Available at: http://www.ictp.trieste.it/~pub_off IC/2004/55

United Nations Educational Scientific and Cultural Organization
and
International Atomic Energy Agency

THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

NUCLEAR INELASTIC SCATTERING CROSS SECTIONS FOR THE TRANSITIONS TO L = 4 AND L = 5 STATES IN NUCLEI

S. Nasmin Rahman
Industrial Physics Division, B.C.S.I.R., Dhaka, Bangladesh,

Sangita Haque
Department of Physics, University of Dhaka, Dhaka, Bangladesh

and

Md.A. Rahman[1]
Department of Physics, University of Dhaka, Dhaka, Bangladesh
and
The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Abstract

The generalized scattering amplitudes of the adiabatic distorted waves theory have been developed by Austern and Blair, connecting the inelastic scattering amplitudes in terms of the elastic scattering amplitudes. Potgieter and Frahn obtained simple, closed and explicit expressions for the inelastic scattering amplitudes, again in the adiabatic approximation, under strong absorption condition. These are analogous to the strong absorption model formulae for elastic scattering. In a previous communication, inelastic scattering cross section leading to the population of L = 2 and L = 3 states in nuclei are derived by us, using the general formulations due to Potgieter and Frahn. We derive here closed expressions for the cross sections for the inelastic scattering, transiting to L = 4 and L = 5 states in nuclei in the same line as before and satisfactory and simultaneous fits to the elastic as well as inelastic angular distributions are obtained.

MIRAMARE – TRIESTE
July 2004

[1] Regular Associate of the Abdus Salam ICTP.

1. Introduction

The decisive advance in correlating inelastic scattering phenomena with elastic scattering phenomena was accomplished by Austern and Blair [1)]. They developed general methods in which the generalized inelastic scattering amplitude of the adiabatic distorted waves theory is expressed as the derivatives of the elastic scattering amplitudes. Potgieter and Frahn[2)] initiated the first step of introducing the strong absorption model (SAM) form of scattering function η_l in the generalized scattering amplitudes of the Austern and Blair theory. The resulting amplitudes could, under strong absorption condition, be recast in an explicit form analogous to that of SAM elastic scattering formalism, where the Coulomb phases, the spherical harmonics and the Clebsch-Gordon coefficients are largely approximated. The imaginary part of η_l is also included in the SAM formalism.

The final result for the inelastic scattering cross section is a simple and analytic expression that shows up characteristic features of the angular distributions for single excitation. The expression depends on four parameters. Three of them characterize the form of the scattering function $\eta_{l,}$ obtained from an analysis of the elastic scattering analysis. The fourth parameter is related to reduced matrix element of the inelastic scattering interaction and is the only free parameter determined by normalizing the theoretical predictions to the experimental angular distributions.

We, in the present communication, use the generalized inelastic scattering formulae of Potgieter and Frahn to obtain the inelastic scattering cross section exciting to L = 4 and L = 5 states in nuclei, which are still to be made available in the SAM formalisms. To illustrate the usefulness of our derived formulae for application to the analysis of inelastic scattering data, we demonstrate simultaneous fits to the elastic and inelastic scattering angular distributions, populating to L = 4 and L = 5 states in nuclei.

2. Inelastic scattering cross section for single, quadrupole and octupole excitations

The formulae for inelastic scattering cross sections have been derived and made explicit by different authors [2, 3)]. We briefly sketch these formulae to present the derivation for L = 4 (hexadecouple excitation) and L = 5 transitions in nuclei. The differential cross section for excitation of multipole order L in the Austern –Blair approximation is:

$$\frac{d\sigma}{d\Omega}\,(0 \rightarrow L) = \Sigma\ |\ f_{LM}(\theta)\ |^{2} \tag{1}$$

The summation in eqn.(1) runs from M = - L to + L. The amplitudes $f_{LM}(\theta)$ have the explicit form as follows :

$$f_{LM}(\theta) = \tfrac{1}{2}\, i^{l+1}\,(2l+1)^{1/2}\, C_L\, \Sigma\, i^{l-l'}\,(2l'+1)^{1/2} \exp i\,(\sigma_l + \sigma_{l'})\ \partial\eta_l/\partial l$$

$$< l'\, L\, 00\ |\ l\, 0 >\ < l'\, L\,,\ -M\ \ M\ |\ l\, 0 >\ Y_{l'}^{-M}\,(\theta, 0) \tag{2}$$

The discrete summation in eqn.(2) runs over l' and l and the derivative of η_l w.r.t. l, where l is understood to be l-bar and the l- bar is assumed to have the values as l- bar = ½ (l + l′). The Coulomb phases are approximated by:

$$\sigma_l + \sigma_{l'} \approx 2\sigma_l \approx 2\sigma_T + (t - T)\,\theta_C \tag{3}$$

where, θ_C = 2 arctg (n / T) and n is the Coulomb parameter.

The above expressions for the amplitudes including the SAM conditions assume a closed and explicit form after performing various steps of approximations and substitutions. Finally the expression for differential cross section eqn.(1) assumes form:

$$\frac{d\sigma}{d\Omega}(0 \rightarrow L) = |C_L|^2 (2l+1)(T^2/16\pi)(\theta/\mathrm{Sin}\,\theta)\{(H_-^2 + H_+^2)\Sigma [\alpha^2_{LM}(\theta) + \beta^2_{LM}(\theta)][J^2_{|M|}(T\theta) + J^2_{|M|-1}(T\theta) + 2H_- H_+ \Sigma [\alpha^2_{LM}(\theta) - \beta^2_{LM}(\theta)]][J^2_{|M|}(T\theta) - J^2_{|M|-1}(T\theta)]\} \quad (4)$$

The explicit forms of $H_{\pm}$, $\alpha_{LM}(\theta)$ and $\beta_{LM}(\theta)$ in the above equation are given elsewhere[2,3)]. The coefficients C_L represent the reduced nuclear matrix elements and are related to various deformation lengths δ_L. For quadrupole deformation i.e. for L = 2, the functions $\alpha_{LM}(\theta)$ and $\beta_{LM}(\theta)$ have the form:

$$\alpha_{2,\pm 2}(\theta) = \tfrac{1}{4}(3/2)^{1/2}(1 + \mathrm{Cos}\theta), \quad \beta_{2,\pm 2}(\theta) = 0,$$

$$\alpha_{2,\pm 1}(\theta) = 0, \quad \beta_{2,\pm 1}(\theta) = \tfrac{1}{2}(3/2)^{1/2}\,\mathrm{Sin}\,\theta,$$

$$\alpha_{2,0}(\theta) = \tfrac{1}{4}(3\,\mathrm{Cos}\theta - 1), \quad \beta_{2,0}(\theta) = 0. \quad (5)$$

and eqn.(4) takes the form :

$$\frac{d\sigma}{d\Omega}(0 \rightarrow 2) = (\delta^2_2)(T^2/64\pi)(\theta/\mathrm{Sin}\,\theta)[(H_- + H_+)^2 \{[\tfrac{1}{4}(3\,\mathrm{Cos}\theta - 1)^2 + 3\,\mathrm{Sin}^2\theta]J^2_0(T\theta) + \tfrac{3}{4}(1+\mathrm{Cos}\,\theta)^2 J^2_2(T\theta)\} + 4(H_- + H_+)^2 J^2_1(T\theta)] \quad (6)$$

The functions $\alpha_{3M}(\theta)$ and $\beta_{3M}(\theta)$ for L = 3 have the form:

$$\alpha_{3,\pm 3}(\theta) = 0.047\,\mathrm{Cos}3\theta/2 + 0.42\,\mathrm{Cos}\theta/2, \quad \beta_{3,\pm 3}(\theta) = 0,$$

$$\alpha_{3,\pm 2}(\theta) = 0, \quad \beta_{3,\pm 2}(\theta) = 0.114\,\mathrm{Sin}3/2\theta + 0.3421\,\mathrm{Sin}\theta/2, \quad (7)$$

$$\alpha_{3,\pm 1}(\theta) = 0.18\,\mathrm{Cos}3\theta/2 - 0.11\,\mathrm{Cos}\theta/2, \quad \beta_{3,\pm 1}(\theta) = 0,$$

$$\alpha_{3,0}(\theta) = 0, \quad \beta_{3,0}(\theta) = 0.07\,\mathrm{Sin}3\theta/2 - 0.375\,\mathrm{Sin}\theta/2.$$

and the differential cross section for the inelastic scattering events leading to octupole (L = 3) excitations in nuclei has the form :

$$\frac{d\sigma}{d\Omega}(0 \to) = (\delta^2{}_3)\ (T^2 / 64\,\pi)\ (\theta / \mathrm{Sin}\,\theta)\ [(H_{-} - H_{+})^2 \{ (J^2_0 (T\,\theta) (2\alpha^2_{31}(\theta) + \beta^2_{30}(\theta)$$

$$+J^2_2(T\,\theta)\ (2\,\alpha^2_{33}(\theta) + 2\,\beta^2{}_{32}(\theta))\} + (H_{-} + H_{+})^2 \{ J^2_1(T\,\theta)\ (2\alpha^2_{31}(\theta)$$

$$+ 2\,\beta^2{}_{32}(\theta) + \beta^2{}_{30}(\theta)) + J^2_3\,(T\,\theta)\ 2\,\alpha^2_{33}\,(\theta)\}] \tag{8}$$

3. Inelastic Scattering Cross section for Hexadecapole (L = 4) Exicitation

We use the general expressions for $\alpha_{LM}(\theta)$ and $\beta_{LM}(\theta)$ to obtain the functions $\alpha_{4M}(\theta)$ and $\beta_{4M}(\theta)$ corresponding to transition L = 4 as follows:

$$\alpha_{4,\pm 4}(\theta) = 0.0654\,\mathrm{Cos}2\theta + 0.2615\,\mathrm{Cos}\theta + 0.2\ ,\quad \beta_{4,\pm 4}(\theta) = 0.0\ ,$$

$$\alpha_{4,\pm 3}(\theta) = 0.0\ ,\quad \beta_{4,\pm 3}(\theta) = 0.185\,\mathrm{Sin}2\theta - 0.37\,\mathrm{Sin}\,\theta\ ,$$

$$\alpha_{4,\pm 2}(\theta) = 0.346\,\mathrm{Cos}2\theta + 0.2\,\mathrm{Cos}\,\theta - 0.15\ ,\quad \beta_{4,\pm 2}(\theta) = 0\ , \tag{9}$$

$$\alpha_{4,\pm 1}(\theta) = 0\ ,\qquad \beta_{4,\pm 1}(\theta) = 0.5\,\mathrm{Sin}2\theta + 0.14\,\mathrm{Sin}\theta\ ,$$

$$\alpha_{4,0}(\theta) = 0.55\,\mathrm{Cos}2\theta - 0.3125\,\mathrm{Cos}\theta + 0.14\,1\ ,\quad \beta_{4,0}(\theta) = 0\ .$$

The differential cross section for the inelastic scattering leading to hexadecapole (L = 4) excitations in nuclei, using eqn.(3) assumes the form:

$$\frac{d\sigma}{d\Omega}(0 \to 4) = \delta^2_4\,(T^2 / 64\,\pi)\,(\theta / \mathrm{Sin}\,\theta)\,[(H_- + H_+)^2 \{ J^2_0(T\,\theta)\,(\alpha^2_{40}(\theta)$$

$$+2\,\beta^2_{41}(\theta)) + J^2_2(T\,\theta)\,(2\alpha^2_{42}(\theta) + 2\,\beta^2_{43}(\theta)) + \tag{10}$$

$$J^2_4(T\,\theta)\,2\alpha^2_{44}(\theta)\} + (H_- - H_+)^2 \{ J^2_1(T\,\theta)\,(2\alpha^2_{40}(\theta) +$$

$$2\alpha^2_{42}(\theta) + 2\,\beta^2_{41}(\theta)) + J^2_3(T\,\theta)\,(2\alpha^2_{44}(\theta) + 2\beta^2_{43}(\theta))\}]$$

Table 1. Summary of SAM parameters for protons elastic and inelastic scattering analyses.

E_{lab} (MeV)	Target	E_x (MeV)	T	Δ	$\mu/4\Delta$	β_4	β_5
66.5	^{154}Sm	0.266	12.75	0.75	0.017	0.07 [a)]	
						0.061-0.66 [b)]	
						0.054-0.069 [c)]	
1044	^{40}Ca	4.49	33	5.25	0.097		0.005 [a)]

The parameter T stands for the cut-off or critical angular momentum.

The parameter Δ stands for the diffuseness of the nuclear surface i.e. it refers to the uncertainties in the localization of the cut-off angular momentum T and the parameter μ takes care of the real nuclear phase shift.

a) Present work .

b) From coupled-channel code ECIS calculations, Ref.[4)].

c) The adopted value from the compilation of Raman *et al.*[8)]

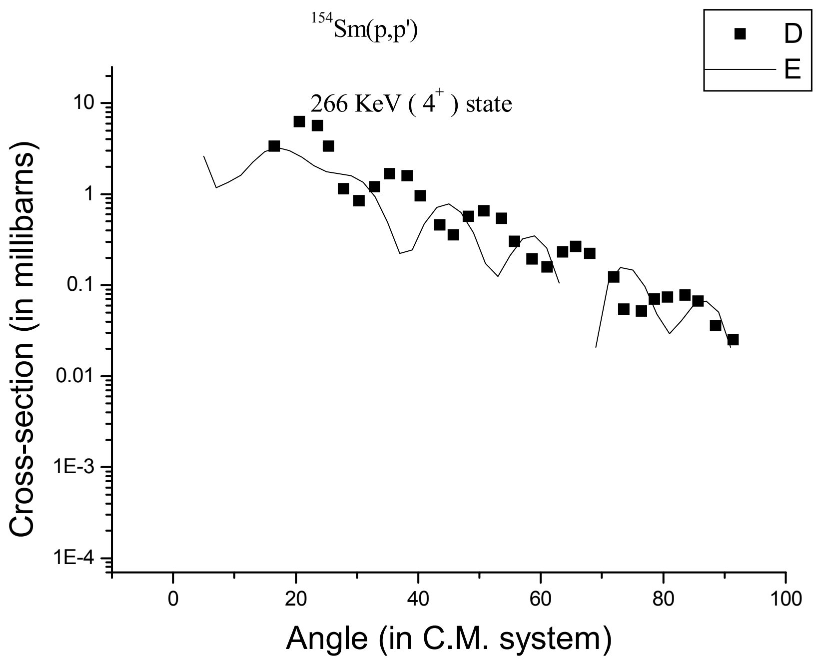

Fig.1. Inelastic scattering of protons leading to L = 4 state in ^{154}Sm (266 KeV (4^+) collective state).

4. Inelastic Scattering Cross section for transition to L = 5 State

Once again the general expression for $\alpha_{LM}(\theta)$ and $\beta_{LM}(\theta)$ are used to determine the functions $\alpha_{5M}(\theta)$ and $\beta_{5M}(\theta)$ corresponding to transitions L = 5 states in nuclei as follows:

$$\alpha_{5,\pm 5}(\theta) = 0.031 \,\mathrm{Cos}5/2\,\theta + 0.155 \,\mathrm{Cos}3/2\,\theta , \quad \beta 5,\pm 5\,(\theta) = 0 ,$$

$$\alpha_{5,\pm 4}(\theta) = 0 , \quad \beta_{5,\pm 4}(\theta) = 0.109 \sin 5/2\,\theta - 0.294 \,\mathrm{Sin}\,3/2\,\theta ,$$

$$\alpha_{5,\pm 3}(\theta) = 0.208 \,\mathrm{Cos}5/2\,\theta + 0.30 \,\mathrm{Cos}3/2\,\theta , \; \beta_{5,\pm 3}(\theta) = 0 ,$$

$$\alpha_{5,\pm 2}(\theta) = 0 , \quad \beta_{5,\pm 2}(\theta) = 0.34 \mathrm{Sin}5/2\,\theta - 0.1132 \,\mathrm{Sin}3/2\,\theta , \tag{11}$$

$$\alpha_{5,\pm 1}(\theta) = 0.45 \,\mathrm{Cos}5/2\,\theta - 0.335 \mathrm{Sin}\,3/2\,\theta . \quad \beta_{5,\pm 1}(\theta) = 0 ,$$

$$\alpha_{5,0}(\theta) = 0 , \quad \beta_{5,0}(\theta) = 0.5 \,\mathrm{Sin}\,5/2\,\theta - 0.2734 \mathrm{Sin}3/2\,\theta .$$

The differential cross section for the inelastic scattering events leading to L = 5 states in nuclei is again, using eqn.(3) has the form:

$$\frac{d\sigma}{d\Omega}\,(0 \rightarrow 5) = (\delta^2_5)\,(T^2/64\,\pi)\,(\theta/\mathrm{Sin}\,\theta)\,[\,(H_- - H_+)^2\,\{\,J^2_0\,(T\,\theta)\,(2\alpha^2_{51}(\theta) + \beta^2_{50}(\theta)) + J^2_2\,(T\,\theta)\,(2\alpha^2_{53}(\theta) + 2\,\beta^2_{52}(\theta)) + J^2_4\,(T\,\theta)\,(2\,\alpha^2_{53}(\theta) + 2\,\beta^2_{52}(\theta)) + J^2_4\,(T\,\theta)\,(2\,\alpha^2_{55}(\theta) + 2\,\beta^2_{54}(\theta))\} + (H_- - H_+)^2\{\,J^2_3\,(T\,\theta)\,(2\,\alpha^2_{55}(\theta) + 2\,\beta^2_{54}(\theta)) + J^2_5\,(T\,\theta)\,(2\alpha^2_{55}(\theta))\}\,] \qquad (12)$$

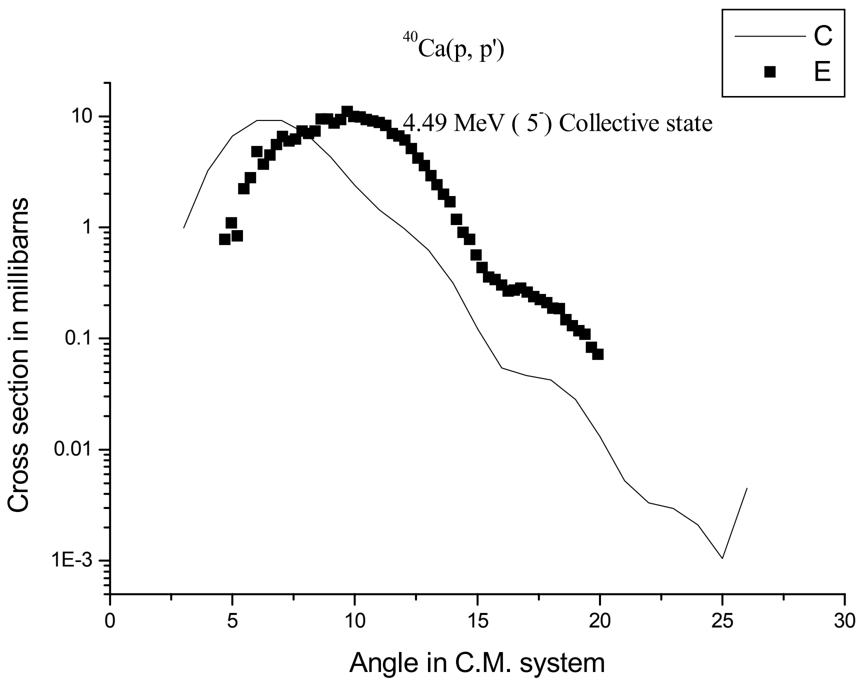

Fig.2 Inelastic scattering leading to L = 5 state in ^{40}Ca (4.49 MeV (5^-) collective state).

5. Comparison with Experiment

We apply eqns. (10) and (12) to calculate the theoretical differential cross section for the inelastic scattering of 66.5 MeV protons from ^{154}Sm [4)] populating 4^+ collective state of 266 KeV excitation energy in ^{154}Sm and for the inelastic scattering of 1044 MeV protons from ^{40}Ca [5)] leading to 5^- collective state of excitation energy 4.49 MeV in ^{40}Ca. The elastic scattering cross sections of 66.5 MeV protons from ^{154}Sm and 1044 MeV protons from ^{40}Ca are computed using the formulations due to Frahn and Venter [6)]. The detailed elastic scattering parameters and the theoretical predictions to the experiments are given in Ref.[7)]. **Table 1.** contains the SAM parameters for the elastic and inelastic scattering. The comparison between theory and experiment are shown in **Figs.1** and **2**. Normalization of theory to experiment is carried out in such a manner that the first peak in the forward angles is reproduced by the theory. The hexadecapole deformation parameter β_4 and β_5 extracted in the present work along with the available literature values are presented in **Table 1.** The quality of description to the experimental inelastic scattering data is reasonable and moderate in the angular distributions of 4^+ and 5^- collective states. The hexadecapole deformation parameter β_4 determined in the present work by employing the model SAM[9)] model is in excellent agreement with the literature quoted values [4,8)]. The deformation parameter β_5 for the collective state labeled by L = 5 in ^{40}Ca is found to be 0.005; which could not be compared with any other values because of the non availability of any other value in the literature. The theoretical prediction to the angular distribution of 5^- collective state is significantly good enough and the value of β_4 deformation parameter in the present analysis by the model SAM is in excellent agreement with the available literature values. It is to further note that the theoretical prediction to the inelastic scattering data has been performed without bringing any change in the elastic scattering parameters. All these speak about the justification of the β_5 – value obtained from the derivation of inelastic formulae in the present work. We are of the confirmed opinion that the inelastic scattering formulae derived here can be applied in suitable cases reliably. Further works on these lines are in progress.

Acknowledgements

This work was accomplished within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. Financial support from the Swedish International Development Cooperation Agency is acknowledged.

References

1) N. Austern and J.S. Blair, *Ann.Phys. (N.Y.),* ***33*** *(1975)15.*

2) J.M. Potgieter and W.E. Frahn, Nucl. Phys. **80** (1966)434.

3) Md.A. Rahman and H.M. Sen Gupta, Nuovo Cim. **105A** (1992) 851.

4) A. Guterman, D.L. Hendrie, P.H. Debenham, K.Kwiatkowski, A. Nadesen, L.W. Woo and R.M. Ronningen, Phys. Rev. **C39** (1989)1730.

5) G.D. Alkhazov, T. Bauer, R. Beurtey, A. Boudard, G. Bruge, A. Chaumeax, P. Couvert, G. Cvijanovich, H.H. Duhm, J.M. Fontaine, D. Garreta, A.V. Kulikov, D. Legrand, J.C. Lugol, J. Saudinos, J. Thirion and A.A. Vorobyov, Nucl. Phys. **A274** (1976) 443.

6) W.E. Frahn and R.H. Venter, *Ann. Phys. (N.Y.)*, **24** (1963) 243.

7) S. Nasmin Rahman, Md. Sanaul Haque, Sangita Haque and Md.A. Rahman, Proton–Nucleus interaction at low and intermediate energies; As I C T P Preprint **IC/2004/36**.

8) S. Raman, C.H. Malarkey, W.T. Milner, C.W. Nestor (Jr.) and P.H. Stelson, Atomic Data & Nuclear Data Tables, **36** (1987) 2.

9) W.E. Frahn and R.H. Venter, Ann. *Phys. (N.Y.),* **24** (1963)243; W.E. Frahn, in *Fundamental in Nuclear Theory* (International Atomic Energy Agency, Vienna, **1967**), p.1.