CONTENTS

	Preface		vi		
	Ac	knowledgments	i		
I.	Introduction				
	0	Motivation, and a Forward Look	:		
	1	Random Variables	4		
	2	Conditional Expectations	13		
	3	Stochastic Processes	16		
II.	Stochastic Integrals				
	1	Stochastic Models and Properties They Should Possess	20		
	2	Definition of the Integral	29		
	3	The Canonical Form	42		
	4	Elementary Properties of the Integral	4		
	5	The Itô-Belated Integral	5		
II.	Existence of Stochastic Integrals				
	1	Fundamental Lemma	59		
	2	Existence of the Stochastic Integral: First Theorem	6:		
	3	Second Existence Theorem	60		
	4	Third and Fourth Existence Theorems	7:		
	5	The Vanishing of Certain Integrals	74		
	6	Special Cases	79		
	7	Examples: Brownian Motions; Point Processes	8		
	8	Extension to the Itô-Belated Integral	89		
IV.	Continuity, Chain Rule, and Substitution				
	1	Continuity of Sample Functions	103		
	2	Differentiation of a Composite Function	114		

vi Contents

	3	Applications of Itô's Differentiation Formula	125	
	4	Substitution	134	
	5	Extension to Itô-Belated Integrals	143	
v.	Stochastic Differential Equations			
	1	Existence of Solutions of Stochastic Differential Equations	152	
	2	Linear Differential Equations and Their Adjoints	160	
	3	An Approximation Lemma	165	
	4	The Cauchy-Maruyama Approximation	176	
VI.	E	quations in Canonical Form		
	1	Invariance under Change of Coordinates	180	
	2	Runge-Kutta Approximations	186	
	3	Comparision of Ordinary and Stochastic Differential Equations	191	
	4	Rate of Convergence of Approximations to Solutions	203	
	5	Continuous Dependence of the Solution on the Disturbance	214	
	6	Justification of the Canonical Extension in Stochastic Modeling	228	
	References		235	
	Subject Index		237	