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THE ABDUS SALAM INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

PROTON-NUCLEUS INTERACTION AT LOW AND INTERMEDIATE ENERGIES

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Abstract

The elastic scattering of protons from a number of nuclei between ^{12}C and ^{208}Pb is the subject matter of the present study at the projectile energies 30.3, 66.5 and 1044 MeV within the framework of the generalized diffraction models of Frahn and Venter. The best fit parameter values, the cut-off angular momentum T, the rounding parameter Δ and the real nuclear phase shift μ are obtained from the elastic scattering analyses of the entrance channel angular distributions. The interaction radius R, diffuseness d and the reaction cross section σ_r , have been estimated from the best fit parameters. Energy dependence of T, Δ and $\sigma_r/\pi R^2$ and mass dependence of R are discussed. Finally, the inelastic scattering of protons exciting to the lowest 2^+ collective states in $^{42,44,48}Ca$, ^{48}Ti and $^{148,154}Sm$ and 3^- collective state in ^{40}Ca are studied to check the validity of the derived elastic scattering parameters. The deformation parameters β_2 and β_3, so extracted are comparable with the available values in literature.

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1. Introduction

The nuclear projectiles n, p, τ, ^3He, ^4He and the like are strongly absorbed by the target nuclei. The diffraction model or the so-called strong absorption model (coined as SAM) was initiated by Frahn and Venter[1)] to study the interaction of hadrons and various other nuclear probes with target nuclei as an alternative to the optical model. The simplicity of the approach is that the elastic scattering phenomena become easily describable without any knowledge of the absorption mechanism. The scattering function η_l of the model is expressed as a function of the angular momentum of the projectile, thereby avoiding the concept of potential as used in the optical model approaches. The scattering function η_l can be made complex to account for the non-elastic processes. The model under discussion has been used by some of the present authors for the description of the interaction of pions, kaons, nucleons, deuterons, $^{3,4}He$ and heavy ions with various target nuclei[2)].

We, in the present communication, study the elastic scattering of protons from several nuclei between ^{12}C and ^{208}Pb at projectile energies 30.3, 66.5 and 1044 MeV [3-5)]. Angular distributions data for the inelastic scattering of protons leading to the lowest 2^+ collective states in $^{42,44,48}Ca$, ^{48}Ti and $^{148,154}Sm$[5)] and 3^- collective states in ^{40}Ca are then studied using the best fit SAM elastic scattering parameters and the corresponding deformations parameters are determined.

2. The SAM formalism

The strong absorption model of Frahn and Venter[1)] is used for the calculation of the differential cross section. The scattering function η_l of the model is parameterized through the angular momentum of the projectile and an analytic expression is attained for differential cross section of the elastic scattering making suitable approximations[6)]. The parameters of the model are T, Δ and μ for the reproduction of the elastic scattering cross section. The first two parameters are related semi-classically to the interaction radius R and surface diffuseness d respectively through the relations:

$$\mathrm{T} = KR\left(1 - \frac{2n}{KR}\right)^{1/2} \quad (1)$$

and

$$\Lambda = Kd\left(1 - \frac{n}{KR}\right)\left(1 - \frac{2n}{KR}\right)^{-1/2} \quad (2)$$

The strong absorption model form of η_l was given by Potgieter and Frahn[7)] in the generalized scattering amplitudes of the adiabatic distorted waves, a pioneering work of Austern and Blair[8)] and an analytic expression for the inelastic scattering amplitude is obtained as the first derivative of the elastic scattering amplitude in terms of the parameters, namely T, Δ and μ, all fixed from the elastic scattering. The fourth parameter, the normalization constant, of theory to experiment, is the only free parameter proportional to the reduced matrix element of the inelastic scattering interaction. The deformation parameter β_L of different multiple modes and connected to reduced matrix element, can be determined from the collective excitation of the target nucleus.

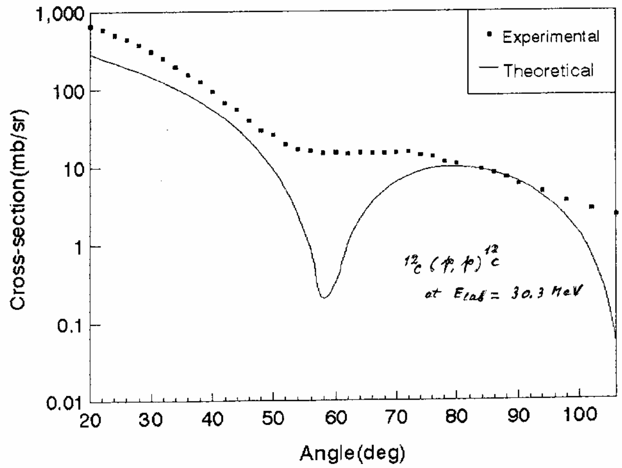

Fig.1. Elastic Scattering of protons from ^{12}C at $E_{lab} = 30.3$ MeV.

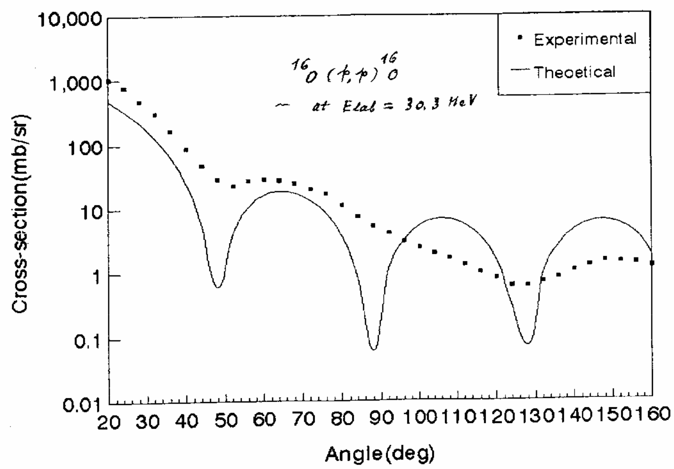

Fig.2. Elastic Scattering of protons from ^{16}O at $E_{lab} = 30.3$ MeV.

3. Results and Discussion

3.1 Elastic scattering analyses

The best fit SAM parameters of the elastic scattering are summarized in **Table** 1.

Table 1. The SAM parameters for the elastic scattering of protons.

SAM parameters					Derived parameters				
Nucleus	E_p (Mev)	T	Δ	μ/4Δ	R (fm)	r_0 (fm)	d (fm)	σ_r (mb)	$\sigma_r/\pi R^2$
^{12}C	30.3	3.75	0.01	0.0	4.60	1.35	.01	600	0.94
^{16}O	30.3	4.60	0.025	0.01	4.27	1.40	0.001	518	0.91
^{40}Ca	30.3	6.30	0.025	0.0	5.87	1.32	0.004	906	0.84
^{56}Fe	30.3	6.70	0.02	0.0	6.33	1.33	0.001	1018	0.81
^{59}Co	30.3	6.90	0.025	0.0	6.52	1.33	0.021	1075	0.81
^{58}Ni	30.3	6.80	0.025	0.01	6.46	1.33	0.021	1045	0.80
^{60}Ni	30.3	6.90	0.03	0.017	6.54	1.33	0.026	1076	0.80
^{62}Cu	30.3	7.1	0.35	0.043	6.73	1.35	0.30	1248	0.88
^{120}Sn	30.3	8.35	0.025	0.01	8.29	1.39	0.021	1546	0.72
^{208}Pb	30.3	10.0	0.25	0.10	10.53	1.52	0.203	2303	0.70
^{148}Sm	66.5	12.5	0.73	0.068	7.76	1.23	0.41	1765	0.93
^{154}Sm	66.5	12.75	0.75	0.017	8.0	1.22	0.42	1837	0.98

Comparisons between the experimental angular distributions and the SAM - predictions in some typical cases are made in **Figs.1-5**. Measured angular distributions display diffractive oscillations in heavier nuclei which wash out as one moves on to lighter nuclei. It may be mentioned that both the elastic and inelastic scattering experimental data are digitized from various refs.[3-5]. The quoted uncertainties range over a value of (5–7) % in the case of elastic scattering and the uncertainties in the case of inelastic scattering range over a value of about (15-20)%. A reasonably good description of the elastic scattering is given by the model. The fit is generally very poor in very light nuclei particularly for ^{12}C at low energies. The description to the data improves as the incident energy increases. This is evident in the description of the proton elastic scattering angular distributions from ^{40,42,44,48}Ca and ^{49}Ti a 1044 MeV. It is observed that the agreement between theory and experiment improves gradually as both the target mass and the projectile energy increase. The poor fit of the model to lighter nuclei at lower projectile energies and the perfect matching between theory and experiment for higher target mass number and at higher projectile energies are in conjunction with the fact that the SAM conditions are ($2\pi T >> 1$, $\frac{1}{2}\pi (\Delta/T) << 1$) are less satisfied in the former case while they are satisfied in the latter case respectively.

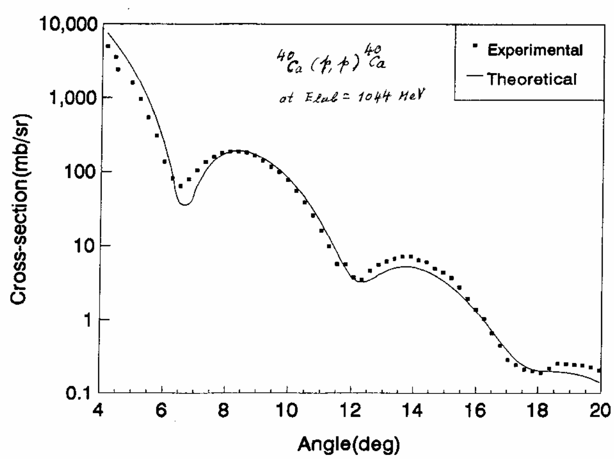

Fig.3. Elastic Scattering of protons from ^{40}Ca at E_{lab} = 1044 MeV.

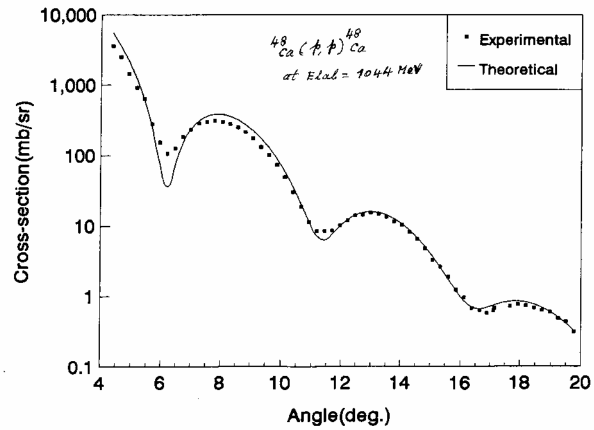

Fig.4. Elastic Scattering of protons from ^{48}Ca at E_{lab} = 1044 MeV.

Table 2. The SAM parameters for the elastic scattering of protons at 1044 MeV.

SAM parameters				Derived parameters				
Nucleus	T	Δ	μ/4Δ	R	r_0	d	σ_r	$\sigma_r/\pi R^2$
		(fm)	(fm)	(fm)	(mb)			
^{40}Ca	33	5.25	0.097	4.8	1.08	0.76	1003	1.39
^{42}Ca	34.1	5.40	0.095	4.95	1.11	0.78	1067	1.38
^{44}Ca	34.4	5.40	0.095	4.92	1.10	0.78	1081	1.38
^{48}Ca	35.5	4.80	0.096	5.15	1.14	0.72	1098	1.32
^{48}Ti	35.75	5.20	0.096	5.18	1.12	0.75	1134	1.36

We then determine the quantities like the interaction radius R, the surface diffuseness d and reaction cross section σ_r. These are tabulated in **Tables** 1 and 2 respectively. The values of R and d are obtained from eqns. (1) and (2) and σ_r is determined from the expression[1)]:

$$\sigma_r = \pi T^2/K^2 \; [\, 1 + 2\,(\Delta/T) + \pi^2/3\,(\Delta/T)^2 - 1/3\,(\pi/\Delta)^2\,(\Delta/T)^2 \,] \qquad (3)$$

The standard nuclear radius parameter r_0 (= $R/A^{1/3}$) from 30.3 MeV data analyses yields a value ≈ 1.37 fm and that from 1044 MeV data analyses gives a value ≈ 1.11 fm. A somewhat smaller value of r_0 at much higher energy is consistent with the smaller de Broglie wave length than at the 30.3 MeV data analysis. The least squares method furnishes us with the following relation for the interaction radius:

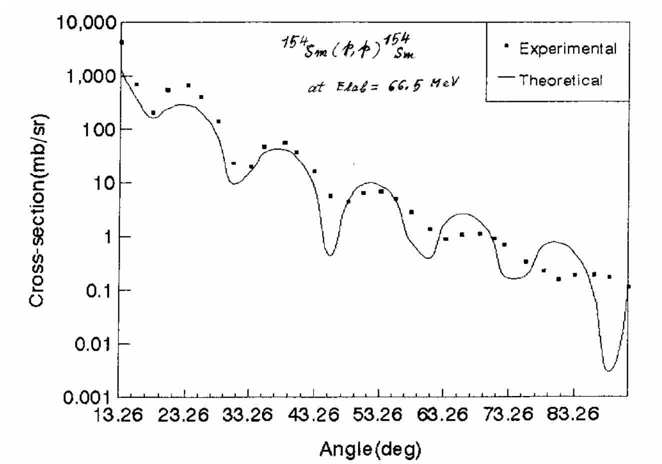

Fig.5. Elastic Scattering of protons from ^{154}Sm at E_{lab} = 66.5 MeV.

$$R = 1.6634\, A^{1/3} + 0.20465 \quad \text{for 30.3 MeV}$$

and

$$R = 1.6094\, A^{1/3} - 0.6805 \quad \text{for 1044 MeV}$$

The diffuseness d of the nuclear surface assumes a negligible value up to 0.30 fm for the present target mass region ^{12}C to ^{208}Pb at 30.3 MeV projectile energy while d remains practically constant at the value = 0.758 ± 0.17 fm for the mass region A= 40- 48 at the proton energy 1044 MeV. The mass number dependence of reaction cross section σ_r, as given by the model, was then studied. Reaction cross section σ_r is dependent both on target mass as well as on the projectile energy. Reaction cross section increases with the increase in target mass number while the projectile energy remains fixed. This fact is substantiated for 30.3 MeV as well as for the 1044 MeV as shown in **Tables** 1 and 2 of SAM parameters. The quantity $\sigma_r/\pi R^2$ is perhaps more meaningful than the quantity σ_r itself as the former remains fairly constant for various target masses at the same projectile energy. The fairly constant value of the quantity $\sigma_r/\pi R^2$ is evident from the present study (**Tables** 1 and 2). The least squares relations for reaction cross section σ_r are as follows:

$$\sigma_r = 0.2044\, A^{2/3} + 3.3684 \quad \text{for 30.3 MeV}$$

and

$$\sigma_r = 0.5448\, A^{2/3} + 25.62 \quad \text{for 1044 MeV}$$

The $A^{2/3}$ dependence of σ_r indicates the strong surface absorption situation.

3.2 Inelastic scattering analyses

The inelastic scattering angular distributions of 65 and 1044 MeV[4,5] leading to collective states, such as the lowest 2^+ and 3^- states in several nuclei, have been undertaken using SAM parameters obtained from the description of the corresponding elastic scattering channels (**Tables 1** and **2**).

Table 3. Quadrupole and octupole deformation parameters from inelastic scattering of protons.

Nucleus	E_p (MeV)	E_x (MeV)	J^π	Deformation parameter β_L	
				(a)	(b)
^{42}Ca	1044	1.524	2^+	0.205±0.08	0.209[b)]
^{44}Ca	1044	1.157	2^+	0.216±0.09	0.218[b)]
^{48}Ca	1044	3.383	2^+	0.0952±0.04	0.0854[b)]
^{48}Ti	1044	0.983	2^+	0.2284±0.06	0.24[b)]
^{148}Sm	66.5	0.055	2^+	0.17	0.12[c)]
^{154}Sm	66.5	0.082	2^+	0.25	0.253[c)]
^{40}Ca	1044	3.737	3^-	0.31042±0.08	0.33-0.36[d)]

a) Present Analysis

b) Summary from electromagnetic measurements Ref.[9)]

c) Ref.[5)]

d) Range of β values as quoted in Ref.[10)]

The fits to the inelastic scattering angular distribution data are displayed in **Figs. 6** and 7. The deformation parameters β_L are obtained from the relation:

$$\delta_L = (T/K)\,\beta_L$$

where, δ_L is found from normalizing the theoretical cross sections to experimental ones in the respective nuclei. The deformation parameter values β_2 and β_3 extracted are summarized in **Table** 3. The β_L values from previous other works are also presented in the **Table** 3 for comparison.

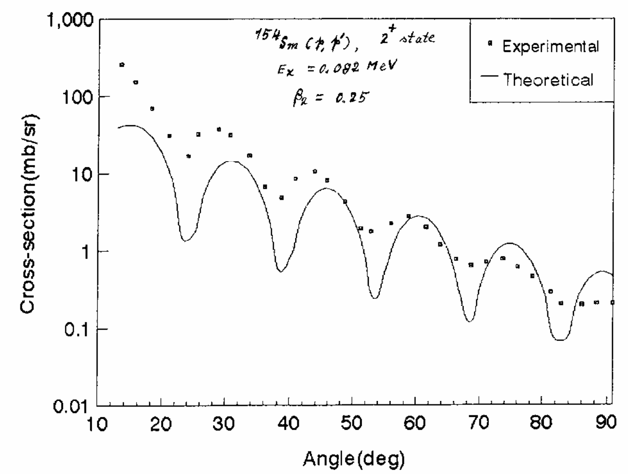

Fig.6. Inelastic Scattering of protons from ^{154}Sm leading to 2^+ collective state in ^{154}Sm.

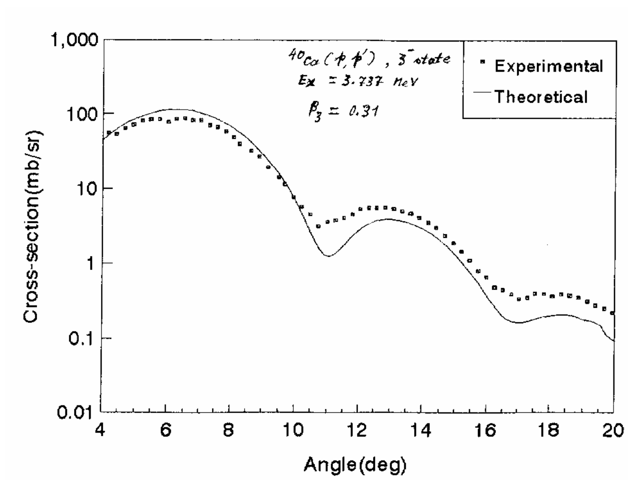

Fig.7. Inelastic Scattering of protons from ^{40}Ca leading to 3^- collective state in ^{40}Ca.

The general characteristics, such as diffractive oscillations in the angular distributions, practically all of them are reproduced by the model throughout the whole angular range. Besides minor discrepancies in the description of the experimental angular distributions of Sm isotopes by the present model, all the other states such as 2^+ states in ^{42,44,48}Ca and ^{48}Ti and 3^- state in ^{40}Ca are fairly well described by the model in comparison with the quality of fit obtained by models like DOMP / CCBA methods[5)]. The latter approaches are more complicated and have many more degrees of flexibility at their disposal. The deformation parameter values β_L for the lowest 2^+ and 3^- collective states are shown in **Table** 3 along with previous other works. The values of the present work compare reasonably well with other values studied through DOMP / CCBA analyses and with adopted values [9,10)].

4. Conclusion

The three parameters version of the simple generalized diffraction model of Frahn and Venter is fairly successful to account for the elastic and inelastic scattering data from several target nuclei between ^{12}C and ^{208}Pb at different projectile energies such as 33.3, 66.5 and 1044 MeV. The fit quality is generally poor particularly at larger angles, at lower energy and lower target mass number (cf. **Figs.**1 and 2). The quality of fit to the angular distribution is excellent at high energy i.e. at 1044 MeV. The SAM is thus successful in extracting standard nuclear radius r_0 to be ≈ 1.37 fm for the mass A = 12-208 at 30.3 MeV projectile energy and this turns out to be 1.10 fm for ^{40,42,44,48}Ca and ^{48}Ti nuclei at 1044 MeV. The so obtained values of r_0 are reasonable and consistent. The mass and energy dependence of reaction cross section are discussed. The quantity $\sigma_r/\pi R^2$ remains fairly constant for various target masses at the same projectile energy, as expected.

The deformation parameters β_2 and β_3 are determined for the available 2^+ collective states in ^{42,44,48}Ca and ^{48}Ti and for the available 3^- collective state in ^{40}Ca respectively. These values are compared with those obtained from the CCBA method and with adopted values. The values of the present analyses are in excellent agreement with the values from other studies. The SAM parameters obtained are thus reliable and credible in reproducing the inelastic scattering data with a minimum number of parameters as against several of the optical model, DOMP, CCBA and other microscopic methods.

Acknowledgements

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