Contents

Physical Applications of Nekhoroshev Theorem and Exponential Estimates
Giancarlo Benettin 1
1 Introduction 1
2 Exponential Estimates 5
3 A Rigorous Version of the JLT Approximation in a Model 23
4 An Application of the JLT Approximation 32
5 The Essentials of Nekhoroshev Theorem 39
6 The Perturbed Euler–Poinsot Rigid Body 49
7 The Stability of the Lagrangian Equilibrium Points $L_4 - L_5$ 62
References 73

The Adiabatic Invariant Theory and Applications
Jacques Henrard 77
1 Integrable Systems 77
1.1 Hamilton-Jacobi Equation 77
Canonical Transformations 77
Hamilton-Jacobi Equation 78
1.2 Integrables Systems 79
Liouville Theorem 79
Stäckel Systems 80
Russian Dolls Systems 81
1.3 Action-Angle Variables 82
One-Degree of Freedom 82
Two Degree of Freedom Separable Systems 86
2 Classical Adiabatic Theory 89
The Adiabatic Invariant 89
Applications 92
The Modulated Harmonic Oscillator 92
The Two Body Problem 93
The Pendulum 93
The Magnetic Bottle 96

3 Neo-adiabatic Theory 101
3.1 Introduction 101
3.2 Neighborhood of an Homoclinic Orbit 102
3.3 Close to the Equilibrium 104
3.4 Along the Homoclinic Orbit 107
3.5 Traverse from Apex to Apex 109
3.6 Probability of Capture 113
3.7 Change in the Invariant 117
3.8 Applications 121
The Magnetic Bottle 121
Resonance Sweeping in the Solar System 122
4 Slow Chaos 127
4.1 Introduction 127
4.2 The Frozen System 128
4.3 The Slowly Varying System 129
4.4 Transition Between Domains 130
4.5 The "MSySM" 133
4.6 Slow Crossing of the Stochastic Layer 136
References 139

Lectures on Hamiltonian Methods in Nonlinear PDEs
Sergei Kuksin 143
1 Symplectic Hilbert Scales and Hamiltonian Equations 143
1.1 Hilbert Scales and Their Morphisms 143
1.2 Symplectic Structures 145
1.3 Hamiltonian Equations 146
1.4 Quasilinear and Semilinear Equations 147
2 Basic Theorems on Hamiltonian Systems 148
3 Lax-Integrable Equations 150
3.1 General Discussion 150
3.2 Korteweg–de Vries Equation 152
3.3 Other Examples 153
4 KAM for PDEs 154
4.1 Perturbations of Lax-Integrable Equation 154
4.2 Perturbations of Linear Equations 155
4.3 Small Oscillation in Nonlinear PDEs 155
5 The Non-squeezing Phenomenon and Symplectic Capacity 156
5.1 The Gromov Theorem 156
5.2 Infinite-Dimensional Case 156
5.3 Examples 159
5.4 Symplectic Capacity 160
6 The Squeezing Phenomenon 161
References 163