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CANCELLATIONS IN THE INCLUSIVE SUM RULE FOR THE TRIPLE POMERON VERTEX

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A B S T R A C T

We show that some multi-Reggeon contributions to the two-particle inclusive cross-section cancel in the sum rule relating the one and two-particle cross-sections. As a result the vanishing of the triple Pomeron coupling, at zero Pomeron mass, does not require a similar vanishing of that part of the Pomeron-Reggeon-particle vertex giving the Pomeron coupling in the total cross-section.

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Unitarity decoupling theorems for a zero mass Pomeron with unit intercept are potentially very serious. If total cross-sections are Pomeron dominated then the inclusive triple Pomeron vertex must vanish [1),2)]. If the Pomeron-Reggeon-particle vertex is then also required to vanish [3)-5)], and if this in turn requires the Pomeron to decouple from total cross-sections [6)], then it would appear that unitarity does not allow what experiment suggests, that is a Pomeron with unit intercept.

In fact the implications of the results of Refs. 3)-5) are much less serious. In a previous paper [7)] it has been shown that if the Pomeron-Reggeon-particle vertex $V(t_1,t_2,\eta)$ is written in the form *)

$$V(t_1,t_2,\eta) = \eta^{-\alpha_R(t_1)} V_1(t_1,t_2,\eta) + \eta^{-\alpha_P(t_2)} V_2(t_1,t_2,\eta) \qquad (1)$$

then the results of Refs. 3) and 4) require only that V_2 vanish at $t_2 = 0$ if $\alpha_P(0) = 1$. Since V_1 contains the poles at $\alpha_R(t_1) =$ integer, this gives no constraint on the Pomeron contribution to the total cross-section. It was stated in Ref. 7) that the results of Ref. 5) also only require the vanishing of V_2. In this paper we shall show that at the "planar" level (when cuts are clearly not involved), the multi-Pomeron-Reggeon contribution to the two-particle inclusive cross-section which contains only V_1, cancels in the integration over the momentum of one produced particle which is necessary to obtain the one-particle cross-section. This confirms that the results of Ref. 5) do not require the vanishing of V_1.

We begin by outlining the argument given by Jones, Low, Tye, Veneziano and Young in Ref. 5) which concludes that $V(t_1,t_2,\eta)$ vanishes at $t_2 = 0$. They consider the sum rule which relates the one and two-particle cross-sections [2)] which is written pictorially in Fig. 1. On taking the limit $s \equiv (p_a + p_b)^2 \to \infty$, $M^2 \equiv (p_a + p_b - p_c)^2 \to \infty$, with $s/M^2 \to \infty$, the leading behaviour of the left-hand side contains the triple Pomeron coupling $\Gamma_{ppp}(t,t,0)$, which vanishes at $t \equiv (p_a - p_c)^2 = 0$. On taking the same limit inside the integral on the right-hand side (with a specific choice of integration variables), the integrand has the form of a triple-Pomeron limit of the

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*) Our notation is the same as that of Ref. 7), but in Ref. 6), V_1 and V_2 are interchanged relative to our notation.

eight-particle amplitude, represented by the diagram of Fig. 2. After extracting $(M^2)^{\alpha_P(0)}(s/M^2)^{2\alpha_P(t)}$, the resulting integral can be written in the form [5]

$$\int d^2\underset{\sim}{p}_{d\perp} \int dy\ (1-y)^{\alpha_P(0)}\, B\left(t, \bar{t}, \frac{1}{1-y}, \phi\right) \qquad (2)$$

where B is a triple Pomeron/two-particle amplitude. Asymptotically, $1 - y = \bar{M}^2/M^2$ where $\bar{M}^2 = (p_a + p_b - p_c - p_d)^2$, and ϕ is a Toller angle. It is then concluded that since B is positive (being obtained from the leading asymptotic behaviour of an inclusive cross-section) the vanishing of Γ implies the vanishing of B at $t = 0$. In the limit $y \to 1$ $(M^2/\bar{M}^2 \to \infty)$ B is approximated by its Regge form (see Fig. 3)

$$(1-y)^{-2\alpha_R(\bar{t})} |V(t, \bar{t}, \eta)|^2\ \Gamma_{RRP}(\bar{t}, \bar{t}, 0) \qquad (3)$$

where η^{-1} is asymptotically linearly related to $\cos\phi$. At $t = 0$, the vanishing of B implies $V(0, \bar{t}, \eta) = 0$.

We explicitly show below, however, that at least one part of $|V|^2$, namely $|V_1|^2$, gives no contribution to the triple Pomeron coupling when integrated over the whole region of the phase space relevant to Fig. 2, and therefore there can be no constraint on it. The implications of this result for the above argument will be discussed at the end.

To discuss separately the contributions of V_1 and V_2 to the eight-particle amplitude, it is necessary to decompose this amplitude into a sum of terms, each of which has simultaneous discontinuities in only specific combinations of invariants, as required by the Steinmann relations [8]. This can be done using the multiple Sommerfeld-Watson representation of the amplitude [9]-[11], as was done for the six-particle amplitude in Ref. 7). We shall not give the details here but simply concentrate on the results of this decomposition. In addition to the variables already defined, we write $s_1 = (p_b - p_d)^2$, $s_2 = (p_a - p_d)^2$, $s_3 = (p_c + p_d)^2$. Taking account of the opposite, $i\varepsilon$, prescriptions which must be used for the equivalent variables on opposite sides of the $\bar{M}^2$ discontinuity, we find that the part of the amplitude which has an $\bar{M}^2$ discontinuity and contains

$V_1(V_1^*)$ has singularities in M^2, s_1 (M^{2*}, s_1^*) but not s_2, s_3 (s_2^*, s_3^*)

$V_2(V_2^*)$ has singularities in s_2, s_3 (s_2^*, s_3^*) but not M^2, s_1 (M^{2*}, s_1^*)

The consistency of this singularity structure can easily be checked by going to the particle poles in $\alpha_R(\bar{t})$ and $\alpha_P(t)$.

To carry out the phase space integration over p_d we find it convenient to use as a complete set of variables *) external variables M^2, s/M^2 and t, and integration variables $\bar{t}$, η and $z \equiv (1-y)^{-1}$. Then in the limit M^2, $s/M^2 \to \infty$ with t, $\bar{t}$, η, z fixed we find

$$\bar{M}^2 \sim z^{-1} M^2, \quad s_1 \sim (1-z^{-1}) M^2, \quad s_2 - s_3 \sim -\eta s/M^2 \tag{4}$$

We shall use the energy component of the inclusive sum rules in the rest frame of $(p_a + p_b - p_c)$. This is simply related to the component chosen in Ref. 5) by a Lorentz transformation. In our frame p_{do} is simply proportional to y and so the phase space integration becomes

$$\int d^4p_d \; p_{do} \, \delta^+(p_d^2 - m_d^2) \to J \int d\bar{t} \, d\eta \, dz \, z^{-2} (1-z)^{-1} H^{-\frac{1}{2}} \theta(H) \tag{5}$$

where

$$H = -\eta (1-z^{-1}) t - \tfrac{1}{4} (t - \bar{t} - \eta + m_d^2)^2 + t m_d^2 \tag{6}$$

and J is a Jacobian depending only on external masses.

Since we are only interested in the way the leading behaviour (as $z \to \infty$) contributes to the sum rule, we can replace $(1 - z^{-1})$ by unity in (5) and (6) and also extend the lower limit of the z integration down to zero. Equation (2) now becomes

$$J \int d\bar{t} \, d\eta \; H^{-\frac{1}{2}} \theta(H) \int_0^\infty dz \; z^{-\alpha_P(0) - 2} \, B(t, \bar{t}, z, \eta) \tag{7}$$

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*) The Feynman variables used in Ref. 5) are appropriate to the case when the particles a and c have equal masses. However, in this case M^2 cannot be taken large when $t = 0$ (in the physical region). To carry through the argument of Ref. 5) it is strictly necessary to consider unequal masses. This has been done in deriving the relations (4)-(6). The Sudakov variables introduced in Ref. 12), Section 2.3, were used in this derivation.

Since B is defined as the asymptotic limit of an $\bar{M}^2$ discontinuity, it can be written as a discontinuity across the z plane cut implied by this $\bar{M}^2$ cut. This cut will extend from $z = 0$ to $+\infty$. It now follows from the Steinmann relations that the part of B which contains only V_1 and V_1^* has only this cut in the z plane and no others, since it has no singularities in M^2 and s_1. The contribution of this part of the amplitude to (7) can therefore be written in the form

$$J \int d\bar{t} d\eta \, H^{-\frac{1}{2}} \Theta(H) \int_C dz \, z^{-\alpha_P(0)-2} A(t, \bar{t}, z, \eta) \tag{8}$$

where the contour C encircles the entire cut of the amplitude A. Since $\alpha_P(0) = 1$ there are no other singularities in the z plane in the integrand of (8), and the contour C can be completed with the circle at infinity to give zero. The convergence at infinity is guaranteed by the Regge power behaviour of A *). The vanishing of (8) will in fact only be true for the "planar" part of the amplitude without simultaneous right and left-hand cuts in $\bar{M}^2$. However, the non-planar part of the eight-particle amplitude will inevitably give rise not only to triple-Pomeron behaviour (of the six-particle amplitude), but also to behaviour involving Regge cuts. In this case, it is clearly no longer possible to neglect cuts as in Ref. 5). Those parts of B containing V_2 or V_2^* (or both) will have extra singularities in the z plane arising from cuts in M^2, M^{2*}, s_1, s_1^*, which will prevent the closure of the contour. The vanishing of these contributions to the triple-Pomeron vertex can be produced by a zero in V_2 alone.

Our results then suggest that there is an inconsistency in the argument of Ref. 5). We believe that the cause of this is the commuting of the external limit M^2, $s/M^2 \to \infty$ with the phase-space integration. It is well-known that the assumption that leading behaviours in particular regions of phase-space integrate up to give the full leading behaviour can lead to erroneous conclusions. The reason lies in the neglect of non-leading contributions. These can be negative and also can persist over far larger regions of phase space than the leading behaviour and so contribute to the same order of asymptotic behaviour of the complete integral. We have studied the planar ladder model in Feynman diagrams as an example of this phenomenon, and will briefly describe our results below. We also note that a similar

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*) The asymptotic behaviour $\sim z^{-\alpha_P(0)-2+2\alpha_R(\bar{t})}$ would only pose a problem if the Reggeons were in fact Pomerons. However, the vanishing of the triple Pomeron coupling at $\bar{t} = 0$ neatly avoids this difficulty.

situation occurs in the isolation of contributions to the two-Reggeon cut through s channel unitarity [13] : leading contributions of order $s^{\alpha_{cut}}$ are positive but occur over only a finite region of phase space ; there are also non-leading contributions which can be negative and which persist over a volume of phase space $O(s)$. Therefore, the complete two-Reggeon cut can be negative. That it actually is negative can be shown by working in the t channel [14].

The six-particle amplitude for the Feynman diagram model we have considered is shown in Fig. 4, sums over the numbers of rungs of the different ladders being implied. This is the simplest set of diagrams whose M^2 discontinuity, when constructed through the sum rule, will involve eight-particle amplitudes with the multi-Reggeon behaviour of Fig. 3. That the model has the triple-Pomeron zero is guaranteed by its planarity and the consequent nonsense zero [15]. It seems that in this model the cancellations between leading and non-leading terms in the various limits are such that not only is $B(0,\bar{t},1/1-y,\emptyset) \neq 0$ but also neither V, V_2 nor V_1 vanish at $t = 0$.

In taking the M^2 discontinuity of this diagram many different cuts have to be taken, three examples of which are shown in Fig. 4. Any one cut represents many contributions from the integral over the eight-point function by taking particle d to be any one of the lines intersected by the cut. In the contributions in which d is the particle exchanged between the ladders, both C_1 and C_2 evidently imply the multi-Reggeon behaviour of Fig. 3, with, however, C_2 giving a non-leading contribution relative to C_1. Integrating this behaviour over p_d would lead to a Regge cut behaviour for Fig. 4 which we know is absent because of its planarity. In fact the non-leading contributions from cuts like C_2 completely cancel the leading behaviour from C_1. The situation parallels that of the cancellation of the AFS cut [16]. The cuts C_2 also imply other types of contribution where particle d is one of the other lines intersected by C_2 some of which have the behaviour of Fig. 2 but not Fig. 3. These non-leading contributions are not cancelled and persist over large regions of phase space. Finally there are the cuts of the form C_3 which involve non-leading contributions as far as the external Reggeons are concerned. It seems essentially that the cuts C_3 give non-leading contributions which allow B not to vanish and the cuts C_1 and C_2 conspire to allow V_2 not to vanish.

A more complete analysis of this model will be given elsewhere. It is difficult to assess the significance of V_2 not vanishing *) since we cannot be sure of the positivity of the model and clearly it is essential for some of the cancellations that the particle d be "inside" the Reggeon in the very specific manner of the ladder model.

In conclusion, we have shown that the argument of Ref. 5) can at most put a constraint on V_2, and this has no consequence for Pomeron couplings in total cross-sections. As we have said the cancellation we have found resembles in some respects the AFS cancellation in s channel unitarity, which is, of course, critical in the study of Regge cuts. If the inclusive sum rules are to be used for a systematic study of Regge singularities then clearly cancellations of this sort must be properly understood first. It would seem that this could only be done by a complete partial wave analysis which effectively diagonalizes the sum rules. The cancellation we have found would then presumably appear via some sort of nonsense zero. However, we have as yet made little progress in understanding the problem from this point of view.

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*) In this context it is perhaps important to note that in the dual resonance model V_2 does vanish but V_1 does not [17].

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FIGURES

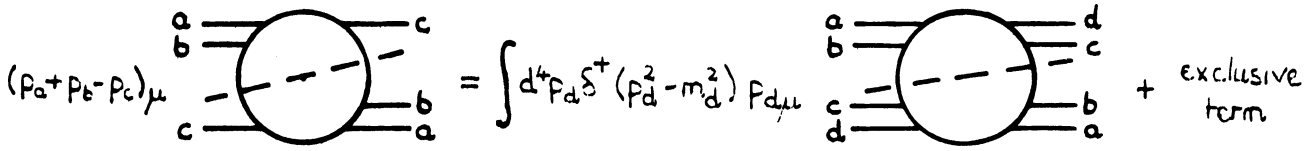

Fig.1

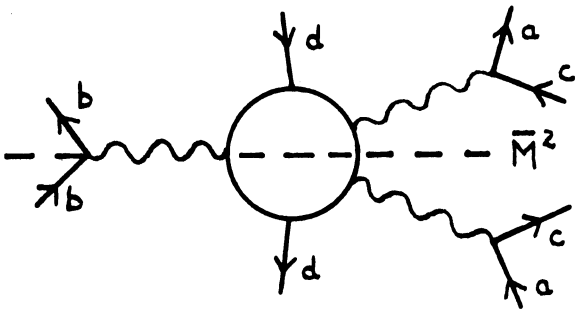

Fig.2

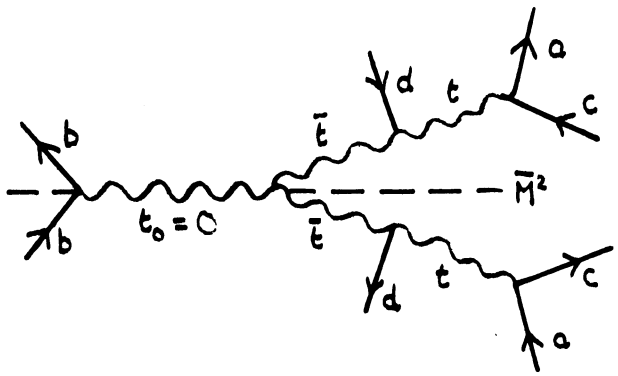

Fig.3

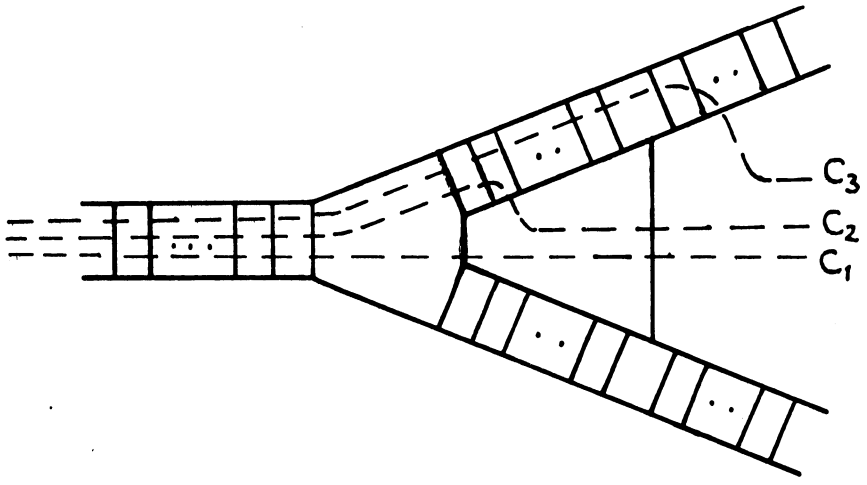

Fig.4