Propagation of seismogenic electric currents through the Earth's atmosphere

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Key Points:

- Lithosphere-Atmosphere-Ionosphere Coupling
- Seismogenic Electric Currents

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Abstract

Seismogenic models have been recently proposed to explain precursors before earthquakes occurrences. Those models refer to physical processes linking the lithosphere, the atmosphere and the ionosphere. We analyze in this work the curl-free current model describing the current flow from the lithosphere to the ionosphere through the atmosphere. We use a numerical simulation based on the finite element method to derive the current between the ground and the ionosphere. We shown that the curl-free approximation of the atmospheric current density leads to significant and unpredictable distortions of the solutions of the electrical conductivity. Hence it incorrectly expands the ionospheric disturbed region associated to lithospheric currents. It is shown that vertical underground external currents can not create currents from ground to the atmosphere.

1 Introduction

Before and after earthquakes some perturbation of the ionospheric electric field are observed. A review of these satellite-based observations is presented by *Zolotov* [2015]. The Lithosphere-Atmosphere-Ionosphere Coupling is a chain of physical processes proposed to explain the ionospheric disturbances recorded before the earthquakes occurrence. Those disturbances are mainly associated to the earthquake preparation zone in the lithosphere.

Until now proposed models can not quantitatively explain the relations between the processes in the lithosphere and in the ionosphere. A review of current state is presented in the paper by *Pulinets et al* [2015] that describes the lithosphere-atmosphere-ionosphere-magnetosphere coupling as a complex dissipative open system. Many researchers try to construct physical and mathematical models, which should explain the perturbation of the ionospheric electric field due to certain physical processes. Proposed models are based on: (a) the radon emanation from the lithosphere affecting the lower atmospheric conductivity ***Harrison et al* [2010]**, (b) the generation of an electric field in the lithosphere due to some physical and chemical processes ***Freund* [2013]**, and (c) atmospheric processes produce acoustic and/or gravitational waves linked to the preseismic preparation region ***Molchanov et al* [2001]**. Main references to the previous models are reported and detailed in *Pulinets and Boyarchuk* [2004], *Molchanov and Hayakawa* [2008], and *Hayakawa* [2015].

The most developed models regard the lithosphere as a generator that creates a quasi-stationary electric current or an electric field in the atmosphere near the surface of the Earth. The appearance of such models is due to the observations of perturbations of the vertical component of the atmospheric electric field before and after earthquakes. In accordance with these observations the strength of such fields reaches 1000 **V/m for very strong earthquakes *Choudhury et al* [2013].**

In this paper we are concentrated on the models which explain the lithosphere-ionosphere coupling by quasi-stationary electric currents through the atmosphere. Some models suggest the presence in the lower atmosphere of external currents which are continued by the conductivity currents and enter the ionosphere through the upper atmosphere *Sorokin et al* [2007].

Because of difficulties in simulation of the penetration of the electric field and current through the atmosphere some models omit this step and take as a given input parameter the electric field in the ionosphere *Namgaladze et al* [2009] or the current from the atmosphere to the ionosphere *Namgaladze et al* [2013] to explain the observed variations of the total electron content associated with earthquakes.

In modeling the currents through the atmosphere, some researchers *Kim et al* [1994], *Sorokin et al* [2007], and *Kuo et al* [2014] explain the electric fields and currents in the ionosphere, corresponding to the observations described in *Zolotov* [2015]. Our analysis *Denisenko et al* [2013], *Denisenko* [2015] of these models showed that excessive simplifications, fundamentally distorting the results, are present in all of them. Other models *Grimalsky et al* [2003], *Hegai* [2015] and *Denisenko et al* [2013] show that the field penetrating the ionosphere is several orders of magnitude smaller than required to explain the satellite-based observations of the ionospheric variations associated with earthquakes.

Recently, a model *Kuo et al* [2014] was developed. The authors derived the current density $\mathbf{j}$ in the atmosphere using the continuity equation $\mathbf{div}\,\mathbf{j} = \mathbf{0}$, and showed how the atmospheric electric currents $\mathbf{j}$ and electric fields $\mathbf{E}$ disturbed the ionosphere above the earthquake preparation zone. *Prokhorov and Zolotov* [2017] criticized the model proposed by *Kuo et al* [2014] and pointed out that the used formula to derive the current $\mathbf{j} = -\,\mathbf{grad}\,\mathbf{\Psi}$ can't reasonably describe the ground-to-ionosphere current of presumably seismic origin. *Kuo and Lee* [2017] replied by considering two approaches to solve the equation $\mathbf{div}\,\mathbf{j} = \mathbf{0}$. The authors also insisted that the existence of a battery/dynamo current source in the lithosphere leads to the presence of current and electric field in the atmosphere and disturbing the ionosphere.

In the work of *Kuo and Lee* [2017] are given arguments in favor to represent the density of atmospheric electric current $\mathbf{j}$ as gradient of a function Ψ:

$$\mathbf{j} = -\mathrm{grad}\,\Psi, \tag{1}$$

and also to consider the presence of an atmospheric electric field due to an underground vertical external current. **A model of underground external current *Freund* [2013] is used. We study such a generator in section 5.**

The main purpose of this paper is to analyze the model *Kuo et al* [2014] and its discussion *Kuo and Lee* [2017]. In section 2 we reproduce the differential equations and the boundary conditions of the model *Kuo et al* [2014] and show that the unexplained boundary condition means existence of an ideal conductor above some height in the ionosphere. In accordance with *Kuo et al* [2014] the new boundary value problem is set in section 3 using (1) to substitute the original electrical conductivity problem. The solutions of these problems are compared in Section 4 to demonstrate their differences. Section 5 is more general. It is devoted to the discussion of atmospheric electric fields which can or can not be created by underground generators. The analysis is more complicated in comparison with *Denisenko* [2015] since the construction of the underground generator proposed in *Kuo and Lee* [2017] is more complicated than that in the model *Kuo et al* [2014]. Nevertheless we obtain the universal result for all 1-D problems: vertical external current does not create electric field and current outside the domain where this external current exists. By this analysis we show that the charge layer model proposed by *Freund* [2013], and used in *Kuo et al* [2014] and *Kuo and Lee* [2017] can't explain an electric field above the ground.

2 The electrical conductivity boundary value problem

In our atmospheric model, the air is considered as an isotropic conductor with a conductivity σ depending only on the height z above ground. The coordinate axes x, y are in the horizontal plane. **In our models (*Denisenko et al* [2013] and *Denisenko* [2015]) we consider scalar conductivity only below 50 km because the geomagnetic field introduces gyrotropy above this height; so the conductivity becomes a tensor. Since the main pur-**

pose of this paper is to analyze the model *Kuo and Lee* [2017] where scalar conductivity is used we do the same unrealistic simplification. By the way it is shown in *Denisenko et al* [2013] that for the electric fields and currents below 50 km the only feature of the ionospheric conductivity are important; the integral conductance of the ionosphere is much larger than the atmospheric one.

The basic equations for the steady state electric field **E** and the current density **j** are Faraday's, charge conservation and Ohm laws,

$$\operatorname{curl}\mathbf{E} = 0, \tag{2}$$

$$\operatorname{div}\mathbf{j} = 0, \tag{3}$$

$$\mathbf{j} = \sigma\mathbf{E}. \tag{4}$$

Because of the equation (2) the electric potential Φ can be introduced as

$$\mathbf{E} = -\operatorname{grad}\Phi. \tag{5}$$

Then, the equation system (2-4) is reduced to the electrical conductivity equation

$$-\operatorname{div}(\sigma\operatorname{grad}\Phi) = 0. \tag{6}$$

The boundary condition at ground means the existence of a vertical current density distribution

$$j_z(x, y, 0) = j_{surf}(x, y)$$

that for potential Φ means

$$-\sigma(z)\left.\frac{\partial\Phi}{\partial z}\right|_{z=0} = j_{surf}(x, y). \tag{7}$$

Some boundary condition in the models of *Kuo et al* [2011] and *Kuo et al* [2014] is set at the upper boundary

$$\left.\frac{\partial j_z}{\partial z}\right|_{z=z_0} = 0. \tag{8}$$

Combining to (3) this condition is equivalent to

$$\left.\left(\frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y}\right)\right|_{z=z_0} = 0. \tag{9}$$

In view of (5) and $\sigma = \sigma(z)$ we obtain

$$-\sigma(z)\left.\left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}\right)\right|_{z=z_0} = 0, \tag{10}$$

that means zero Laplassian of the function Φ at the plane $z = z_0$. Only the constant $\Phi = \Phi_0$ has such a property among boundered functions. From physical point of view it is obvious since this equation simulates conducting film with zero current source. We can take state that $\Phi_0 = 0$ since such constant is of no value for $\mathbf{E}$ because $\text{grad}\,\Phi$ (5) does not vary when a constant is added to Φ. Therefore the boundary condition (8), sometimes used without justification, is equivalent to

$$\Phi(x, y, z_0) = 0, \tag{11}$$

describing an ideal conductor at $z > z_0$. It is valid for an arbitrary function $\sigma(x, y, z)$. **The equivalence of (11) to (8) can also be shown for tensor conductivity with vertical magnetic field. We stress that there must be infinite conductivity in horizontal directions above this boundary to set the condition (11). Of cause there is no conductor of this kind in the real ionosphere. Such an approach was used in *Grimalsky et al* [2003] and the introduced error was analysed in *Denisenko et al* [2013]**.

We are interested in the solution that decreases far of the domain $|x| < a, |y| < b$ where $j_{surf}(x, y) \neq 0$. We can approximately solve the equation (6) not in the infinite flat layer $0 < z < z_0$, but in a parallelepiped $|x| < x_0$, $|y| < y_0$, $0 < z < z_0$ with additional conditions applied at the four sides (i.e. $x = \pm x_0$ and $y = \pm y_0$) of the parallelepiped:

$$\Phi(\pm x_0, y, z) = 0, \quad \Phi(x, \pm y_0, z) = 0, \tag{12}$$

where x_0 and y_0 are large enough. These conditions are not considered in *Kuo and Lee* [2017] and also in previous papers (i.e. *Kuo et al* [2011] and *Kuo et al* [2014]), but we suppose that they also used some method to reduce the infinite domain to a finite one. The elliptical boundary value problem (6, 7, 11, 12) has a unique solution that is numerically resolved and reported in *Kuo et al* [2011]. We refer it as $\Phi-$problem.

3 The model of curl-free current

A new approach has been developed in the model of *Kuo et al* [2014]. The use of the equation (1) lead to re-write (3) and boundary conditions (7, 8) as

$$-\Delta\Psi = 0, \quad -\frac{\partial\Psi}{\partial z}\bigg|_{z=0} = j_{surf}(x, y), \quad \Psi(x, y, z_0) = 0. \tag{13}$$

The last condition is derived from (8) in the same way as (11). We already mentioned that such a boundary condition would be valid if conductivity in horizontal directions above this boundary is infinite. However that is not valid for the real ionosphere. We reduce the infinite domain to a finite one by similar conditions like in (12):

$$\Psi(\pm x_0, y, z) = 0, \quad \Psi(x, \pm y_0, z) = 0. \tag{14}$$

The elliptical boundary value problem (13, 14) has also a unique solution. We refer it as $\Psi-$ problem. **The original equation (2) can be satisfied only occasionally in some specific cases described in the next section since it was not taken into account while the equations (13, 14) were derived.**

4 Numerical example

It is important to note that the solutions for the $\Phi-$ and $\Psi-$ problems are valuably different, as discussed in *Denisenko et al* [2016] as well as in *Prokhorov and Zolotov* [2017]. However this statement is contested in *Kuo and Lee* [2017] by the analysis of one example. Of course similarity of the solutions for one case does not prove the equivalency of the equations. Such equivalency exists when the conductivity σ is constant (*Prokhorov and Zolotov* [2017]) and in some specific cases (*Denisenko* [2015]). There are 1-D problems among them for example $\sigma = \sigma(z)$ and vertical **E** supposed to be independent of x, y coordinates. Nevertheless let us analyze the case *Kuo and Lee* [2017].

We construct numerical solutions for the $\Phi-$ and $\Psi-$ problems in the $2-D$ approximation where functions are independent of y coordinate. Since the solution in *Kuo and Lee* [2017] is elongated in the $y-$ direction, our solutions do not differ much from them at the plane $y = 0$ as we show hereafter. It is not difficult to get rather precise solutions in such a case. We use finite element method based on minimization of the energy functional. More about this method and its accuracy is detailed in *Denisenko* [1998]. Following *Kuo and Lee* [2017], we use **the exponential conductivity height distribution** $\sigma(z) = \sigma_0 \exp(-z/h)$**, where** $\sigma_0 = 2 \cdot 10^{-14}$ **S/m,** $h = 6$ **km, and** a 1-D current distribution in the fault region with $a = 200$ km:

$$j_{surf}(x) = j_{max}(1 + \cos(\pi x/a))/2. \tag{15}$$

Thin lines in Fig. 1 show the solution for the $\Psi-$problem with z_0 equal to 200km. Also we only display half-plane $x > 0$ because of symmetry. Current between neighbor lines is equal to $\delta I = I_0/10$ where the total current I_0 is defined as the integral of $j_{surf}(x)$. In view of charge conservation law (3) it can only decrease with height because of partial closer in the atmosphere. So the 3% increase in Fig. 1(d) in *Kuo and Lee* [2017] is due to an error in their numerical method. **It can be mentioned that both $\Phi-$ and $\Psi-$ problems can be solved in a simple manner by using the Fourier Series since conductivity depends only on the height. This method was applied for a 2-D problem *Ampferer et al* [2010] and for a 3-D problem with tensor conductivity *Denisenko et al* [2013]**. In $3-D$ case the current lines show the direction of current, but the current between neighbor current lines is not constant, contrary to $2-D$ case. May be this is the reason why *Kuo and Lee* [2017] started the lines with an equal δx distance at $z = 0$. Dark dots in Fig. 1 show few points of two current lines which start at $x = 80$ km and $x = 120$ km, similar to those in Fig. 3(a) of *Kuo and Lee* [2017]. As we see the lines in our $2-D$ approach are rather similar to $3-D$ ones at $y = 0$ plane for $x < a$. The difference between 3-D and 2-D solutions is increasing for $x > a$.

Dashed lines in Fig. 1 displays current lines of our solution for the $\Psi-$problem with $z_0 = 130$ km. Since our solution looks like one in *Kuo and Lee* [2017] when $z_0 = 200$ km, this leads to suppose that the solutions are similar when $z_0 = 130$ km, and also similarity of these dashed lines with the current lines in Fig. 2(a) of *Kuo and Lee* [2017]. It does not happen. We think that by mistake Fig. 2(a) is just a copy to Fig. 1(a), at least one can not find any difference. It is also the case Fig. 2(b) and Fig. 1(b).

Thick lines in Fig. 1 show current lines for the $\Phi-$problem with $z_0 = 200$ km. Almost the same lines for $z_0 = 130$ km. They are close to verticals and are similar to ones in Fig. 1(a) *Kuo and Lee* [2017]. To demonstrate the similarity we put one light circle that corresponds to the line which starts from $x = 80$ km, $z = 0$ km as shown in Fig. 1(a). Another line which begins from $x = 120$ km, $z = 0$ km in Fig. 1(a) differs more from our one which starts from the same point. The last line started from $x = 200$ km, $z = 0$ km must be horizontal because no current goes through the ground surface to the atmosphere at the points $x > 200$ km.

This line in Fig. 1(a) demonstrates error of the numerical method by going up. We agree with the authors of *Kuo and Lee* [2017] that the numerical method used in *Kuo et al* [2011] was not convenient. Nevertheless we see that the solutions of *Kuo and Lee* [2017] for $\Phi-$problem are in some agreement with our ones. Analysis of Fig. 1 shows valuable difference between currents obtained in $\Phi-$ and $\Psi-$problems. For example current from ground at the interval $0 < x < 80$ km enter the ionosphere at the chosen height $z = 85$

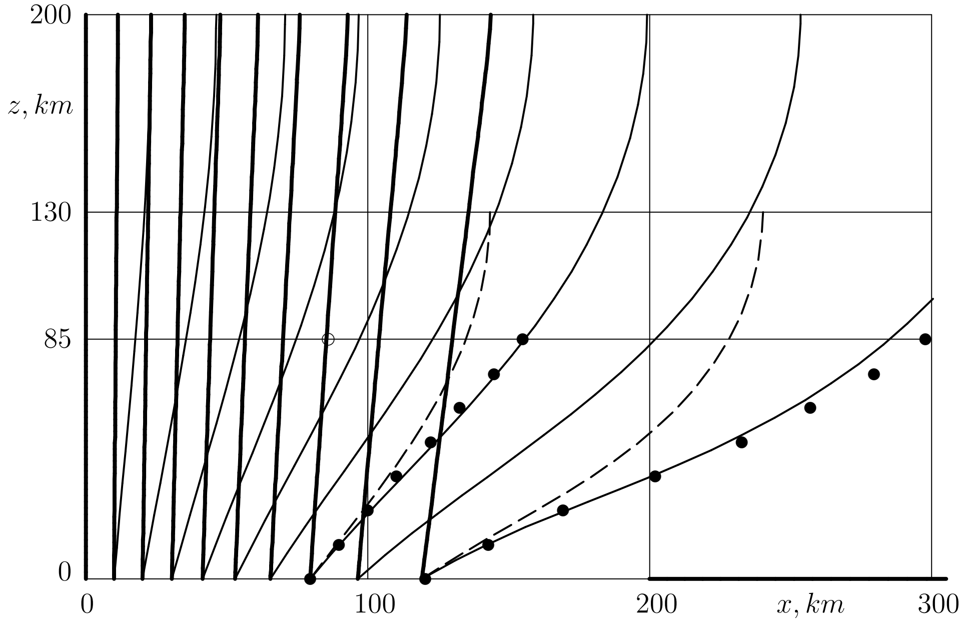

Figure 1. Thin and thick lines displays the current lines for **j** derived, respectively, from the Ψ− and Φ− problems where z_0 is equal to 200 km. The interval between neighbor lines is equal to $I_0/10$ where I_0 is the total current. Black circle points are associated to two current lines which start at 80 km and 120 km for Ψ− problem presented in Fig. 3(a) of *Kuo and Lee* [2017]. Dashed lines displays the current **j derived from the Ψ− problem with** z_0 equal to 130 km. Light circle corresponds to the line started from $x = 80$ km, $z = 0$ km for Φ− problem presented in Fig. 1(a) of *Kuo and Lee* [2017].

km through the region $0 < x < 86$ km in Φ−problem *Kuo and Lee* [2017] and $0 < x < 155$ km or $0 < x < 135$ km in dependence of the chosen height $z_0 = 200$ km or $z_0 = 130$ km in Ψ−problem. This is 70% of the total current. The increase of the interval 1.8 or 1.6 times, and its dependence on an arbitrary selected parameter means just valuable difference and contradicts the conclusion of *Kuo and Lee* [2017] about similarity of the solutions for Φ− and Ψ−problems.

The decision to solve another problem instead of using a not convenient numerical method for the original problem has given even worse result and the conclusion of *Kuo and Lee* [2017]: "the result of Ψ Method can provide a good approximation for upward currents that flow into the ionosphere obtained from the Φ Method" contradicts the obvious difference of the results.

5 Underground external currents

Here we analyze the reply of *Kuo and Lee* [2017] to our critics (see *Denisenko* [2015]) of the model of current flow from ground to atmosphere (*Kuo et al* [2011]; *Kuo et al* [2014]). Fig. 4 of *Kuo and Lee* [2017] is reproduced in Fig. 2 with additional objects which were initially not shown but definitely exist. In Fig. 2(a) we plot only sum electric field **E**, designate charge densities at the planes as $\pm\Sigma$, and add external current J_0 which is necessary for steady state existence of such a electrical construction. As it was mentioned in *Kuo and Lee* [2017] without J_0 the charges would decrease because of conductivity current $j = \sigma E$ with relaxation time τ of about 10^{-9} s (there is a misprint 10^{-10} in *Kuo and Lee* [2017] for $\sigma_l = 0.01$ S/m).

The current $J_0 = \sigma_l E$ is necessary for stationarity state, where σ_l is the conductivity in the layer 3, .i.e. in the interval $-z_2 < z < -z_1$. Nothing is wrote in *Kuo and Lee* [2017] about this layer despite the existence of the current J_D. They call it the dynamo current J_D inside the battery. We suppose the same conductivity as σ_l in this layer, but any other value instead of this σ_l can also be considered. Let us consider the process of transition to a stationary state after the starting of an external current J_0 at the time $t = 0$. The equations (2, 3, 4) become more complicated. If the process is slow enough to neglect electromagnetic induction, they are

$$\operatorname{curl}\mathbf{E} = 0, \quad \operatorname{div}\mathbf{E} = \rho/\varepsilon_0, \quad \frac{\partial\rho}{\partial t} + \operatorname{div}\mathbf{j} = 0, \quad \mathbf{j} = \sigma\mathbf{E} + \mathbf{J}_0, \tag{16}$$

where ρ is the charge density and ε_0 is the dielectric permeability of vacuum. If charged surfaces exist the volume density ρ is substituted by the surface density Σ, and the second and the third equations (16) can be written as

$$E_+ - E_- = \Sigma/\varepsilon_0, \quad \frac{\partial\Sigma}{\partial t} = -j_+ + j_-, \tag{17}$$

where indexes $\pm$ indicate the values of the normal component of a vector at opposite sides of the surface. Let $\mathbf{J}_0 = 0$ for $t < 0$ and for $t > 0$ the vector $\mathbf{J}_0$ has only the z−component J_0 in the interval $-z_2 < z < -z_1$, where J_0 is a given constant. It is simple to check that the following functions give the solution for the equations (16, 17) with zero electric field and charge density at $t = 0$:

$$\Sigma = \Sigma_0(1 - \exp(-t/\tau)), \quad E = -E_0(1 - \exp(-t/\tau)), \quad j = J_0\exp(-t/\tau) \tag{18}$$

inside the domain $-z_2 < z < -z_1$ and equal to zero outside. Here $E_0 = J_0/\sigma_l$, $\Sigma_0 = \varepsilon_0 E_0$, $\tau = \varepsilon_0/\sigma_l$. The relaxation time $\tau \simeq 10^{-9}$ s is the same as mentioned above when $\pm\Sigma$ de-

Figure 2. Lithospheric charge layers. E and J indicate, respectively, the electric field and the current. Positive and negative charge densities at the surfaces are designated by $+\Sigma$ and $-\Sigma$. Crosshatching region corresponds to the lithosphere layer. Panel (a) and framed part of panel (b) reproduce Fig. 4 (a) and (b) in *Kuo and Lee* [2017]. The electric field and current system (c) is the difference between the systems (b) and (a).

crease when $J_0 = 0$. For $t >> \tau$ the solution becomes a stationary one with the values Σ_0, E_0 and zero total current $j = J_0 - \sigma_l E_0 = 0$. As it is shown in Fig. 2(a). It ought be stressed that the charge density ρ is not used in the steady state equations of electrical conductivity (6) or magnetohydrodynamics, but it can be calculated after solving a problem as $\varepsilon_0 \mathrm{div}\,\mathbf{E}$. Framed part of Fig. 2(b) reproduces Fig. 4(b) of Kuo and Lee [2017] in similar manner. Here we add designations for the layer 3. An arbitrarily direction of E_3 is chosen, may be $E_3 < 0$. Outside the frame we add a layer with charge density $-\Sigma_a$ that is somewhere above the atmosphere, and a layer with charge density $+\Sigma_l$ that is below the shown part of the lithosphere. From a mathematical point of view these layers can be at $\pm\infty$. These charged layers are necessary for existence of the electric fields E_a and E_2. Also the current J is added. Without it the charges $-\Sigma_a$ and $+\Sigma_l$ would disappear as well as $\pm\Sigma$ in Fig. 2(a) while much slower, since the atmospheric conductivity σ_a is much smaller than the lithospheric one σ_l. The charge conservation law (3) for this construction means that the total current is independent of height:

$$\sigma_a E_a = \sigma_l E_1 = J_D + \sigma_l E_3 = \sigma_l E_2 = J. \tag{19}$$

The solution for this system is

$$E_1 = E_2 = J/\sigma_l, \quad E_a = J/\sigma_a, \quad E_3 = (J - J_D)/\sigma_l. \tag{20}$$

It is simple to find all surface charge densities by these electric fields, if they are of interest. For example, $\Sigma_s = \varepsilon_0(E_a - E_1) = \varepsilon_0 J(1/\sigma_a - 1/\sigma_l)$. It looks similar to the equation (12) in *Kuo and Lee* [2017] but here must be just J instead of J_D. As we see the current J_D has only an effect on the electric field in the layer 3. We can present the construction shown in Fig. 2(b) as the composition of the constructions shown in Fig. 2(a) and Fig. 2(c). If the current J is absent, $E_a = E_l = 0$ or in detailed form $E_a = E_1 = E_2 = 0$ and E_3 is the same as in Fig. 2(a) with $J_0 = J_D$. The construction presented in Fig. 4(b) *Kuo and Lee* [2017] is more complicated than one in Fig. 1 in *Kuo et al* [2011]. So our actual analysis is longer than it was in *Denisenko* [2015]. Nevertheless we obtain the same result. It is universal for all 1-D problems: vertical external current can not create electric field and current outside the domain where this external current exists. Here for simplicity we use constant values of atmospheric and lithospheric conductivities σ_a, σ_l, but similar analysis with the same conclusion can be done for any height distributions $\sigma_a(z)$, $\sigma_l(z)$.

There must be a current like J which moves charges from the ionosphere to ground. Since the current J moves charges upstream electric field it can not be a conductivity current. There is no such a current in the models *Kuo et al* [2011], *Kuo et al* [2014], *Kuo and Lee* [2017], but the absence of J means no atmospheric electric field. Charged layer $-\Sigma_a$ must be somewhere above the atmosphere. Figuratively speaking, a field line starts at a positive charge and finishes at negative one. Some current must bring back positive charges from the ionosphere to the lithosphere to keep $-\Sigma_a$ not variable.

There is a simple way to create such a current by underground generator. Such a generator must flow charges of different signs to different parts of the ground surface, as it is in our model *Denisenko et al* [2013]. There is no explanation of such generator, but other kinds in the lithosphere can not generate the atmospheric electric field. Our models do not prove its existence. We only show, that if such a generator provides current from ground to the atmosphere with density of a few pA/m^2, the vertical electric field near ground has strength of about hundred V/m before earthquakes occurrence, as reported in the literature. However only negligible electric field and current appear in the ionosphere in frame of our models. In contrast with *Prokhorov and Zolotov* [2017], we think that additional external atmospheric current created by moving of charged aerosols does not help. Critical analysis of this kind of models (e.g. *Sorokin et al* [2007]) is discussed in *Denisenko et al* [2013] and *Denisenko* [2015] . We believe that such ionospheric models (e.g. *Kuo et al* [2011] and *Namgaladze et al* [2013]) have no atmospheric origin.

It ought be mentioned that the lithosphere can vary atmospheric electric field without underground electric generators. For example radon emanation increases atmospheric conductivity near ground, that locally varies the electric field of the Global electric circuit as reported by *Harrison et al* [2010].

Conclusions

The curl-free presentation of the atmospheric electric current of *Kuo et al* [2014] gives solutions which valuably differ from the solutions of the electrical conductivity problems. The explanations and additional proofs of the curl-free presentation in the model *Kuo et al* [2014] which are re-considered in the paper of *Kuo and Lee* [2017] are not accurate since such a key parameter as the size of the ionospheric region where current enters ionosphere is distorted up to twice, and there is no proof that it can not be worse in other cases.

The model of appearance of the atmospheric electric field due to vertical underground generator (*Kuo et al* [2011] and *Kuo et al* [2014]) contains inaccuracy. Precise analysis of the proposed construction shows zero field above ground. The lithospheric and atmospheric parts of the models of *Kuo et al* [2011] and *Kuo et al* [2014] yield zero current to the ionosphere after the correct consideration.

Basing on the results of this paper, also the conclusions of *Hegai* [2015] and our previous analysis of many models (*Denisenko et al* [2013] and *Denisenko* [2015]) it is hard to imagine a valuable electric current in the ionosphere penetrating through the

atmosphere and generated by lithospheric physical processes. It is necessary to study other atmospheric physical processes to explain the lithospheric influence on the ionosphere; may be gravity waves as was proposed in *Molchanov and Hayakawa* [2008].

Acknowledgments

This work is supported by grant 18-05-00195 from the Russian Foundation for Basic Research. We acknowledge support by the Austrian Academy of Sciences and the Russian Academy of Sciences. Part of this research was done during academic visits of V.V. Denisenko to the Space Research Institute of the Austrian Academy of Sciences in Graz. H. Lammer, acknowledge support by the Austrian Science Fund (FWF) NFN project S11607-N16 "Particle/Radiative Interactions with Upper Atmospheres of Planetary Bodies under Extreme Stellar Conditions"

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