

## ON THE ROLE OF MATHEMATICAL LANGUAGE CONCEPT IN THE THEORY OF INTELLIGENT SYSTEMS

H.Andreka, T.Gergely, I.Nemeti  
Hungarian Academy of Sciences, Budapest, Hungary

### Abstract

The aim of our paper is to show that a general theory of Intelligent Systems /IS/ can be based on the mathematical logic and on its model theory. A descriptive definition of IS is given. One can see that the investigation of some important aspects of IS's functioning lead us to realize the necessity of a general language concept. The definition of the language is given as a triple:  
language = <syntax, set of possible worlds, validity>.

The usefulness of this language concept for describing the IS functioning is shown. Mathematical background of this concept gives the possibility for the foundation of the formal IS's theory.

### Introduction

The research of Artificial Intelligence /AI/ is highly ramified as it can be seen from [1] and it doesn't have a unified theory that would coordinate the different branches of researches and that would provide the compatibility of results

One of the main reasons of the superficial connection between the different research branches of AI is that they make use of and develop the methods of various disciplines, i.e. psychology, mathematical logic, programming theory, algorithmic theory, probability theory, control theory, linguistics, etc. Of course this diversity of disciplines may also be explained by the fact that much effort is made on the satisfaction of actual practical requirements in the development of AI.

In our opinion it is necessary to establish a main theory based on mathematics so as to bring these diversive researches onto common base or at least to have well definable common points in

them. Such role could be fulfilled by the so called theory of intelligent systems. This would provide methods for the investigation of the natural and artificial interpretation of the intelligence\* This theory presupposes the abstract definition of the intelligence as well, as it provides means for the description of the structure and operation of such system.

In this study we give a descriptive definition of the intelligence and we introduce such a definition of the language that provides a useful means for the mathematical foundation of the theories of intelligent systems.

### Intelligent systems /IS/

So far there has been innumerable literature published in connection with IS, and its brief summary can be found in [2].

Since the only one "intelligent" system so far known is the human being therefore he or, the society is focused in the concepts, and the cognitive process has the chief role respectively.

Here we try to give the intuitive level, descriptive definition of the IS. First we define the concept of the system. We mean by system the manifold of such elements or units among which there is a regulated transport /of energy, information e.g./ and its elements are differentiated and specialized. In other words there is a division of functions among the elements of a system. The system is open if it is in mutual effect with its environment and it is closed in the contrary case. An open system may be passive while its functioning if this mutual effect

realises only on behalf of the environment and it may be active if the system is able to effect its environment. The functioning of a system may occur on several different levels /energy, information, e.g./.

From the point of view of the intelligence of the system the functioning on the informative level has a decisive role. The quality of this functioning level differentiates the IS from other systems even more, this differentiates between the interpretations of IS-s. So the IS can be created by the description of functioning on the informative level. In the following we shall always mean informative level by functioning unless otherwise defined. Information is meant on the intuitive level and therefore it isn't defined separately.

A system is intelligent, if

- its functioning ensures /provides/ the storing elaboration, transformation and generation of information and it can be represented through problemsolving processes.
- its functioning always aims to reach certain actualized goals. This means that the system has a hierarchy of goals out of which several may be actualized or may be dominant in a given period. These goals influence the functioning of the system.
- its functioning is controlled predictably i.e. the elaboration of the functioning plan precedes the execution of a concrete procedure. This will build up the actual functioning from its components /elementary actions, e.g./ and it will consider their expected effect from the point of view of the actualized goals.

- the system is double - it may be open or closed.

As an open system it may be either active or passive. Its functioning is characterized by the fact that the information obtained from an open system is processed as a closed system. From the structure of the IS it is only required to ensure above functioning.

Here we note that specifications of IS such as learning, or evolution suppose actualization of certain goals and a functioning to reach these. The functioning of the IS is connected with its environment. Therefore to obtain its actualized goals it is connected in most cases with certain knowledge acquired about the environment. We consider cognitive process as a function collecting information about the environment. The cognitive process always aims to describe a kind of object, we note here that the cognition of IS may direct to itself\* The IS may be in connection with other systems, thus there can be cooperative, conflicting IS-s. In our present study we investigate the functioning of an IS in its effect on its environment and we neglect the fact that this IS may be in complicated relation with other IS.

#### Intuitive description of functioning of IS

As we have already mentioned the functioning of IS is connected with the cognition of its environment. Therefore the functioning will be discussed from the point of view of such an imaginary observer whom about we suppose that it possesses the necessary and complete description of the IS and of its environment. First let's see the environment of the IS from the point of view of the observer. This is part of some kind of a world.

Instead of defining the idea of world we give some examples\* A tree, a room, the nature can all be worlds. The investigated IS can also be the part of the world containing its environment. The IS does not know the world in its totality and it cannot get acquainted with it. The reason of this is that the world is infinitely complex, e.g. in its material structure. But in the same time this world is the element of an infinite set of worlds. E.G. such set of worlds are the possible trees, possible rooms, etc. The IS can examine a finite subset of the set of worlds in its cognitive process and as we have mentioned it can investigate only that much, as the environment reflects from them. Instead of using the expression environment we shall be using the expression world, because cognition is focused on the world and the environment is a part of it accessible for the IS. Considering the characteristics of IS we face the following dialectic contradiction:

/ i / the world is infinite, infinitely complex etc.

/ iii / the IS is finite but extendable without limits. In other words we can view the system as having a finite alphabet and a finite memory e.g. a finite number of sheets of papers with marks, sentences etc. on it. The system can always produce new sheets. write new sentences on these and invent new symbols into its alphabet but nevertheless the information actually written down in papers is finite.

/See for analogy a Turing machine\*/

The representation of the world, and the representation of the connection between the world and the IS should explicitly reflect the contradiction described

above. Thus the representation should reflect the information the system has about the world and also the uncertainty /lack of information/ of the system about the world\* We can represent this by defining the class of all possible worlds which are compatible with the system's present knowledge.

So if we are given the system's present knowledge in internal terms, that is in terms of thoughts /strings of symbols and algorithms which are again strings of symbols/, the external representation of the system's uncertainty is the class of all worlds which are possible from the system's subjective point of view. The system only knows that the real world it lives in is one of these possible worlds. /This uncertainty is not always negative, since there are things which the system does not want to know about simply because they are utterly irrelevant for all its purposes. This positiveness of uncertainty will be described later as the "choice of the adequate abstraction"./ During knowledge acquisition the class of possible worlds gets smaller and smaller. However this class will never be finite and the goal of the system is not simply to make the class of possible worlds as simple as possible but rather to make this class to have certain properties. Of course this will be done by making the class of possible worlds smaller, but then there will always be an orientation in it which is more subtle.

Let us now investigate the role of a general concept of language in the description of the cognitive process of an IS.

Language must be able to describe the possible worlds to some extent, hence it must be based on a finite set of symbols and rules /grammar/ that underline the formation of expressions suitable for the description of the different properties of

some or other worlds.

Thus a language must consist of a syntax /grammar/ and a semantics /meaning, denotation/. Syntax consists of a set of strings of symbols /texts/ and the system of rules working on these texts. Semantics of the language characterizes the connection between the syntactically expressions and the possible worlds. It consists of a pair of the set of all possible worlds and validity. Consequently a language must contain the following components:

Language = \syntax, set of possible worlds, validity> •

That is symbols a language L is a triple:

$L = \langle F, M, k \rangle$  where P is the set of the possible thoughts /sentences/, M is the class of all possible worlds and k defines the meaning /the denotation/ of the sentences in the worlds. Generally k is a function with domain  $F \times M$  which to each thought - world pair gives the meaning Of that thought in that world. For example the meaning of a declarative sentence in a world might simply be a truth value /a yes or a no/; the meaning of a noun /in a world/ might be an object in that world.

To apply mathematics we should represent every object of the system in consideration by a mathematical object. That is F should be a set of mathematical objects /this is easy to satisfy/, M should be a class of mathematical objects /and k is already defined as a function so it is a mathematical object/. To choose M is the most difficult and most important step in mathematical logic. To choose the mathematical objects constituting M means to choose the mathematical representation of the world /or more precisely the class of possible worlds/. For example the difference between classical logic, modal logic, temporal logic etc, can be originated

from choices of M. Having chosen M, we find that F /and k/ is similar to the problem of finding an adequate form to a content. E.g. different F-s might be more or less adequate to M and also might have more or less expressive power for M. The mathematical objects in M are called models, because they are only the mathematical models of the possible real worlds.

Accordingly "Model Theory" is that branch of mathematical logic which concentrates on the models /on M/ and investigates the relationship between the models and the syntax of the language. In other words Model Theory is the mathematical science of semantics. The study of LI in its own right /without considering its role in the language and in the thinking process/ is called universal algebra. The study of F is a purely syntactical exercise, and it involves the theory of algorithms and grammars.

Some of the failures of early automatic theorem proving were caused by the fact that the logicians concentrated much more on the syntax /F/ than on the semantics. /Of course, by purely syntactical means, by simply combining the elements of F into new elements we get conginatorical algorithms which are rather inefficient/.

The importance of the choice of M is also well illustrated by some recent critics of logic. Usually the criticism was that this and this cannot be represented /investigated/ by logic. But in all such cases a certain fixed system was understood by logic and the cause of the impossibility of representation was the given choice of M. Mathematical logic however is not devoted to the investigation of a unique, pre-specified language  $\langle F, M, k \rangle$  but instead,

it investigates the properties of such languages /and also systems of such languages/ in general and gives tools to construct that concrete language which is adequate in some given situations or problem domain.

These tools give the pragmatic aspect of language. It is also true that in doing so Mathematical logic investigates some fixed languages in very great detail, the reasons for which is that certain languages turned out to be rather practical kernel-stones for nearly all other languages. Such a kernel-stone is the classical first-order logic,

Now we turn back to the concept of language. We note, that the concept of language in itself is not enough to the investigation of the IS, but is an indispensable building-block in this investigation. The concept of a deductive logic will be much closer, but it will be still a building block only, We also note that an IS is not supposed to function in one fixed language, it might have many different languages or hierarchies of languages and the ability of generating new ones, etc,

A state of knowledge of an IS, using the language  $\langle F, M, K \rangle$  is represented by a set of sentences, that is by a subset  $\Sigma$  of  $F$ .  $\Sigma$  is that explicit knowledge which has already been formulated by the IS. Now the set of possible worlds for the IS is the set of those elements of  $M$  which are compatible with  $\Sigma$ . /Since the IS already knows that  $\Sigma$ ./

So far we have described the basic reason why the whole structure of logic is based upon is a class of possible worlds. We said that the IS simply does not know which is the real one /and in certain respects the IS does not want to know/. However, there are more subtle reasons

for having a class of possible worlds and also these classes play more subtle roles in the investigation of cognition.

It might be the case that the IS investigates many different parts of the world independently. It treats these parts of the world as independent worlds. /Of course this means that the IS is not using always the same language since one language is used to "live in these worlds" and another is needed to go to one of these world, then leave it, go to another or do quite different things, e.t.c./

If the IS considers these worlds analogous somehow and decides to make use of this analogy, then it forms a language the models of which are these worlds, together with potentially existing parts of the world which might be visited by the IS sometimes in the future. Thus the variety of different situations where the IS wants to apply the same knowledge. E.g. for a carpenter each piece of furniture is a different world where the carpenter applies the same knowledge. Of course there might be special pieces of furniture which require some additional specific knowledge /and language, of course/ but the role of the language is not changed by this, only the complexity of the system of interacting languages has risen a bit.

All this means that the formation of a new language /from the existing ones/ during some problemsolving processes i.e. during the operation is more frequent. Of course formation of a new language is also caused by the basic process, when the IS investigates the same world and the variety is purely caused by absence of knowledge.

Now we only mention that there are other still more subtle generators of the variety of possible worlds. One of these is the following: The IS can /and in many situations must/ view the same piece of

the real world in infinitely many different ways /e.g. we can investigate the atomic structure of a tree, its statical properties, we can view the armchair as a world in which lots of different bugs can live, or its sensitivity for fire, how it conducts heat or electricity, etc./. From the logical point of view these are different worlds /certainly they give rise to different models/ which in the same time are just aspects and abstraction levels of the same complex world, that is they can /and should be synthesised, or organised into one.

Of course these different ways of looking upon the same object might give birth to different languages, but there also might be different worlds of the same language /as in the case of interdisciplinary sciences happens/, and also might be synthesised into one complex world of one language.

#### Conclusion

As it could be seen from above the new language concept can provide adequate methods for the formal description of cognitive process of the IS.

By this we wished to point out that the research of IS can be the real user of present researches in mathematical logic, The newest achievements of mathematical logic may become a suitable base for creating a unified theory of IS. This theory can only be accomplished by the coordination of researches in mathematical logic and in IS.

#### References

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