

Table of Contents

PART I.

PRELIMINARIES

1. INTRODUCTORY REMARKS ON QUADRATIC FORMS	1
2. ALGEBRAIC BACKGROUND	6
A. Factorial rings (ufd)	6
B. Integral elements	9
C. Euclidean domains	10
D. Modules and ideals	11
E. Principal ideal domains (pid)	12
F. Rational integers	15
3. QUADRATIC EUCLIDEAN RINGS	17
4. CONGRUENCE CLASSES	24
A. Norm and phi-function	24
B. Module operations	25
C. Chinese remainder theorem	27
D. Euler phi-function and Möbius mu-function	28
E. Rational residue class groups	29
F. Quadratic reciprocity	31
5. POLYNOMIAL RINGS	34
A. Factorization properties	34
B. Finite fields	37
C. Abstract model and automorphisms	38
6. DEDEKIND DOMAINS	42
A. Prime and maximal ideals	42
B. Noether axioms	44
C. Sufficiency of axioms	46
D. Equivalence classes	48
7. EXTENSIONS OF DEDEKIND DOMAINS	53
A. Validity of axioms	55
B. Root-discriminant	56
C. Basis of theorems of Hermite and Smith	62

8. RATIONAL AND ELLIPTIC FUNCTIONS	66
A. Rational functions	66
B. Elliptic functions	67
C. Riemann surfaces	67
D. Ideal structure	68
E. Principal ideals (Abel's theorem)	70
PART II.	
IDEAL STRUCTURE IN NUMBER FIELDS	
9. BASIS AND DISCRIMINANT	72
A. Free nonsingular basis	72
B. Norm and trace	73
C. Conjugates	74
D. Basis and discriminant computation	75
E. Quadratic field $\mathbb{Q}(\sqrt{D})$	78
F. Pure cubic field $\mathbb{Q}(\sqrt[3]{m})$	79
G. Cyclotomic field $\mathbb{Q}(\exp 2\pi i/m)$	80
H. Ring index	83
10. PRIME FACTORIZATION	86
A. Main theorem	86
B. Ring ideals	88
C. Quadratic field $\mathbb{Q}(\sqrt{m})$	90
D. Kronecker symbol	92
E. Pure cubic field $\mathbb{Q}(\sqrt[3]{m})$	93
F. Cyclotomic field $\mathbb{Q}(\exp 2\pi i/m)$	96
G. Discriminantal divisors	97
11. UNITS	102
A. Quadratic fields	102
B. Pell's equation	105
C. Dirichlet theorem	108
D. Imbeddings of \mathcal{O} and \mathcal{O}^*	109
12. GEOMETRY OF NUMBERS	114
A. Convex bodies	116
B. Existence theorem	118

C.	Parallelopiped applications	119
D.	Octahedron (norm) applications	121
E.	Volume coordinates	122
13.	FINITE DETERMINATION OF CLASS NUMBER	125
A.	Primary associates	125
B.	Norm estimates and class number	128
C.	Norm density	131
D.	Zeta function	133
E.	Quadratic case	134
PART III.		
INTRODUCTION TO CLASS FIELD THEORY		
14.	QUADRATIC FORMS, RINGS AND GENERA	137
A.	Forms and modules	138
B.	Strict equivalence	141
C.	Ring equivalence	143
D.	Genus equivalence	144
E.	Number of genera	146
F.	Quadratic reciprocity	148
G.	Genus characters	150
H.	p -adic numbers	153
I.	Norm-residue theory: Hilbert symbol	156
15.	RAY CLASS STRUCTURE AND FIELDS, HILBERT CLASS FIELDS	163
A.	Ray modulus semigroup	164
B.	Ray number groups	166
C.	Ray ideal groups	168
D.	Conductor and maximal ray ideal group	170
E.	Weber-Takagi correspondence	171
F.	Rational base field	173
G.	Genus extension field	176
H.	Hilbert class field	177
I.	Ring class fields	180
16.	HILBERT SEQUENCES	188
A.	Galois groups	188
B.	Classical examples	191
C.	Relative norms	192

D.	Definition of Hilbert sequence	195
E.	Illustrations (and quadratic reciprocity again)	202
F.	Tchebotareff monodromy theorem	206
17.	DISCRIMINANT AND CONDUCTOR	208
A.	Relative different and discriminant	208
B.	Kronecker's theory of forms	210
C.	Hensel's local theory	213
D.	Relative quadratic fields	215
E.	Ramification in Hilbert sequence	216
F.	Conductor-discriminant relation	218
G.	Relative bases	221
18.	THE ARTIN ISOMORPHISM	226
A.	Artin symbol	226
B.	Illustrations	227
C.	Artin reciprocity	229
D.	Automorphisms of base fields	232
E.	Arithmetic invariants	233
F.	Group extensions and class field transfers	236
G.	Dirichlet genus characters	242
19.	THE ZETA-FUNCTION	248
A.	Class number relations	249
B.	Unit relations	251
C.	Hecke L-function	254
D.	Tchebotareff density theorem	256
E.	Analytic motivation of class field	258
F.	Artin L-function	260
BIBLIOGRAPHY	267	
APPENDICES (by Olga Taussky)	275	
LECTURES ON CLASS FIELD THEORY BY E. ARTIN (GÖTTINGEN 1932)		
Notes by O. Taussky	277	
INTRODUCTION INTO CONNECTIONS BETWEEN ALGEBRAIC NUMBER THEORY AND		
INTEGRAL MATRICES (Appendix by Olga Taussky)	305	
SUBJECT MATTER INDEX	327	