaching the stationary mode of redistribution was greater because of complication of automata behaviour due to the differences of their individual experience. "FISHING" is a Goor game where the payoffs of the participants depend not on the strategy chosen by some participant (in our case not on the choice of the place for fishing) but on participants' distribution over the possible strategies. Introducing into the Goor's game the common bank according to which the payoffs of all participants are being added up and divided equally among them at each step of the game may help to obtain the maximum of the overall payoff for the whole of the collective (in our case to obtain the over-all towns' goal).

The disadvantage of common the common bank is the system's central distribution, to implement which there should be a special clearing house. For the "FISHING" model this means that all the fishermen should deliver their catch of the day to one center wherefrom they receive an N -th part of the overall catch in return.

Introduction of nonuniformity has helped to do without such central distribution body. Let us consider the results of a simulation experiment. During it an additional assumption was made that the choice of the fishing place with no fish gives a fisherman a constant loss reducing his mean income collected on the previous days of the season. Besides a certain threshold was assumed to exist, Exceeding this by the amount of catch gives a fisherman some income proportional to this excess. With the

catch lower than this threshold a fisherman has neither loss nor income. A case was considered with $M = 4$, $N = 32$, $a_1 = 0,25$, $a_2 = 0,75$, $b_1 = 8$ and $b_2 = 24$. The simulation variants differed in the fishermen collective structure.

For a uniform collective consisting exclusively of objective fishermen or frantic optimists the mean input of each member was 118 conditional units after the setting of redistribution process. The uniform collective consisting of frantic pessimists managed to get an income of 134 units. All other uniform collectives obtained worse results in comparison with these first. Mixed collectives in a number of cases showed much better results. An experimentally composed collective of 8 first-rank pessimists (careful ones) and 24 objectivists could gain a mean income of 158 units which corresponds to the maximal possible mean input in this game.

Study with different parameters and estimating functions (e.g. disjunctive) has shown that a nonuniform collective providing a global income with the help of a common bank procedure can always be composed.

The "FISHING" model is easily transformed into various models or decentralised resource allocation or into the models of the bulk service or queueing theory.

The "NEIGHBOURS" model

In a droughty place some country cottages are located round the foot of a hill. There is a water source on the hill which can be quite sufficient with reasonable distribution of watering time among the cottage tenants. But in this problem the tenants do not wish or just cannot arrange such a reasonable timetable. They simply do not seek any kind of contacts most probably considering it to be the best way of keeping neighbourly. The low fences between cottages allow

each tenant to watch the others and thus adjust his own watering activity. For purposeful use of the water supply it is necessary that only one half of the cottage ground be watered at a time. Whenever more than a half the tenants open the water a certain number of grounds are being automatically switched off from water supply.

To put things in order the water supply service has decided that all the tenants whose cottages have even numbers should water their grounds before the noon, and all the rest, after the noon. But the cottage tenants would not obey this order of the water supply service. Then it was decided that all the tenants violating the order should be fined with switching off their grounds from water supply for a certain time period. After several fines they stopped supplying such cottage ground with water at all.

A natural question is: can these conditions and such a technique enforce order in water supply and not ruin any of the cottage grounds by leaving it without water.

Let us formulate the problems in a more formal way. There is a chain of $2N$ automata connected in a ring. Each automaton has information on its closer neighbours' states. There are only two such states, 0 and 1. The collective's task is to transform the automata into the states, forming a sequence of $a_1,$

$a_2, a_3, \dots, a_{2N}, a_1$ of the form of 0, 1, 0, ..., 1, 0 on this ring. The task of any of them is to minimise the overall fine for exceeding a certain fine threshold with its overall fine results in the automaton.

At each cycle the automaton may choose one of the two possible actions corresponding to the choice of this or that state. In the simplest case this choice is made through the analysis of the neighbour automata states at a time neglecting the past history of the automaton. This choice is made using the Betow following table. When the choice is not single-valued the automaton chooses either state with equal probability. The initial state distribution is given at random. The question of setting the necessary order in such a collective may be answered analytically through the analysis of the initial situation. In order to avoid bulky computations of the corresponding Markov system states one can simulate the problem on a computer. As a result not a very consoling fact would emerge that the system of this kind almost never settle in a desired way. The model may be slightly improved by introducing the fines received from the environment (the Water Supply Service). If after the automaton has made its choice situation 000 or 111 arises, the automaton is fined. After a certain number of fines it is switched off and

state of neighbour on the left	state of neighbour on the right	own state	choice
0	0	0	1
0	0	1	1
0	1	1	1 or 0
0	1	0	1 or 0
1	0	1	1 or 0
1	0	0	1 or 0
1	1	0	0
1	1	1	0

its state kept fixed during the subsequent cycles. But even after that the whole system does not work very satisfactory.

Let us introduce the nonuniformity into the automata collective under study using the concept of reflection ranks, thoroughly analysed in [3] by V.A. Lefevre. Following this concept let us inductively introduce the reflection rank for the automata in our model. We shall consider the automaton to have a reflection rank of zero if its functioning is completely defined by the above mentioned system state table. The automaton's reflection rank is taken for 1 when it works in the following way. First it watches the neighbouring automata change their states and then chooses its own state through forecasting its neighbours' behaviour. In general we shall say that the automaton has a reflection rank equal to l if it considers his neighbours to be automata with reflection ranks equal to $(l - 1)$.

The increase of the reflection rank in our model results from the increase of the amount of information coming at the input of the automaton under study. The automaton with zero reflection is only informed of the states of his nearest neighbours. The automaton with a reflection rank of 1 should possess the information on the states of both two of his righthand and two of his lefthand neighbours. The automaton with a reflection rank of l should use the information on the states of $(l + 1)$ neighbouring automata, both righthand and lefthand.

Two models may be studied. The first does not change the automata reflection ranks in the process of their functioning. The second may change the automata reflection rank by the fining signals from the environment. The first model appears to be greatly dependent on the random reflection rank distribution over the automata collective. To solve our

problem of the automata collective's achieving the sequence of states of $0, 1, 0, \dots, 0, 1$ it is most desirable to have the reflection ranks of 0 and 1 distributed among the automata in the same order. In this case with any distribution of the initial states over it the collective still comes into the desired global point.

The second-type model always solves the problem providing that one may use zero and first rank reflections. Use of other reflections does not always bring the desired results to the collective. The collective having automata with zero and second rank reflection behaves in a less purposeful way than a uniform collective with zero reflection. For the first type model one may show such a reflection rank distribution among the automata which zeroes the collective into the desired point.

We shall postpone until a little later the discussion of the results obtained from the "NEIGHBOURS" model and now switch over to one which is a direct generalisation of it.

The "PATTERN" model

Let the automata now be located not in the form of a ring, as in the "NEIGHBOURS" model, but fill in some torus. Let us consider that each of the automata with zero rank reflection has eight neighbours the complete information of the states of which is available at any time. (Again we assume that there are only two states, 0 and 1). The automata collective's task is to design a certain pattern on the torus defined by its elementary part being represented by a 3×3 matrix. This matrix is known to the automata. Each automaton's task is to choose a state that would allow it and its eight neighbours to design a pattern after the model known. The initial automata states at the torus are taken arbitrarily.

The direct generalisation of the "NEIGHBOURS" model for the torus is the model where the automata should design a chess-board pattern. The algorithm of each automaton's work is defined as follows. The automaton compares its own state with that of the central square of the model matrix. After that it counts up the number of its neighbours with the states differing from those of the model matrix. The automaton changes its state with a probability proportional to this overall number.

The computer simulation made by Ye.T. Semionova has shown that the uniformity of a collective led to obtaining only the simplest patterns like those of a chess-board, or of horizontal and vertical stripes. More complicated patterns, for instance, the one given by a matrix

$$\begin{matrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{matrix}$$

are not obtainable. But the introduction of the non-uniformity over the reflection ranks both in the "NEIGHBOURS" and in the "PATTERN" models has resulted in obtaining practically any kind of patterns given by a 3×3 matrix.

Conclusions

All above allows making important conclusions on the advantages of a non-uniform collective of automata when solving many practical problems. The problems of the "FISHING" type were already mentioned. The "NEIGHBOURS" and "PATTERN" type models can easily be transformed into many models having great importance when applied to constructing the uniform arrays and most probably in biological and microbiological models. Sociological analogues of the models considered are also easy to be shown.

Another conclusion is that the control in nonuniform collectives is most effective when the control over the entire

distribution of this nonuniformity is possible. Notice that in performing creative tasks animals and people should adapt to the environment obtaining from it the necessary control for a nonuniform collective (see, for instance, [4], [5], [6]).

All this shows the models of collective behaviour of the nonuniform community to be of greater interest for all the scientists engaged in any way in solving the problems of artificial intelligence for a collective of individuals.

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