SOME REMARKS ABOUT THE PROCESS $e^+ + e^- \rightarrow$ HADRONS

IN THE CASE OF POLARIZED ELECTRONS AND POSITRONS

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A B S T R A C T

General expressions for the inclusive
cross-section and some exclusive cross-sections
have been worked out for the annihilation of
polarized electron-positron pairs.

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1. The radiative polarization of electrons and positrons moving in the storage ring was theoretically predicted more than a decade ago[1]. This polarization was very recently definitely observed at the VEPP-2M storage ring of the Novosibirsk Institute of Nuclear Physics[2] (direct measurement of the polarization of an electron beam, reaction $e^+ + e^- \to \mu^+ + \mu^-$) and at the SPEAR storage ring of SLAC[3-5] (reaction $e^+ + e^- \to \mu^+ + \mu^-$, inclusive cross-section $e^+ + e^- \to h + X$, jet structure analysis). Thus, colliding-beam experiments with polarized particles become reality. Taking this into consideration we will write below some formulae, which describe the annihilation of the polarized electron-positron pair.

2. Let us consider the process:
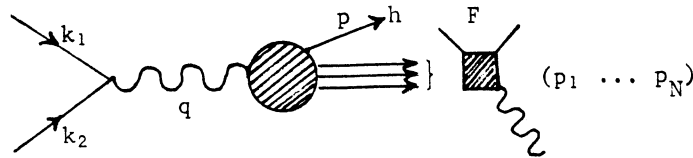
The cross-section of this process can be represented in the form

$$d\sigma = \frac{\alpha^2}{(k_1 k_2) q^4} \left| \bar{v}(k_2) \gamma^\mu u(k_1) \langle p, F | J_\mu(o) | o \rangle \right|^2 (2\pi)^3 \, \delta(q - p - \textstyle\sum p_i) \frac{d^3 p}{2\epsilon_p} \prod_{i=1}^{N} \left[\frac{d^3 p_i}{2\epsilon_i} \frac{1}{(2\pi)^3} \right]$$ (1)

As a consequence of polarization of the initial electrons and positrons, the cross-section of the process becomes substantially deformed as compared to the cross-section of the process for unpolarized particles. For the two-particle annihilation cross-section this was shown in an earlier paper[6]. Some results for resonance production are given in Schilling[7].

Consider the inclusive cross-section, which can be obtained from Eq. (1) in a straightforward manner, using the standard definition of the Bjorken structure functions $\tilde{W}_1$ and $\tilde{W}_2$

$$\sum_{F, s_p} \langle p, F | J_\mu(o) | o \rangle \langle p, F | J_\nu(o) | o \rangle^* (2\pi)^3 \, \delta(q - p - \textstyle\sum p_i) = \tilde{W}_{\mu\nu}$$

$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \tilde{W}_1 + \frac{1}{m_h^2} \left(p_\mu - q_\mu \frac{qp}{q^2} \right) \left(p_\nu - q_\nu \frac{qp}{q^2} \right) \tilde{W}_2$$ (2)

where m_h is the mass of particle h. However, one can also use the general formula, obtained in Ref. 6 [see Eq. (7)], which turns out to be valid for the inclusive cross-section of the process $e^+ + e^- \to h + X$, if one makes the replacement of notation of the structure functions used in this formula

$$D_1 \to \frac{\tilde{W}_1}{4} \quad , \quad D_2 \to -\frac{\tilde{W}_2}{8 m_h^2} \quad ,$$ (3)

and gets back the δ-function $\delta(q^2 - 2qp)$, that is, makes the substitution

$$\frac{|\vec{q}|d\Omega}{8\epsilon} \longrightarrow \frac{d^3p}{2\epsilon_p} \tag{4}$$

Finally, we obtain (we omit the terms $\sim m^2/\epsilon^2$, where m is the electron mass, ϵ is the initial energy of the electron)

$$d\sigma = \frac{\alpha^2}{16\epsilon^4}\frac{d^3p}{2\epsilon_p}\left\{2\,\tilde{W}_1\left(1+\frac{(\vec{\zeta}\vec{k})(\vec{\zeta}_1\vec{k})}{\epsilon^2}\right)+\frac{\tilde{W}_2}{m_h^2}\left[\vec{p}^2\sin^2\theta\,(1+\vec{\zeta}\vec{\zeta}_1\right)-\right.$$
$$\left. 2\,(\vec{p}\vec{\zeta})(\vec{p}\vec{\zeta}_1)+2\frac{(\vec{k}\vec{p})}{\epsilon^2}\left((\vec{k}\vec{\zeta})(\vec{p}\vec{\zeta}_1)+(\vec{k}\vec{\zeta}_1)(\vec{p}\vec{\zeta})\right)-2\frac{(\vec{k}\vec{p})^2}{\epsilon^4}(\vec{k}\vec{\zeta})(\vec{k}\vec{\zeta}_1)\right]\right\} \tag{5}$$

where $\vec{k}_1 = -\vec{k}_2 = \vec{k}$, θ is the angle between the vectors $\vec{k}$ and $\vec{p}$, and $\vec{\zeta}$, $\vec{\zeta}_1$ are the vectors of the electron and positron polarization in their rest frame.

The structure functions $\tilde{W}_1$ and $\tilde{W}_2$ can be related in a known way to partial decay widths of the polarized (transverse T and longitudinal L) time-like photons [see, for example, Gourdin[8)]]

$$d\Gamma_{T(L)} = \frac{\alpha}{2\pi}\frac{1}{2\epsilon}\frac{d^3p}{2\epsilon_p}\,e^\mu_{T(L)}\,e^{\nu\,*}_{T(L)}\,\tilde{W}_{\mu\nu} \tag{6}$$

From Eqs. (6) and (2) we obtain the relations

$$\tilde{W}_1 = \frac{2\epsilon}{\alpha|\vec{p}|}\frac{d\Gamma_T}{d\epsilon_p}\quad,\quad \tilde{W}_2 = \frac{2\epsilon\,m_h^2}{\alpha|\vec{p}|^3}\left(\frac{d\Gamma_L}{d\epsilon_p}-\frac{d\Gamma_T}{d\epsilon_p}\right) \tag{7}$$

After the integration over the energy ϵ_p, the cross-section (5) in terms of Γ_L, Γ_T takes the form

$$\frac{d\sigma}{d\Omega} = 2\sigma_T\left(1+\frac{(\vec{\zeta}\vec{k})(\vec{\zeta}_1\vec{k})}{\epsilon^2}\right)+(\sigma_L-\sigma_T)\left\{\sin^2\theta\,(1+\vec{\zeta}\vec{\zeta}_1)-\right.$$
$$\left. 2\frac{(\vec{p}\vec{\zeta})(\vec{p}\vec{\zeta}_1)}{\vec{p}^2}+2\frac{(\vec{k}\vec{p})}{\vec{p}^2\epsilon^2}\left[(\vec{p}\vec{\zeta}_1)(\vec{k}\vec{\zeta})+(\vec{p}\vec{\zeta})(\vec{k}\vec{\zeta}_1)\right]-2\frac{(\vec{k}\vec{p})^2}{p^2\epsilon^4}(\vec{k}\vec{\zeta})(\vec{k}\vec{\zeta}_1)\right\} \tag{8}$$

where $\sigma_{T(L)} = (\alpha/16\epsilon^3)\Gamma_{T(L)}$.

In the actual case of transverse antiparallel polarization of the initial particles we have the form used in Ref. 4

$$\frac{d\sigma}{d\Omega} = \sigma_T+\sigma_L+(\sigma_L-\sigma_T)\cos^2\theta+|\vec{\zeta}||\vec{\zeta}_1|(\sigma_T-\sigma_L)\sin^2\theta\cos 2\phi \tag{9}$$

where an azimuthal angle ϕ is measured from the plane perpendicular to the vector ζ (orbit plane).

For the longitudinal polarization of the initial particles [this polarization can really be obtained in the beam intersecting region, see other papers[9,10])]
we have

$$\frac{d\sigma}{d\Omega} = \left(1 + \vec{\zeta}\,\vec{\zeta}_1\right)\frac{d\bar{\sigma}}{d\Omega} \tag{10}$$

This result is obvious if one takes into account the fact that in the ultrarelativistic limit there is annihilation only for an electron and a positron of opposite helicities.

The above result reveals that experiments involving polarized initial particles will yield no essentially new information on cross-sections σ_T, σ_L, other than that which can be provided by experiments with unpolarized particles. This was pointed out in Ref. 6. However, under actual experimental conditions, only polarization permits the differentiation between σ_T and σ_L. A knowledge of the interrelation between σ_T and σ_L is very important for theoretical analysis, particularly for the parton model.

3. From current conservation $q^\mu J_\mu = 0$, it follows that in the c.m. frame the transition current has only the space components $\langle F|\vec{J}|0\rangle \equiv \vec{J}$. Consider the case of n-pion production ($e^+ + e^- \to$ n-pions). Here $\vec{J}$ is a polar vector if n is even and $\vec{J}$ is an axial vector if n is odd. In the c.m. frame the lepton current can be written in a simple two-component form

$$\vec{j} = 2ie\,\phi_p^+\left[\,\vec{k}\,\vec{\sigma}\,\right]\phi_e \tag{11}$$

where ϕ_p, ϕ_e are two-component spinors which describe particle polarization. Starting from Eq. (1) and using Eq. (11), one easily obtains for transverse antiparallel (parallel) polarization

$$d\sigma_{\substack{\uparrow\downarrow\\\downarrow\uparrow}} = \frac{\alpha^2}{16\varepsilon^4}\,|\vec{J}|^2\,\sin^2\theta\,(2\pi)^6\,\delta(k_1+k_2-\Sigma\,p_i)\prod_{i=1}^{N}\left[\frac{d^3p_i}{2\varepsilon_i}\frac{1}{(2\pi)^3}\right]\left(1\mp|\vec{\zeta}||\vec{\zeta}_1|\cos 2\phi\right) \tag{12}$$

$$= d\sigma / \left(1\mp|\vec{\zeta}||\vec{\zeta}_1|\cos 2\phi\right)$$

For n = 2 $[\vec{J} = 2\vec{p}F_\pi(q^2)]$, we obtain the known result[6], for n = 3, $\vec{J} = 2\varepsilon H(\vec{p}_1 \times \vec{p}_2)$, where $\vec{p}_1$, $\vec{p}_2$ are the momenta of the charged pions and H is the "form factor". Putting this $\vec{J}$ into Eq. (12), we find the deformed cross-section [the number of events increases when the vector $\vec{p}_1 \times \vec{p}_2$ is situated in the plane $(\vec{k}, \vec{\zeta})$, etc.]. This effect should be observed at Novosibirsk.

Thus, we see that every exclusive cross-section of pion production becomes substantially deformed as compared to the cross-section of the process for un-polarized particles. This fact must be taken into consideration in the analysis of the experimental data.

4.   If jets are produced through a pair of partons, then we must consider the cross-section of two-particle annihilation (spin?). A general formula for this case, when the summation is taken over spins of the final particles, has been obtained in Ref. 6. As it was pointed out, this expression has the same form as the inclusive cross-section. So one can use for this case just the formula (8), but σ_T, σ_L are now the cross-sections of pair production through the virtual photon with T, L polarization. Angular distribution of the produced partons can also be written as a Feynman-type formula[11]

$$D_{1/2} \left[1 + \cos^2 \theta + |\vec{\xi}||\vec{\xi}_1| \sin^2 \theta \cos 2\phi \right] + \frac{D_0}{2} \left(1 - \cos^2 \theta \right) \left(1 - |\vec{\xi}||\vec{\xi}_1| \cos 2\phi \right) \tag{13}$$

where $D_{1/2}$ and D_0 are proportional to the charge of partons of spin $1/2$ and 0, respectively.

Acknowledgement

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