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OVERLAP ANALYSIS OF TWO-BODY SCATTERING AMPLITUDES *)

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ABSTRACT

The relationship between multiparticle final states and two-body amplitudes, implied by s channel unitarity, is studied. After giving model-independent estimates, their realization in specific models and in actual data is discussed.

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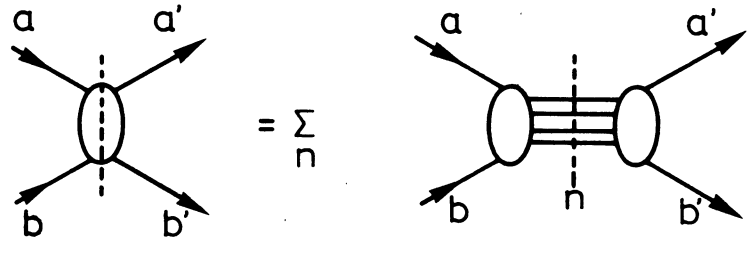
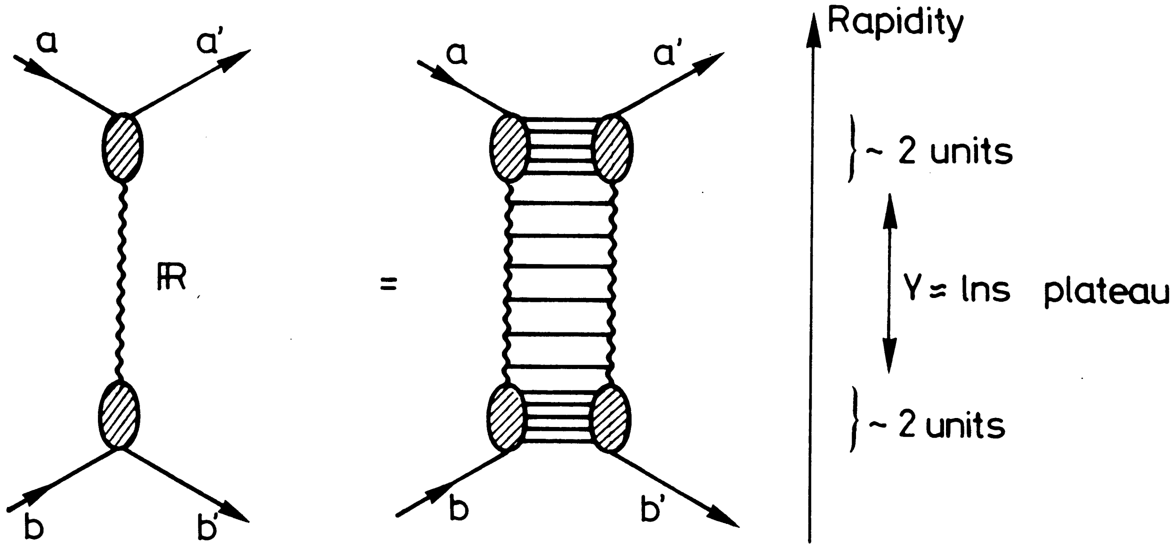
Any model for multiparticle production yields, via unitarity, also some prediction for two-body amplitudes at high energies. This connection is particularly useful if inclusive correlations are of short range, in which case we can picture the unitarity equation



symbolically as



Notice that we need not restrict ourselves here to the Pomeranchuk amplitude, but we can study in this way also any quantum number exchange, in which case the output Reggeon will depend on the way this quantum number is distributed among the produced particles in the central region. The fragmentation regions influence only the couplings to the incoming particles.

This had been used in model calculations for a long time, though with varying success, in particular what concerns the slope of elastic scattering [1),2)]. Only in the last years, however, it was realized by several groups [3)-5)] that this connection can be studied model-independently, to some extent. In the following, we shall discuss mainly the case of momentum exchange, i.e., the elastic scattering at small angles. Only at the end we shall make some comments about charge exchange. Out treatment will closely follow the one in the second paper of Ref. 5). A more general derivation can be found in the papers of Krzywicki and Weingarten [4].

The most important parameter for elastic scattering is, apart from the integrated cross-section, the forward slope

$$b = 2\frac{d}{dt}\ln A(s,t)\Big|_{t=0} = \frac{1}{2}\langle\vec{B}^2\rangle , \tag{1}$$

where $\vec{B}$ is the impact parameter distance of the incoming particles

$$(B_1, B_2) = \frac{1}{p_{in}}(-J_2, J_1)$$

(J is the angular momentum operator). The expectation value is to be taken with respect to the scattered state $T|\alpha_{in}>$. So the problem of estimating the elastic slope is equivalent to get an estimate on the average angular momentum of the produced multiparticle state. The simplest way consists in searching for an observable A whose expectation value is highly non-invariant under rotations, i.e., $[\vec{J},A]\neq 0$. Then the uncertainty principle tells us that

$$\langle B_i^2\rangle = \frac{1}{p_{in}^2}\langle J_i^2\rangle \geq \frac{1}{4p_{in}^2}\frac{|\langle[J_i,A]\rangle|^2}{\langle A^2\rangle-\langle A\rangle^2} . \tag{2}$$

Furthermore, it will be advantageous to use for A a single-particle operator, since then the right-hand side of Eq. (2) will be expressible in terms of one- and two-body inclusive densities.

One useful choice for A, which leads to expressions resembling estimates of multiperipheral models, is the integral over one component of the transverse momentum transfer,

$$A = \int_{-\infty}^{\infty} dy\, K_1(y) ,$$
$$\vec{K}_\perp(y) = \int_y^{\infty} d\eta \int d^2p_\perp\, \vec{p}_\perp\, n(\eta,\vec{p}_\perp) , \tag{3}$$

where $n(y,\vec{p}_\perp)$ is the particle density operator such that $\langle n(\vec{p})\rangle$ is the single-particle density, $\langle :n(\vec{p})n(\vec{k}):\rangle$ the two-particle density, etc.

Up to terms $O(1/\ln s)$, the total impact parameter and the momentum transfer across any rapidity in the central plateau are canonically conjugate [6),7)]

$$[B_i , K_j(y)] = i\delta_{ij} \quad , \quad |y| \lesssim Y/2 \, , \tag{4}$$

and we get for the slope parameter

$$b \gtrsim \frac{(\ln s)^2}{2 \int\int_{-\infty}^{+\infty} dy\, dy' \, \langle \vec{K}_\perp(y) \cdot \vec{K}_\perp(y') \rangle} = \frac{(\ln s)^2}{| \int\int dy\, dy' (y-y')^2 \langle \vec{p}_\perp(y) \cdot \vec{p}_\perp(y') \rangle |} \, . \tag{5}$$

Here we have used Eq. (3) to replace the transfer correlations by the inclusive $\vec{p}_\perp$ correlations

$$\langle \vec{p}_\perp(y) \cdot \vec{p}_\perp(y') \rangle \equiv \int d^2p_\perp d^2k_\perp \, \vec{p}_\perp \cdot \vec{k}_\perp \;\; C(\vec{p}_\perp, y \, ; \vec{k}_\perp, y') \, . \tag{6}$$

This expression shows first of all the big influence of the fragmentation regions, due to the factor $(y-y')^2$, unless the $p_\perp$ correlations are strictly of short range. In the latter case, if their range λ is finite, the denominator increases like $\ln s$, and we get a Regge-like behaviour with the following bound for the Pomeron slope

$$\alpha'_P \gtrsim \frac{1}{4\lambda^2 \langle \vec{p}_\perp^{\,2} \rangle \cdot \langle n \rangle / \ln s} \, . \tag{7}$$

In naive multiperipheral models, the correlation length is comparable to the mean distance of neighbouring particles, $\lambda \approx \ln s/\langle n \rangle \approx 1/3$, and we get the absurd result $\alpha'_P \gtrsim 4$ [see also Ref. 1)], compared to the phenomenological value $\alpha'_P \approx 0.2 - 0.3$. Indeed, the latter puts a non-trivial lower bound on the $\vec{p}_\perp$ correlation length, $\lambda \gtrsim 1.5$. Indeed, there are various mechanisms possible that give similar estimates in more sophisticated multiperipheral models [2),7)-10)].

Experimental data on $\langle\vec{p}(y)\cdot\vec{p}_\perp(y')\rangle$ are very scarce. There are only preliminary data from Serpukhov [11], shown in Fig. 1. They indeed show a very long range, extending over the whole available rapidity range (indeed, the angular correlations also shown in Fig. 1 indicate that the slight decrease at large rapidities is only due to the fact that $\langle p_\perp^2\rangle$ is smaller there). Thus, not only the bound is amply fulfilled, but one might even have to look for some other mechanism to explain the Pomeron shrinkage, e.g., phases or highly spinning clusters.

Before doing this, one should check whether the above bound is really optimal, i.e., whether it makes the best use of multiparticle production data. The first restriction was that we used the uncertainty principle in connection with a single-particle operator A. One could relax this by using a two-particle operator, but one would need higher order inclusive correlations which will not be available for some time. Another possibility consists of taking for A an observable which is not defined inclusively, but for exclusive events [3),9),12),13)]. Then one is, however, experimentally constrained to a few channels only, so one cannot calculate the overlap function, though the information gained in this way is extremely useful in its own right. A further advantage of considering completely constructed events might be that one can take more sophisticated inequalities, instead of the uncertainty principle.

Sticking to a single-particle ansatz

$$A = \int \frac{d^3p}{p_0}\, w(\vec{p})\, n(\vec{p}) \tag{8}$$

with some arbitrary weight function $w(\vec{p})$ -- the previous ansatz corresponded to $w = y\cdot p_1$ -- the best one can do is to optimize the bound with respect to $w(\vec{p})$. In general, this is not easy, except in the case of purely long range correlations, corresponding to an uncorrelated jet model:

$$\langle \vec{p}_\perp(x)\cdot \vec{p}_\perp(x') = -\frac{\langle \vec{p}_\perp^{\,2}\rangle\, \varrho(x)\,\varrho(x')}{\langle n\rangle} \,. \tag{9}$$

In this case, the best bound is got with [3),13)]

$$w(\vec{p}) = p_1\cdot x \,,$$

and one finds

$$b \geq \frac{1}{4\langle p_\perp^2 \rangle} \cdot \int_{-1}^{+1} dx\, \varrho(x)\, x^2 - \frac{1}{4 p_{in}^2} \, . \tag{10}$$

It is amusing that this is indeed saturated in the uncorrelated jet model [14], up to contributions from phase or spin effects.

Everything we have done so far inclusively can, of course, also be done for exclusive channels or, disregarding neutrals, for events with fixed prong number. This would provide us with incomplete, but still interesting, information on the geometrical structure of multiparticle production [3),12),13)]. We shall not go into details, but instead conclude the discussion with a word of warning: one can try to apply the above bounds also to estimate the mean impact parameter of inelastic two-body cross-sections, as e.g., $\pi^- p$ charge exchange or $\pi^- p \to \pi^0 n$. In these cases, however, good amplitude analyses are available, which show that the bounds are off by at least a factor of 3, mainly due to the neglect of the nucleon spin. If a similar situation holds in multiparticle production, it might be very difficult to get realistic model-independent estimates.

Let us now go over to charge exchange, in addition to $p_\perp$ exchange. Our basic assumption shall be that charge correlations are of short range. By charge correlation we mean a quantity analogous to Eq. (6), but with $p_\perp$ replaced by the electric charge,

$$C_q(y, y') = \sum_{c,d} q_c q_d \; C_{cd}(y, y') \, . \tag{11}$$

Experimentally, this seems definitely to be true [15], in contrast to the $p_\perp$ case. We need it, since only in this case the properties of the Reggeon are determined by the total quantum number flow only. Pirilä et al. [16] have pointed out that even then the situation is not completely clear, and we refer to their paper for a detailed discussion.

Neglecting problems due to the discreetness of the charge (there is no <u>infinitesimal</u> charge transfer), we can use the uncertainty principle as before, to get [4]

$$\alpha_p(0) - \alpha_{\Delta q}(t) \geq \frac{|t|}{2 c_1/\ln s} + \frac{(\Delta q)^2}{4 c_2/\ln s} - \frac{|t|\,(\Delta q)^2 c_3}{8 c_1^2 c_2^2/\ln^3 s} \pm \cdots \tag{12}$$

where c_1 is the double integral appearing in Eq. (5), c_2 is the analogous integral with the $\vec{p}_\perp$ correlation replaced by the charge correlation defined in Eq. (11). The constant c_3 finally measures the combined $\vec{p}_\perp$ and charge transfer correlations. Keeping only the lowest order inclusive correlations, we can write for it

$$c_3 \cong \frac{1}{2}\iint dy\, dy' \,(y-y')^4 \Big\{ \langle \vec{p}_\perp(y)\cdot\vec{p}_\perp(y')\rangle_{--} - \langle \vec{p}_\perp(y)\cdot\vec{p}_\perp(y')\rangle_{+-} \Big\} . \tag{13}$$

From this we see that the sign of c_3 depends on whether the $\vec{p}_\perp$ correlations are stronger between particles of equal or of opposite charge. It would be extremely interesting if $c_3 < 0$, since this could (with many caveats) explain why $\alpha'_\rho > \alpha'_P$. There exist now preliminary data on $\langle \vec{p}_\perp(y)\cdot\vec{p}_\perp(y')\rangle_{\pm\cdot -}$ from Serpukhov [11], shown in Fig. 2. Though they still have huge errors, they clearly indicate that $c_3 > 0$, so our hope was false and the bigger slope of α_ρ must be explained differently.

Finally, one can evaluate Eq. (13) at forward direction, obtaining thus a bound on the ρ intercept. Unfortunately, inserting FNAL data [15], one gets $\alpha_\rho(0) \lesssim 0.9$, which is not too encouraging. Again one concludes that other effects must play a substantial role, which is indeed borne out in simple multiperipheral models [16),*)].

In conclusion, it seems to us that the method discussed is most useful for analyzing the interdependence of the slope of elastic scattering and $\vec{p}_\perp$ correlations in multibody production. On the one hand it shows clearly the physical basis of various model calculations, and in particular, it shows unambiguously that the troubles in obtaining the correct Pomeron slope in naive multiperipheral models are due to the fact that the $p_\perp$ correlations are wrongly described in these models. On the other hand, the method can hopefully be used -- eventually after optimizing it -- to estimate the impact parameter structure of multiparticle final states.

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*) The discussion in the second paper of Ref. 5) is essentially wrong in this respect.

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FIGURE CAPTIONS

Fig. 1 : $\vec{p}_\perp$ correlations, as defined in Eq. (6), in pp collisions at 69 GeV/c [11]. Also shown are the azimuthal angle correlations, defined similarly with $\vec{p}_\perp \cdot \vec{k}_\perp$ replaced by $\cos(\vec{p}_\perp, \vec{k}_\perp)$.

Fig. 2 : Difference of $\vec{p}_\perp$ correlations between particles with equal resp. opposite charge, from Ref. 11).

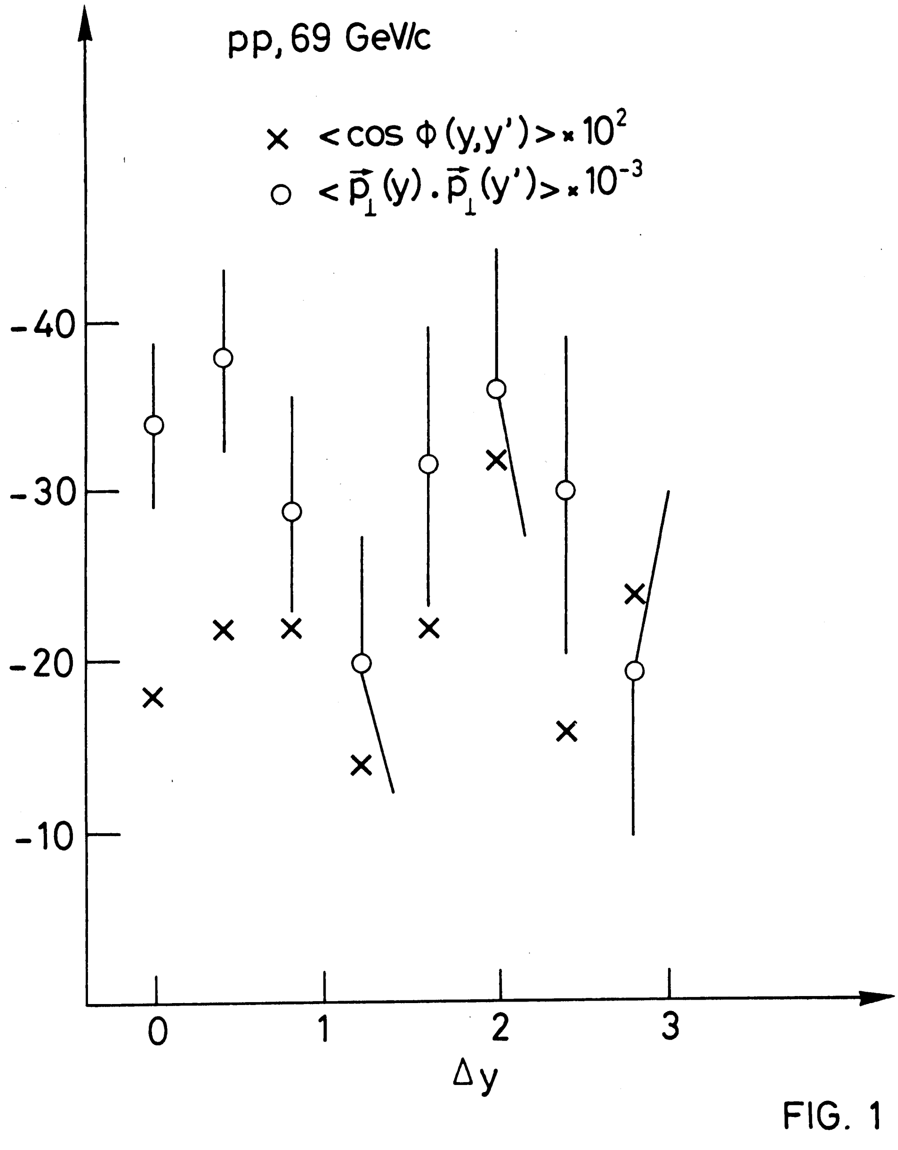
pp, 69 GeV/c
× <cos φ (y,y')> × 10^2
○ <$\vec{p}_\perp(y) \cdot \vec{p}_\perp(y')$> × 10^{-3}
-40
-30
-20
-10
0
1
2
3
Δy

FIG. 1

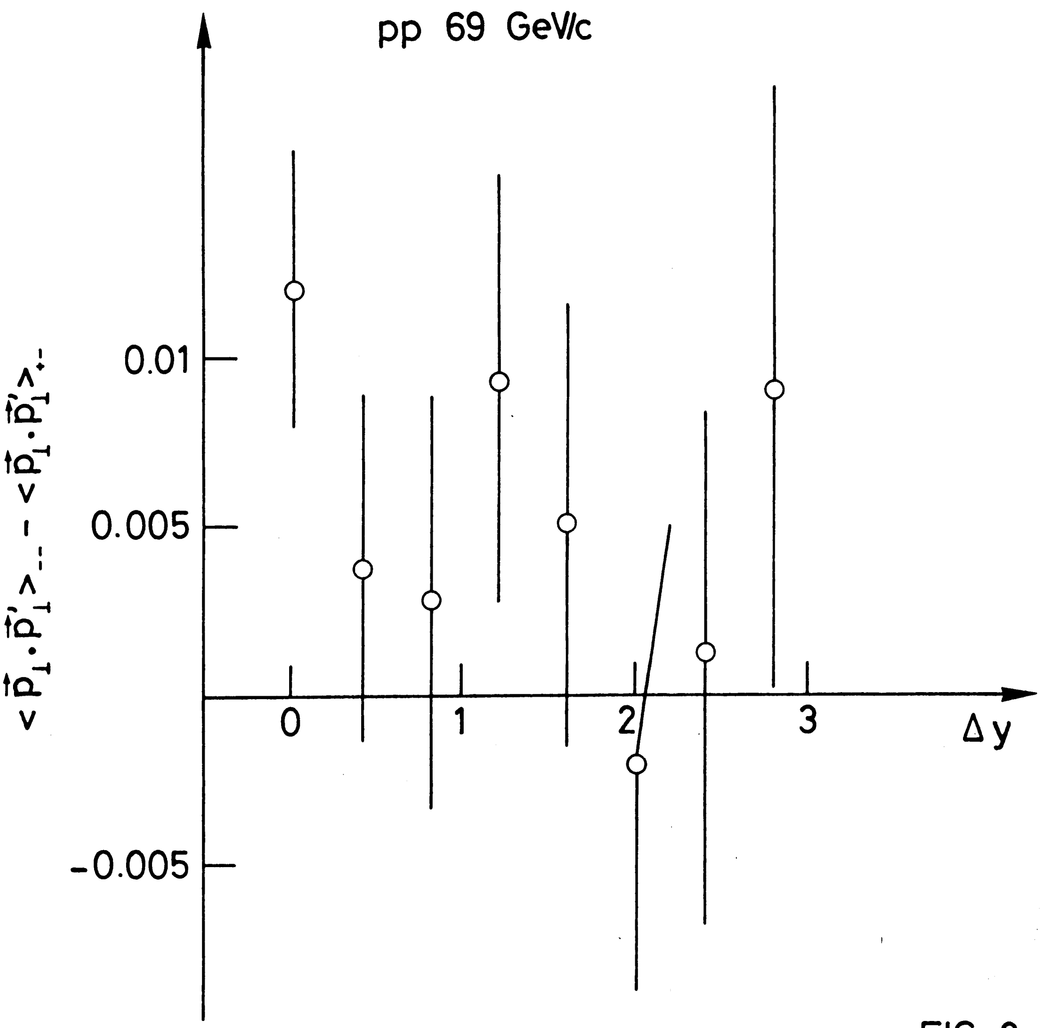
pp 69 GeV/c
0.01
0.005
-0.005
0
1
2
3
Δy
$<\vec{p}_\perp \cdot \vec{p}'_\perp>_{--} - <\vec{p}_\perp \cdot \vec{p}'_\perp>_{+-}$

FIG. 2