APPLICATION OP METHODS OP ADAPTIVE LEARNING TO THE PROBLEM OF AUTOMATIC DECISION MAKING UNDER THE CONDITIONS OP APRIORI UNCERTAINTY

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The paper deals with the adaptive approach to deoision making as applied to the problems of biological and medical cybernetics characterised by apriori uncertainty. A number of algorithms is suggested which, being based on the an allsis of aposteriory information, yield the most adequate decision rule to a specific problem. The learning is achieved by the stochastic approximation method and by means of the correction of the decision rule structure The methods discussed have been realized on a computer and used in a cybernetic medical centre.

introduction

The basic difficulties of the artificial intelligence simulation in biological and medical applications are connected with problem of choosing of techniques and algorithms of decision making. These problems are characterised by a lack of apriory information which prevetns the use of the classical techniques of the theory of automatic decision making. A way out may be found in the application of the ideas of adaptive learning. Let us consider the formal approach to the problem.

Let $\{\Omega, \mathcal{A}\}$ and $\{\Delta, \mathcal{E}\}$ - two measurable space, i.e. the observation space and the decision space, respectiveli. OnΩ a finite family $\mathcal{P}$ of probability measures is given. Based on a certain rule $\mathcal{S}$ probability $P_\omega(\cdot)$ is built which is a measurable image from Ω to$\{\Delta, \mathcal{E}\}$. $\omega \in \Omega$ being fixed then $P_\omega(\cdot)$ is a probability measure on$\{\Delta, \mathcal{E}\}$. In case the family $\mathcal{P}$ is fully defined the construction of the decision rule $\mathcal{S}$ is clear. When, however, the family has undefined parameters, the decision rule will have the same degree of uncertainty, to be specified with the accoxnulation of observation data.

Adaptive procedure for the estimation of the potential function parameters

Let the probability measure on $\{\Delta, \mathcal{E}\}$ is built by the procedure of the potential function method. As is well known [1] there are two realisation of potential function method, i.e. the perception and the computer ones. the perception realisation deals with a system of basic function $\mathcal{U} = [\mathcal{Y}_i(\omega), \omega \in \Omega, i = 1, 2, \dots]$,while the computer realisation deals with a potential function $K(\omega_1, \omega_2)$, which ie a symmetrical kernal on $\Omega \times \Omega$ and is connected with the system $\mathcal{Y}(\omega)$ through the relation

$$K(\omega_1, \omega_2) = (\mathcal{Y}(\omega_1), \mathcal{Y}(\omega_2))$$

If $f(\omega)$ is any function on Ω , approximated using observation $\omega \in \Omega$ and $C = \{c_1, c_2, \dots\}$ parameter vector, then the condition for choosing the system $\mathcal{Y}(\omega)$ is the nearness of $f(\omega)$ to its approximation

$$\hat{f}(\omega) = (C, \mathcal{Y}(\omega))$$

The procedure of the function approximation when using computer realisationis as follows

$$\hat{f}^{n+1}(\omega) = \hat{f}^{n}(\omega) + \gamma_n [z(f(\omega^{n+1}), \hat{f}^{n}(\omega^{n+1}))] K(\omega, \omega^{n+1}) \quad (1)$$

where $z(f, \hat{f}^n)$ -is a certain function of two variables;
γ_n -is a nonnegative sequence, such as

$$\sum_{n=1}^{\infty} \gamma_n = \infty, \quad \sum_{n=1}^{\infty} \gamma_n^2 < \infty$$

The lack ofapriori information on the family $\mathcal{P}$ prevents the use of the recommendations in [1] for the constructior of a suitable potential function.

However, one may restrict oneself to a certain parametrs family $\mathcal{R}_\alpha$, having the necessery properites. It is, for example, the families of unimodal functions of distance in Ω ,such as

$$\mathcal{R}_\alpha(\omega_1, \omega_2) = \lambda \exp\{-\alpha \rho^2(\omega_1, \omega_2)\}$$

or

$$\mathcal{R}_\alpha(\omega_1, \omega_2) = \lambda \{1 + \rho^2(\omega_1, \omega_2)\}^{-1}$$

where λ - a constant;
$\rho(\omega_1, \omega_2)$ - the distance in Ω .

In this case the best potential function construction is reduce to the estimation of the parameter α , optimal in a certain sense. Let the decision set Δ constists of two elements and the approximated function $f(\omega)$ satisfied the relation

$$f(\omega) = \begin{cases} > 0 & \omega - \text{corresponds to } \delta_1 \\ < 0 & \omega - \text{corresponds to } \delta_2 \end{cases} \quad (2)$$

Then functional

$$I_\alpha(\omega) = -\hat{f}_\alpha(\omega)[\operatorname{sign} f(\omega) - \operatorname{sign} \hat{f}_\alpha(\omega)],$$

can be defined, where
$\hat{f}_\alpha(\omega)$ - the functional from (1) obtained with the potential function $\mathcal{R}_\alpha(\omega_1, \omega_2)$ and satisfying the condition samilar to (2).

Obviously, $I_\alpha(\omega)$ has the following properties:
$I_\alpha(\omega) > 0$, if $\operatorname{sign} f(\omega) \neq \operatorname{sign} \hat{f}_\alpha(\omega)$ i.e. when the decision is wrong
$I_\alpha(\omega) = 0$, if $\operatorname{sign} f(\omega) = \operatorname{sign} \hat{f}_\alpha(\omega)$ i.e. whenthe

decision is correct.

The optimal value of will be defined by the conditions of the functional minimum

$$J_\alpha = E_\omega I_\alpha(\omega) = E_\omega \left\{ -\hat{f}_\alpha(\omega) [\operatorname{sign} f(\omega) - \operatorname{sign} \hat{f}_\alpha(\omega)] \right\} \quad (3)$$

The Kifer-Wolfowits procedure can be used here

$$\alpha_{n+1} = \alpha_n + \frac{\gamma_n}{c_n} \left[I_{\alpha_n + c_n}(\omega^n) - I_{\alpha_n - c_n}(\omega^n) \right], \quad (4)$$

where γ_n, c_n - are the non-negative sequence satisfying the conditions

$$\sum_{n=1}^{\infty} \gamma_n c_n < \infty \qquad \sum_{n=1}^{\infty} \left(\frac{\gamma_n}{c_n} \right)^2 < \infty$$

If instead of this we use the regression equation

$$\nabla_\alpha J_\alpha = 0$$

then the solution is given by the Robbins-Monro procedure

$$\alpha_{n+1} = \alpha_n + \gamma_n \nabla_\alpha I_\alpha(\omega^n), \quad (5)$$

where

$$\sum_{n=1}^{\infty} \gamma_n = \infty \qquad \sum_{n=1}^{\infty} \gamma_n^2 < \infty$$

For a more precise account taken of local properties of the observation space Ω the family of subsets $A = \{A_i, i = \overline{1..}\}$ can be defined, such as

$$\bigcup_{i=1}^{N} A_i = \Omega \qquad A_i \cap A_j = \emptyset \quad i \neq j$$

Then α can be considered as the function of two variables denoting

$$\alpha(\omega_1, \omega_2) = \alpha_{ij}, \text{ if } \omega_1 \in A_i \quad \omega_2 \in A_j$$

It is only natural to suppose $\alpha_{ij} = \alpha_{ji}$ and changing the learning aim (3), to estimate sequentially the symmetrical parameters matrix $A(N \times N)$ by (4) or (5).

The optimal choice of the threshold function

Usually the procedure of decision making ends with the construction of the probability measure $P_\omega(D)$ on the decision space and with the choice of the decision $D \in \mathfrak{D}$ with the maximum value $P_\omega(D)$. However, in concrete problems this condition is insufficient. It becomes necessary to introduce the threshold function $T = T(\omega)$ and the decision $\delta(\omega)$ will be accepted if

$$P_\omega(\delta(\omega)) \geq T(\delta) \quad (6)$$

and otherwise rejected. Actually, if a certain measurable partition Ω, such as

$$\delta^{-1}(\Delta) = \Omega \quad \delta^{-1}(D) \in A, \quad \forall D \in \mathfrak{D},$$

then the introduction of the threshold is eqeivalent to the introduction of a subset $A^* \subset A$, such as

$$P_\omega(\delta(\omega)) < T(\delta), \quad \forall \omega \in A^*$$

and of a corresponding additional decision $D^* = \delta(A^*)$ i.e. refusal of decision making.

The introduction of the threshold function is similar to the threshold system in [2], where the algorithm for searching such system is given based on the gradient searching of optimum of any convex criterion. Below the algorithm is suggested for the determination of thresholds, different from that of [2].

The algorithm suggested is connected with a definete correlation between the number of errors of the first and second types for the accepted decision.

It is clear based on the analisis of a verified sample of elements $\omega \in \Omega'$ the true decision $D_0 = \delta_0(\omega)$ is known. Suppose basing on the observation $\omega \in \Omega'$ the hypothisis on the decision making $D^*(\omega)$ was suggested, and for arbitrary $D \neq D^*$

$$P_\omega(D^*) > P_\omega(D) \qquad D, D^* \in \mathfrak{D}$$

Let the error of the first type for the rule (6) be the event

$$\left\{ \{ P_\omega(D^*(\omega)) < T(D^*) \} \cap \{ D^*(\omega) = D_0(\omega) \} \right\}$$

and the error of the second type the event

$$\left\{ \{ P_\omega(D^*(\omega)) > T(D^*) \} \cap \{ D^*(\omega) \neq D_0(\omega) \} \right\}$$

Consider a certain decision $D^*(\omega)$ and determine for it the value of the threshold. For the decision D^* the upper threshold be

$$T^u_{D^*} = \inf_{\omega \in \Omega} \{ P_\omega(D^*) \} \qquad D_0(\omega) = D^*(\omega)$$

and the lower threshold for the decision D^* be

$$T^l_{D^*} = \sup_{\omega \in \Omega} \{ P_\omega(D^*) \} \qquad D_0(\omega) = D^*(\omega)$$

Thus substitution in the right part of the inequality (6) of $T^u_{D^*}$ will lead to the fact that will be no errors of the first type and the substitution $T^l_{D^*}$ will lead to the fact that be no errors of the second type.

The properties will obviously hold true for any $t \leq T^u_{D^*}$ and $t \geq T^l_{D^*}$.

Assuming the set Ω' not larger then countable, define the function of waste $N^1_{D^*}(t)$ equal to the number of errors of the first type for the set Ω' depending of the threshold value and $N^2_{D^*}(t)$ equal to the number of errors of the second type. These functions are characterised by the following properties:

$N^1_{D^*}(t) \equiv 0$ when $t < T^u_{D^*}$ and it is decreasing function when $t > T^u_{D^*}$

$N^2_{D^*}(t) \equiv 0$ when $t > T^l_{D^*}$ and it is increasing function when $t < T^l_{D^*}$

Let $Q^1_{D^*}$ and $Q^2_{D^*}$ is the decreasing functions, which are the waste of the first and secons type errors when dealing with the decision D^*. The total waste for the rule D^* when the

threshold value $T(D^*)=t$ is

$$Q_{D^*}(t) = Q^1_{D^*}(N^1_{D^*}(t)) + Q^2_{D^*}(N^2_{D^*}(t))$$

then the threshold value T_{D^*} will follow from $Q_{D^*}(T_{D^*}) = \min\limits_{t \in (T^\ell_{D^*}, T^u_{D^*})} Q_{D^*}(t)$

The area where the minimum is searched, is determined the following. If $T^\ell_{D^*} < T^u_{D^*}$, then

$$Q_{D^*}(t) = \text{const} \qquad t \in (T^\ell_{D^*}, T^u_{D^*})$$

$$Q_{D^*}(t') < Q_{D^*}(t) \qquad t' \in (T^\ell_{D^*}, T^u_{D^*})$$

Then the threshold value T_{D^*} may be chosen any of $T_{D^*} \in (T^\ell_{D^*}, T^u_{D^*})$, if $T^u_{D^*} < T^\ell_{D^*}$, then taking T_{D^*} external to the interval $(T^\ell_{D^*}, T^u_{D^*})$ we shall obtain an increase in the number of errors of one type without the decrease in the number of errors of the other type.

The thresholds thus determined are equal to those in [2], if

$$Q^i_{D^*}(N^i_{D^*}(t)) = N^i_{D^*}(t) \qquad i = 1,2$$

Applications

The adaptive learning techniques suggested above have been realized as algoritms and routines for the computer ODRA-1204 and have been used in the research centre of the biological and medical cybernetics in Leningrad in the dessing of the cybernetical medical centre.

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