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DEEP INELASTIC PHENOMENOLOGY AND LIGHT CONE PHYSICS

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I. DEEP INELASTIC PHENOMENOLOGY

This Chapter will be devoted to a review of our present understanding of inelastic electron-nucleon scattering phenomenology. In this phenomenological approach, we shall not attempt to "explain" the experimental data in terms of some underlying physical theory, but shall simply attempt to summarize and parametrize these data. Fundamental physical principles, such as Poincaré invariance and causality, and theoretical ideas, such as scaling, Regge behaviour, and duality, will of course be heavily used in these parametrizations. It will be particularly stressed how the Regge and duality concepts borrowed from purely hadronic physics are extremely useful in this analysis. Indeed, the success of these concepts here is perhaps even more striking than in hadronic physics because of the extra variable (photon mass) involved. The field theoretic causality principle will be seen to play an important rôle in relating the hadronic concepts to behaviours in the electroproduction scaling (deep inelastic) limit.

A. Introduction

A.1 Kinematics

We begin as usual with a review of the relevant kinematics. Because of the large number of such reviews already in the existing literature [1)-5)], a brief summary will suffice here.

The total electron-proton cross-section in order α^2 can be written

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^2\frac{\theta}{2}}\left[W_2(\kappa,\nu)\cos^2\frac{\theta}{2} + 2W_1(\kappa,\nu)\sin^2\frac{\theta}{2}\right], \tag{1}$$

where E, E' and θ are, respectively, the electron initial energy, final energy, and scattering angle

$$\kappa = q^2 = -4EE'\sin^2\frac{\theta}{2} \tag{2}$$

is the square of the momentum transferred to electron, and

$$\nu = q\cdot p = E - E' . \tag{3}$$

Throughout this section, we take the initial proton to be at rest and to have mass 1, so that the four-momentum p is

$$p^\mu = (1,0,0,0) \, . \tag{4}$$

The process is illustrated in Fig. 1.

The structure functions W_i can be defined from the forward spin-averaged current-proton scattering amplitude (Fig. 2)

$$\begin{aligned} T_{\mu\nu} &= i\int d^4x \, e^{iq\cdot x}\,\theta(x_0)\langle p|[J_\mu(x), J_\nu(0)]|p\rangle + \textit{polynomial} \\ &= \left(p_\mu - \frac{\nu q_\mu}{\varkappa}\right)\left(p_\nu - \frac{\nu q_\nu}{\varkappa}\right)T_2(\varkappa,\nu) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{\varkappa}\right)T_1(\varkappa,\nu) \end{aligned} \tag{5}$$

by

$$\begin{aligned} W_{\mu\nu} = \frac{1}{\pi}\,\mathrm{Im}\,T_{\mu\nu} &= \frac{1}{2\pi}\int d^4x \, e^{iq\cdot x}\langle p|[J_\mu(x), J_\nu(0)]|p\rangle \\ &= \left(p_\mu - \frac{\nu q_\mu}{\varkappa}\right)\left(p_\nu - \frac{\nu q_\nu}{\varkappa}\right)W_2(\varkappa,\nu) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{\varkappa}\right)W_1(\varkappa,\nu), \end{aligned} \tag{6}$$

so that

$$W_i = \frac{1}{\pi}\,\mathrm{Im}\,T_i \, . \tag{7}$$

The positivity condition

$$W_i \geq 0 \tag{8}$$

follows from (6). The transverse and longitudinal cross-sections σ_T and σ_L are related to the W_i by

$$\begin{aligned} W_1 &= \frac{\sigma_T}{4\pi^2\alpha}\left(\nu - \tfrac{1}{2}|\varkappa|\right), \\ W_2 &= \frac{(\sigma_T + \sigma_L)(\nu - \frac{1}{2}\varkappa)}{4\pi^2\alpha\,(1 + \nu^2|\varkappa|^{-1})} \, . \end{aligned} \tag{9}$$

Equation (5) becomes the physical Compton amplitude for $\varkappa \to 0$, and we have

$$W_1 \xrightarrow[\varkappa \to 0]{} \frac{\nu \sigma_\gamma}{4\pi^2 \alpha} , \qquad W_2 \xrightarrow[\varkappa \to 0]{} \frac{|\varkappa| \sigma_\gamma}{4\pi^2 \alpha \nu} , \tag{10}$$

where σ_γ is the total photon-proton cross-section.

It is convenient to introduce additional functions V_i defined by

$$\begin{aligned} W_{\mu\nu} = &\left[\varkappa P_\mu P_\nu - \nu(P_\mu q_\nu + q_\mu P_\nu) + g_{\mu\nu} \nu^2\right] V_2(\varkappa, \nu) \\ &- (\varkappa g_{\mu\nu} - q_\mu q_\nu) V_1(\varkappa, \nu) . \end{aligned} \tag{11}$$

Comparison with (6) gives

$$W_2 = \varkappa V_2 , \qquad W_1 = \varkappa V_1 - \nu^2 V_2 . \tag{12}$$

We will be mainly concerned with the amplitude $W_2(\varkappa, \nu)$. The product νW_2 will often occur and so we call it simply W :

$$W(\varkappa, \nu) \equiv \nu W_2(\varkappa, \nu) . \tag{13}$$

W will vanish when the s channel variable

$$s \equiv (p+q)^2 = \varkappa + 2\nu + 1 \tag{14}$$

is less than the nucleon mass squared 1, the lowest mass intermediate state, i.e., when $\varkappa + 2\nu < 0$. The contributions of the nucleon intermediate state are the Born diagrams shown in Fig. 3. For electroproduction $\varkappa$ is always negative and for convenience we will always choose ν to be positive :

$$\varkappa \leq 0 , \qquad \nu \geq 0 . \tag{15}$$

Negative values of ν can be reached via the crossing relation

$$W(\varkappa, \nu) = + W(\varkappa, -\nu) . \tag{16}$$

A.2 Scaling

The most prominent feature of the electroproduction data is its exhibition of scaling. The structure function $W(\mathcal{H},\nu)$ is dimensionless and so, if masses are in some sense unimportant for large $\mathcal{H}$ and ν, it might be expected to approach a function of only the dimensionless ratio

$$\rho = -\frac{2\nu}{\mathcal{H}} = \frac{1}{\omega} \tag{17}$$

in the Bjorken (A) limit $|\mathcal{H}|\to\infty$ with ρ fixed [6]:

$$W(\mathcal{H},\nu) \xrightarrow[A]{} F(\rho) \; . \tag{18}$$

We will often write $F(\omega)$ instead of $F(\rho)$ - this should cause no confusion. Experimentally [7),8)], the scaling behaviour (18) is well satisfied, beginning at the surprisingly low values $\mathcal{H} \sim -2\ \mathrm{GeV}^2$, $\nu \sim 3\ \mathrm{GeV}^2$ (precocious asymptopia) (see Fig. 4). We will here assume that (18) is exactly satisfied and study some of its consequences. Note that since W vanishes for $\nu < |\mathcal{H}/2|$, F vanishes for $\rho < 1$. In Chapter II, we will review some attempts to understand the dynamical reasons for scaling.

The experimental scaling data are shown in Fig. 4. A numerical fit is provided by the form [9)]

$$\tilde{F}(\rho) = \sum_{i=1}^{3} a_i \left(1-\frac{1}{\rho}\right)^{2+i} , \tag{19}$$

with $a_1 = 1.650$, $a_2 = 1.325$, $a_3 = -1.399$. There are only scaling data (i.e., data taken for $|\mathcal{H}| \geq 2\ \mathrm{GeV}^2$, $\sqrt{s} \geq 2\ \mathrm{GeV}$) for $\rho < 7$, where (19) fits the data with $\chi^2 = 619$ for 397 degrees of freedom [10)]. We can therefore take

$$F(\rho) = \tilde{F}(\rho) , \quad 1 \leq \rho \leq 7 . \tag{20}$$

A plot of this function is shown in Fig. 5.

Scaling is found to set in even earlier if the scaling variable ρ is replaced by the new variable [7),9),11)]

$$\rho' = \rho - \frac{1}{\mathcal{H}} \tag{21}$$

With (21), scaling begins at $|\mathcal{H}| \sim 1\ \mathrm{GeV}^2$. Using (21) is therefore convenient since it increases the amount of scaling data. Since

$$F(\rho') = F(\rho) - \frac{1}{\varkappa} F'(\rho) + \dots , \tag{22}$$

the use of (21) amounts to keeping some non-leading terms in the A limit. This will be discussed in Section II.A.2. Note that the distinction between ρ and ρ' is only important for small $\varkappa$ and small ρ, i.e., in the threshold region $\rho \sim 1$ for $|\varkappa| < 5$. A fit to the data of the form (19) using ρ' gives $a_1' = 0.687$, $a_2 = 1.773$, $a_3 = -2.242$ [10].

In discussing the scaling curve, it is convenient to divide it into several different pieces which emphasize distinct characteristics. We will call the large ρ ($\rho > 16$, see Section I.B.1) region the <u>Regge region</u>, the small ρ region the <u>threshold region</u> ($1 < \rho < 2$), and the region $2 < \rho < 5$ will be called the <u>resonance region</u>. We will see that $F(\rho)$ has Regge-like behaviour in the Regge region, has form-factor-like behaviour in the threshold region, and is correlated to the s channel resonances in the resonance region. We will refer to such properties of the scaling curve, which have to do with its behaviour in a restricted range of ρ, as <u>local properties</u>. These will be considered in Section I.B. There are other properties, typically integrals of the form $\int_1^\infty d\rho\, \rho^{-n}[F(\rho)-\dots]$, which have to do with the entire scaling curve. We call such properties <u>global properties</u>. These will be discussed in Section I.C.

The amplitude T_2 [Eq. (5)] whose absorptive part is W_2 need not scale even if W_2 does. We have the dispersion relation

$$T_2(\varkappa, \nu) = \frac{1}{2} \int_{(\frac{\varkappa}{2})^2}^{\infty} d\nu'^2 \, \frac{W_2(\varkappa, \nu')}{\nu'^2 - \nu^2} + \text{polynomial}, \tag{23}$$

the convergence of the integral being suggested by Regge theory and by experiment (see Section I.B.1). If the additive polynomial is absent, then T_2 should scale as

$$\nu T_2(\varkappa, \nu) \xrightarrow[A]{} G(\rho) . \tag{24}$$

We will have occasion to make the extra assumption (24). It is consistent with, but not implied by, the present data. If (24) is valid, we have

$$G(\rho) = \frac{1}{2} \int_1^\infty d\rho'^2 \, \frac{F(\rho')}{\rho'^2 - \rho^2} . \tag{25}$$

Simple dimensional analysis suggests that the second structure function $W_1(\varkappa, \nu)$ should obey the scaling law [6)]

$$W_1(\varkappa, \nu) \xrightarrow[A]{} F_1(\rho) \quad . \tag{26}$$

This behaviour also seems to be experimentally correct [7)], although W_1 is not known nearly so well as W_2 since most of the data are taken at small angles [cf., Eq. (1)]. We will therefore consider only W_2 in this review.

We have so far discussed only electron-proton scattering. There is also some electron-neutron scaling data [7),8)]. The quality of the neutron data is not equal to that of the proton data, and deuteron correction subtleties may also be present [12)]. Scaling is expected to be valid also for neutrons and the data are consistent with this. In the scaling region, the simple relation [13)]

$$F_2^{(n)}(\rho) \simeq \left(1 - \frac{1}{\rho}\right) F_2(\rho) \tag{27}$$

seems to be valid [8)]. It is hoped that (27) is <u>not</u> valid down to $\rho = 1$ because of the bound

$$F_2^{(n)}(\rho) / F_2(\rho) \geq \frac{1}{4} \tag{28}$$

suggested by some theoretical considerations [14)]. The present data are consistent with (28).

The scaling laws described above can be simply illustrated in free field theories. Consider a free scalar field theory. The amplitude T_f for the scattering of a scalar current $(\sim \varphi^2)$ from the scalar particle is given by the Feynman diagrams of Fig. 3 (with no form factor at the vertices). Essentially

$$T_f(\varkappa, \nu) = \frac{1}{\varkappa + 2\nu + i\epsilon} + \frac{1}{\varkappa - 2\nu - i\epsilon} = \frac{2\varkappa}{\varkappa^2 - 4\nu^2 + i\epsilon\nu} \tag{29}$$

and

$$W_f(\varkappa, \nu) \equiv \frac{1}{\pi} \operatorname{Im} T_f(\varkappa, \nu) = \delta(\varkappa + 2\nu) - \delta(\varkappa - 2\nu) = 2\varkappa\, \varepsilon(\nu)\, \delta(\varkappa^2 - 4\nu^2). \tag{30}$$

We have the explicit scaling laws

$$\nu T_{L}(\varkappa,\nu) \xrightarrow[A]{} G_{L}(\rho) = \frac{1}{\rho^{2}-1} \quad , \tag{31}$$

$$\nu W_{L}(\varkappa,\nu) \xrightarrow[A]{} F_{L}(\rho) = \frac{1}{2}\left[\delta(\rho-1)+\delta(\rho+1)\right] . \tag{32}$$

For free scalar or spinor theories with vector currents ($\varphi^{+}\overleftrightarrow{\partial}_{\mu}\varphi$ or $\bar{\Psi}\gamma_{\mu}\Psi$), essentially the same functions result for the amplitudes T_2 and W_2.

A.3 Causality

The principle of causality is basic to a field theoretic description and its constraints will be extremely useful to us. The causality requirement on the local electromagnetic currents,

$$\left[J_{\mu}(x),\ J_{\nu}(0)\right] = 0 \quad for \quad x^{2} < 0 \ , \tag{33}$$

is reflected through (22) into constraints on the structure functions $W_i(\varkappa,\nu)$. These constraints can best be met and exploited by representing the W_i in a manifestly causal form. We will therefore work with a manifestly causal integral representation - the DGS representation [15]. Although not a strict consequence of quantum field theory, the representation is correct in every order of perturbation theory [15]. We could equally well employ the rigorous JLD representation [16], but this is less convenient.

Although $W_{\mu\nu}$, as defined by (22), is the Fourier transform of a causal commutator, the W_i will not in general be causal because of the $\varkappa^{-1}$ factors in (22). This is why the V_i were introduced in (27) [17].

The Fourier transform of (6) is

$$\begin{aligned}\widehat{W}_{\mu\nu} &\equiv \frac{1}{2\pi}\langle p|\left[J_{\mu}(x),\ J_{\nu}(0)\right]|p\rangle \\ &= \left[-\Box p_{\mu}p_{\nu}+(p\cdot\partial)(p_{\mu}\partial_{\nu}+\partial_{\mu}p_{\nu})-g_{\mu\nu}(p\cdot\partial)^{2}\right]\widehat{V}_{2}(x^{2},p\cdot x) \\ &\quad -\left(-\Box g_{\mu\nu}+\partial_{\mu}\partial_{\nu}\right)\widehat{V}_{1}(x^{2},p\cdot x) ,\end{aligned} \tag{34}$$

where

$$V_{i}(\varkappa,\nu) = \int d^{4}x\ e^{-iq\cdot x}\ \widehat{V}_{i}(x^{2},p\cdot x) . \tag{35}$$

Note that

$$\hat{V}_i(x^2, -p\cdot x) = -\hat{V}_i(x^2, p\cdot x). \tag{36}$$

The $\hat{V}_i$ are locally related to $\hat{W}_{\mu\nu}$, and it can be shown that the $\hat{V}_i$ are themselves causal [18]. Thus we have the DGS representations [19]

$$V_i(\varkappa,\nu) = \int_0^\infty da \int_{-1}^1 db\, \sigma_i(a,b)\,\delta(\varkappa + 2b\nu - a)\,\varepsilon(\nu + b). \tag{37}$$

In configuration space these become

$$\hat{V}_i(x^2, p\cdot x) = -\frac{i}{2\pi}\int_0^\infty da \int_{-1}^1 db\, \sigma_i(a,b)\, e^{-ibp\cdot x}\,\Delta(x; a + b^2), \tag{38}$$

where

$$i\Delta(x;a) = \frac{1}{(2\pi)^3}\int d^4p\, e^{-ip\cdot x}\,\delta(p^2 - a)\,\varepsilon(p^0) \tag{39}$$

is the usual mass $a^{\frac{1}{2}}$ free field commutator function :

$$\Delta(x;a) = 0 \qquad \text{for } x^2 < 0. \tag{39}$$

We shall always take $\nu > +1$ so that $\varepsilon(\nu + b)$ can be replaced by $+1$ in (37). We can now use (12) to write representations for the W_i [17] :

$$W_2 = \varkappa \int_0^\infty da \int_{-1}^1 db\, \sigma_2(a,b)\,\delta(\varkappa + 2b\nu - a), \tag{40}$$

$$W_1 = \int_0^\infty da \int_{-1}^1 db \left[\varkappa\sigma_1(a,b) - \nu^2\sigma_2(a,b)\right]\delta(\varkappa + 2b\nu - a). \tag{41}$$

We emphasize here that the above representations incorporate, in addition to causality, the constraints arising from the spectrum of the allowed intermediate states. These spectral conditions restrict the supports of the spectral functions $\sigma_i(a,b)$, as embodied in the integration limits in (37). These constraints, specifically the boundedness of the b integration range, will be crucial to our analysis.

The reasons for the validity of (37) are apparent from the x space representation (38), which expresses $\hat{V}(x^2, p\cdot x)$ in what is essentially a Fourier representation in the $p\cdot x$ variable and a Bessel representation in the x^2 variable. The Bessel functions $\Delta(x;\mu^2)$, $0 \leq \mu^2 \leq \infty$, are complete for sufficiently smooth distributions $f(x^2)$ which vanish for $x^2 < 0$. The completeness of the exponentials $e^{-ibx\cdot p}$ actually requires that the b integration in (38) run from $-\infty$ to $+\infty$. It is the spectral conditions which allow the integration range to be restricted to $-1 \leq b \leq 1$. It is clear from (37) that this restriction is sufficient to guarantee the vanishing of $V(\varkappa, \nu)$ for $|\nu| < |\varkappa/2|$. The generality of the restrictions for all Fourier transforms of causal commutations obeying the spectral conditions is shown in Ref. 15).

To gain familiarity with the representations (37) and (40), we consider again the free field function (30). The function (30) can clearly be represented as (37) with spectral function

$$\delta(a)\left[\delta(b-1) + \delta(b+1)\right]. \tag{42}$$

The representation of the form (40) has the spectral function

$$\sigma_f(a,b) = \delta'(a)\,\theta(b^2-1), \tag{43}$$

as can be seen by explicitly performing the a and b integrations.

The causal representation for $W \equiv \nu W_2$ we will use is thus

$$W(\varkappa,\nu) = \varkappa\nu\int_0^\infty da\int_{-1}^{1} db\, \sigma(a,b)\,\delta(\varkappa + 2b\nu - a), \tag{44}$$

where we have dropped the index 2 on σ. It may not seem that (44) accomplishes much since it expresses the <u>a priori</u> unknown function $W(\varkappa,\nu)$ of two variables in terms of the new unknown function $\sigma(a,b)$ of two variables. However, in attempting to describe the experimental data for $W(\varkappa,\nu)$, a parametrization must be made and it is very likely that an arbitrary parametrization will <u>not</u> be consistent with causality. If a parametrization is instead made for $\sigma(a,b)$, the resulting parametrization for $W(\varkappa,\nu)$ obtained from (44) will be guaranteed to be consistent with causality (and spectrum conditions). This then is the virtue of doing phenomenology in terms of (44). Another important advantage of using (44) is that it provides relations between the behaviour of W in different limits. Several examples of this will be given below.

If we were to consider also negative ν , we must use

$$W(x,\nu) = x\nu \int da \int db\, \sigma(a,b)\, \delta(x + 2b\nu - a)\, \varepsilon(\nu + b) . \qquad (45)$$

Note that the crossing relation (16) gives

$$\sigma(a,b) = \sigma(a,-b) , \qquad (46)$$

so that

$$\sigma(a,b) = \sigma(a,|b|) \qquad (47)$$

The b integration in (44) can be explicitly performed using the δ function, which gives

$$b = \frac{a - x}{2\nu} , \qquad (48)$$

to obtain

$$W(x,\nu) = \frac{x}{2} \int_0^\infty da\, \sigma\left(a, \frac{a-x}{2\nu}\right) , \qquad (49)$$

with the understanding that $\sigma(a,b)$ vanishes for $|b| > 1$. The form of (49) and the precocity of the scaling limit strongly suggest that $\sigma(a,b)$ decreases rapidly for large a. Indeed, without this rapid decrease for large a, (49) would only be consistent with the scaling behaviour (18) if $\sigma(a,b)$ had very pathological behaviour. The mere existence of the equal time commutator $[J_i(0,\underset{\sim}{x}), J_j(0)]$ is in fact effectively enough to guarantee a sufficient decrease. We will therefore always make this assumption [17),20)]:

$$\sigma(a,b) \xrightarrow[a\to\infty]{} 0 \qquad \text{fast} . \qquad (50)$$

In the A limit, (49) gives

$$W(x,\nu) \xrightarrow[A]{} \frac{x}{2} \int da\, \sigma(a,\omega) , \qquad (51)$$

and so, to obtain the scaling behaviour (18), we must have [17)]

$$\int da\, \sigma(a,\omega) = 0 , \qquad 0 \leq \omega \leq 1 . \qquad (52)$$

Given (52), (18) will be satisfied with [17)]

$$F(\omega) = -\frac{\omega}{2}\int da\, \sigma'(a,\omega)\, a\ , \tag{53}$$

where

$$\sigma'(a,\omega) = \frac{\partial}{\partial\omega}\sigma(a,\omega)\ . \tag{54}$$

The condition (52), a consequence of scaling, is the first restriction on $\sigma(a,b)$ we have deduced from the data.

The simplest [19] integral representation for

$$T(\varkappa,\nu) \equiv \nu T_2(\varkappa,\nu) \tag{55}$$

is

$$T(\varkappa,\nu) = \varkappa\nu\int_0^\infty da\int_{-1}^{1} db\, \frac{\sigma(a,b)}{\varkappa+2b\nu-a+i\varepsilon}\ . \tag{56}$$

An additive polynomial is in principle also allowed, but its presence would violate scaling and so we shall assume it is not present. With (52), (56) gives the scaling behaviour (24) with

$$G(\omega) = -\frac{\omega}{2}\int_0^\infty da\int_{-1}^{1} db\, \frac{\sigma(a,b)}{(b-\omega)^2}\ . \tag{57}$$

As we will see in Section I.B.3, $\sigma(a,\pm 1)=0$ and so (57) can be written

$$G(\omega) = -\frac{\omega}{2}\int da\int db\, \frac{\sigma'(a,b)}{b-\omega}\ . \tag{58}$$

The absorptive part of (58) is (53).

The spectral function (43) in the free field model clearly obeys (52) and the structure function (32) satisfies (53). Equations (56) and (57) are also explicitly verified in the free field case.

B. Local properties

B.1 Regge behaviour

The first local property of the scaling curve we shall consider is its Regge behaviour. We assume Regge behaviour for $W(\varkappa, \nu)$ in the Regge (R) limit $\nu \to \infty$ with $\varkappa$ fixed :

$$W(\varkappa, \nu) \xrightarrow[R]{} \beta_P(\varkappa) + (2\nu)^{-\frac{1}{2}} \beta_T(\varkappa) . \tag{59}$$

Here we have included the Pomeron (P) and tensor meson ($T = f^0 + A_2$) Regge pole contributions with $t = 0$ intercepts $\alpha_P = 1$, $\alpha_{f0} = \alpha_{A_2} = \frac{1}{2}$ [21]. It is clear from (44) and (48) that the behaviour of $W(\varkappa, \nu)$ in the R limit is controlled by the behaviour of $\sigma(a,b)$ in the $b \to 0$ limit. In order to obtain (59), we must take [17),22)]

$$\sigma(a,b) \xrightarrow[b\to 0]{} \sigma_P(a) \ln b + \sigma_T(a) b^{1/2} . \tag{60}$$

Note that (52) requires

$$\int_0^\infty da\, \sigma_P(a) = \int_0^\infty da\, \sigma_T(a) = 0 , \tag{61}$$

Then (44) gives the behaviour (59) with [17)]

$$\begin{aligned} \beta_P(\varkappa) &= \frac{\varkappa}{2} \int_0^\infty da\, \sigma_P(a) \ln(a-\varkappa) , \\ \beta_T(\varkappa) &= \frac{\varkappa}{2} \int_0^\infty da\, \sigma_T(a) (a-\varkappa)^{1/2} . \end{aligned} \tag{62}$$

The representations (62) obtained for the Regge residue function imply the expected result that the residues are analytic functions of $\varkappa$ apart from right-hand cuts. It further follows from (50) and (61)-(62) that [17)]

$$\begin{aligned} \beta_P(\varkappa) &\xrightarrow[\varkappa\to\infty]{} -\frac{1}{2} \int_0^\infty da\, \sigma_P(a) a \equiv \beta_P \\ \beta_T(\varkappa) &\xrightarrow[\varkappa\to\infty]{} (-\varkappa)^{1/2} \frac{1}{4} \int_0^\infty da\, \sigma_T(a) a \equiv (-\varkappa)^{1/2} \beta_T . \end{aligned} \tag{63}$$

The large $\varkappa$ behaviours of the residues are thus specified. It is of course possible that β_P and/or β_T vanish.

An important further consequence of our analysis is that the small b behaviour (60) of $\sigma(a,b)$ determines, via the representation (53), the large ρ behaviour of $F(\rho)$. Equations (60) and (61) give [17),23)]

$$F(\rho) \xrightarrow[\rho\to\infty]{} \beta_P + \rho^{-\frac{1}{2}}\beta_T \equiv F_R(\rho) . \tag{64}$$

We thus obtain a commutativity between the large $\mathcal{H}$ behaviour of the R limit and the large ρ behaviour of the A limit. This can be summarized in the commutative diagram

$$\begin{array}{ccc} W(\mathcal{H},\nu) & \xrightarrow[R]{\qquad} & \beta_P(\mathcal{H}) + (2\nu)^{-\frac{1}{2}}\beta_T(\mathcal{H}) \\ \Big\downarrow A & & \Big\downarrow \mathcal{H}\to\infty \\ F(\rho) & \xrightarrow[\rho\to\infty]{\qquad} & \beta_P + \rho^{-\frac{1}{2}}\beta_T \end{array} . \tag{65}$$

It is at present not possible to reliably compare (64) with experiment because of the absence of scaling data for large ρ. [The data for $\rho > 7$ correspond to values of $\mathcal{H}$ which are too small to be in the scaling region and so are artificially decreased by the kinematical vanishing of $W(\mathcal{H},\nu)$ at $\mathcal{H} = 0$.] This difficulty can be overcome if one is willing to make a simple parametrization of the residues (62). We will take [10)]

$$\beta_P(\mathcal{H}) \simeq \frac{-\mathcal{H}}{-\mathcal{H}+\mu^2}\beta_P \quad , \quad \beta_T(\mathcal{H}) \simeq \frac{-\mathcal{H}}{(-\mathcal{H}+\mu^2)^{1/2}}\beta_T \; , \tag{66}$$

which is the simplest parametrization consistent with scaling, [(63)] and the linear vanishing of $W(\mathcal{H},\nu)$ at $\mathcal{H} = 0$, Eqs. (66) correspond to the parametrizations

$$\sigma_P(a) \simeq 2\beta_P\,\delta'(a-\mu^2), \qquad \sigma_T(a) \simeq 4\beta_T\,\delta'(a-\mu^2), \tag{67}$$

in the representation (62). Note that precocity suggests that $\mu^2 \lesssim \frac{1}{4}\mathrm{GeV}^2$ in order that the scaling limit (63) is approximately obtained for $|\mathcal{H}| \sim 1$ or 2 GeV^2.

Using (66), the Regge behaviour (59) can be compared with all of the data for all ν and $|\mathcal{H}|$ [10)]. It was asked for which ν and $\mathcal{H}$ are (59) and (66) valid and for which (constant) values of β_P, β_T, and μ^2. It was found that (59) and (66) give an excellent fit to all of the data for

$$\rho \geq \rho_0 \cong 16 \tag{68}$$

with the values [24]

$$\mu^2 \cong 0.25, \quad \beta_P \cong 0.29, \quad \beta_T \cong 0.28. \tag{69}$$

The fit is shown in Fig. 6.

Let us now comment on the result of this fit. The value 0.25 for μ^2 is quite consistent with the precocity requirement discussed above. The value 16 for the magnitude ρ_0 of ρ above which the Regge behaviour (64) sets in may seem rather large. The reasonability of this value can, however, be understood as follows. The Compton amplitude $\lim_{\varkappa\to 0} [|\varkappa|^{-1} W(\varkappa,\nu)]$ is known to be well approximated by the Regge form (59) for $\nu \geq \nu_0 \equiv 2\ \mathrm{GeV}^2$ [25),26)]. Using (44) with (60), (63) and (67) gives

$$\rho_0 = \frac{2\nu_0}{\mu^2} \tag{70}$$

as the value of ρ above which $F(\rho)$ is well approximated by the Regge form (64) [10)]. It is thus the small mass μ^2 which determines when Regge behaviour sets in for $F(\rho)$. Numerically, (70) gives precisely $\rho_0 \cong 16$. The general magnitude of ρ_0 is, in fact, independent of the particular approximation (67) and such a large value of ρ_0 could have been anticipated <u>a priori</u>. With this value of ρ_0, the values of β_P and β_T in (69) are seen (Fig. 6) to be such that the Regge asymptotic form $F_R(\rho) = \beta_P + \rho^{-\frac{1}{2}}\beta_T$ everywhere lies above the scaling curve $F(\rho)$ [27)].

In the R limit, the amplitude $T(\varkappa,\nu)$ whose absorptive part is $W(\varkappa,\nu)$ will also receive contributions from the Pomeron and tensor meson Regge poles. In addition, T can have a $J = 0$ fixed pole [28),29)] contribution. Thus

$$T(\varkappa,\nu) \xrightarrow[R]{} \gamma_P(\varkappa) + (2\nu)^{-\frac{1}{2}}\gamma_T(\varkappa) + \nu^{-1}(-\varkappa)\gamma_F(\varkappa), \tag{71}$$

where $\gamma_P(\varkappa)$, $\gamma_T(\varkappa)$, and $-\varkappa\gamma_F(\varkappa)$ [30)] are, respectively, the Pomeron, tensor, and fixed pole residues. We have for $\varkappa < 0$

$$\frac{1}{\pi}\,\mathrm{Im}\left\{\gamma_P(\varkappa),\ \gamma_T(\varkappa),\ \gamma_F(\varkappa)\right\} = \left\{\beta_P(\varkappa),\ \beta_T(\varkappa),\ 0\right\}. \tag{72}$$

We assume that T, as well as W, scales and the validity of (56). Then, just as for W, we obtain commutativity as in (65), so that

$$\left\{\gamma_P(\varkappa),\ \gamma_T(\varkappa),\ \gamma_F(\varkappa)\right\} \xrightarrow[\varkappa\to\infty]{} \left\{\gamma_P,\ |\varkappa|^{1/2}\gamma_T,\ \gamma_F\right\} \tag{73}$$

for some constants γ_P, γ_T, γ_F. Clearly

$$\frac{1}{\pi}\,\mathrm{Im}\{\gamma_P,\gamma_T,\gamma_F\} = \{\beta_P,\beta_T,0\}. \tag{74}$$

If $\gamma_F = \gamma_F(\infty) \neq 0$, then there is a $J=1$ fixed pole in deep inelastic electroproduction. Similarly, if $\gamma_F(0) \neq 0$, then there is a $J=1$ fixed pole in Compton scattering.

Let us now see explicitly how the behaviours (71)-(74) arise from the integral representation (56) for $T(\varkappa,\nu)$. To this end, we consider a class of model functions $(\nu > 1)$

$$T^{(\alpha)}(\varkappa,\nu) = \varkappa\nu\int_0^\infty da\int_{-1}^{1} db\,\sigma^{(\alpha)}(a,b)\,(\varkappa + 2b\nu - a + i\varepsilon)^{-1}, \tag{75}$$

where

$$\sigma^{(\alpha)}(a,b) = \sigma_\alpha(a)|b|^\alpha, \qquad \alpha \geq 0, \tag{76}$$

with

$$\int_0^\infty da\,\sigma_\alpha(a) = 0 \tag{77}$$

and $\sigma_\alpha(a)$ of rapid decrease for $a\to\infty$ in order to have scaling. For $\alpha > 1$, the $2b\nu$ term in the denominator in (75) dominates for large ν and we obtain [using (77)]

$$T^{(\alpha)}(\varkappa,\nu) \xrightarrow[R]{} \frac{\varkappa}{2\nu}\frac{1}{\alpha-1}\int_0^\infty da\,\sigma_\alpha(a)\,a, \qquad (\alpha > 1). \tag{78}$$

We thus get a decrease in the R limit at least as fast as ν^{-1}. If $\int_0^\infty da\,\sigma_\alpha(a)a \neq 0$, we have a $J=0$ fixed pole with a residue linear in $\varkappa$. Clearly, $W^{(\alpha)} \equiv \frac{1}{\pi}\,\mathrm{Im}\,T^{(\alpha)} \underset{R}{\to} 0/\nu$ so that W will not have this ν^{-1} behaviour.

Consider next the range $0 < \alpha < 1$. In this case the large ν behaviour of (75) can be obtained by changing integration variables from b to $x \equiv 2b\nu(\varkappa - a)^{-1}$. The result is

$$T^{(\alpha)}(\varkappa,\nu) \xrightarrow[R]{} \frac{\varkappa}{(2\nu)^\alpha}C_\alpha\int_0^\infty da\,\sigma_\alpha(a)(a-\varkappa)^\alpha - \frac{\varkappa}{2\nu}D_\alpha\int_0^\infty da\,\sigma_\alpha(a)\,a, \qquad (0<\alpha<1), \tag{79}$$

where

$$C_\alpha = \lim_{N\to\infty} \int_{-N}^{N} dx\, |x|^\alpha (x+1)^{-1} = \frac{e^{\pi i\alpha}-1}{\sin\pi\alpha} ,$$

$$D_\alpha = \frac{1}{\alpha} + \frac{1}{1-\alpha^2} . \tag{80}$$

So now $T^{(\alpha)}$ exhibits leading Regge pole behaviour. In particular, we get the tensor meson trajectory behaviour in (71) for $\alpha = \frac{1}{2}$, as in (60). Taking $\sigma_{\frac{1}{2}}(a) = \sigma_T(a)$, (79) gives an expression for $\gamma_T(\varkappa)$ whose absorptive part for $\varkappa < 0$ is $\beta_T(\varkappa)$, as given in (62). $T^{(\alpha)}$ also exhibits a $J = 0$ fixed pole with residue linear in $\varkappa$.

For $\alpha = 0$, (79) does <u>not</u> give the Pomeron exchange behaviour in (71) because, according to (77), the coefficient vanishes. The Pomeron behaviour is actually provided by the term $\sigma_P(a)\ln b$ in the small b limit, as in (60). The actual large ν behaviour of $T^{(0)}$ is simply obtained by explicitly performing the b integral in (75). The result is

$$T^{(0)}(\varkappa,\nu) \xrightarrow[R]{} \frac{\varkappa}{2\nu} \int_0^\infty da\, \sigma_0(a)a . \tag{81}$$

We therefore obtain again a $J = 1$ fixed pole behaviour with residue linear in $\varkappa$.

The final case to be considered is $\alpha = 1$. Explicitly performing the b integration gives

$$T^{(1)}(\varkappa,\nu) = \frac{\varkappa}{2\nu} \int da\, \sigma_1(a) \left(\frac{a-\varkappa}{2}\right)\left[\ln\left(\left(\frac{\varkappa-a}{2\nu}\right)^2 - 1\right) + 2\ln\left(\frac{\varkappa-a}{2\nu}\right)\right] . \tag{82}$$

We see that, in order to have pure Regge pole behaviour, we require the <u>two</u> conditions

$$\int da\, \sigma_1(a) = \int da\, \sigma_1(a)a = 0 , \tag{83}$$

and not just (77). Taking (83), we obtain

$$T^{(1)}(\varkappa,\nu) \xrightarrow[R]{} \frac{\varkappa}{2\nu} \int da\, \sigma_1(a)\,(a-\varkappa)\ln(a-\varkappa) . \tag{84}$$

This is again a $J = 0$ fixed pole behaviour, but now the residue is <u>not</u> simply a polynomial in $\varkappa$. The residue has a non-vanishing discontinuity for $\varkappa > 0$ [unless $\sigma_1(a)$ vanishes identically]. Note, however, that because of (83), $T^{(1)}$ vanishes in the A limit :

$$T^{(1)}(\varkappa,\nu) \xrightarrow[A]{} 0 . \tag{85}$$

Note also that, since (84) has no discontinuity for $\varkappa < 0$, the fixed pole asymptotic behaviour is not exhibited by $W^{(1)}$. It is, in fact, not possible for W to have the fixed pole behaviour if T has pure Regge pole behaviour. A non-pure Regge term of the form $\nu^{-1}\ln\nu$ in T would be required to produce a ν^{-1} term in W. We will not consider such possibilities since such contributions are absent or at worst small at $t = 0$ in hadronic reactions [31].

From the above examples, it follows that the most general form of $\sigma(a,b)$ consistent with (71) and our other assumptions is

$$\sigma(a,b) = \sigma_0(a) + \sigma_P(a)\ln|b| + \sigma_T(a)|b|^{1/2} + \sigma_1(a) + \bar{\sigma}(a,b), \tag{86}$$

where

$$b^{-1}\bar{\sigma}(a,b) \xrightarrow[b\to 0]{} 0 . \tag{87}$$

Then (71) is satisfied with

$$\gamma_P(\varkappa) = \pi i \varkappa \int da\, \sigma_P(a) \ln(a-\varkappa), \tag{88}$$

$$\gamma_T(\varkappa) = \varkappa C_{\frac{1}{2}} \int da\, \sigma_T(a)(a-\varkappa)^{1/2}, \tag{89}$$

$$\gamma_F(\varkappa) = \int da\Big[-\tfrac{1}{2}\sigma_0(a) + \tfrac{1}{2}D_{\frac{1}{2}}\sigma_T(a) + \sigma_1(a)\left(\frac{a-\varkappa}{a}\right)\ln(a-\varkappa) - \tfrac{1}{4}\int db\, \bar{\sigma}(a,b)b^{-2}\Big]a . \tag{90}$$

Although W does not have the $J = 0$ fixed pole asymptotic contribution, the value of the fixed pole residue $-\varkappa\gamma_F(\varkappa)$ in T can be obtained from W in the usual way. The quantity $T(\varkappa,\nu) - \gamma_P(\varkappa) - (2\nu)^{-\frac{1}{2}}\gamma_T(\varkappa)$ satisfies the unsubtracted dispersion relation

$$T(\varkappa,\nu) - \gamma_P(\varkappa) - (2\nu)^{-\frac{1}{2}}\gamma_T(\varkappa)$$

$$= 2\int_0^\infty d\nu' \left[W(\varkappa,\nu') - \beta_P(\varkappa) - (2\nu')^{-\frac{1}{2}}\beta_T(\varkappa)\right](\nu'-\nu)^{-1} \; . \tag{91}$$

Taking the $\nu \to \infty$ limit, using (71), gives

$$-2\int_0^\infty d\nu \left[W(\varkappa,\nu) - \beta_P(\varkappa) - (2\nu)^{-\frac{1}{2}}\beta_T(\varkappa)\right] = -\varkappa\gamma_F(\varkappa) \; , \tag{92}$$

which expresses the fixed pole residue in terms of W. Note that the absence of ν^{-1} terms in W guarantees the existence of the integral in (92). As a special case of (92), we can consider its scaling limit. We divide (92) by $\varkappa$ and let $-\varkappa \to \infty$ [recalling that $W(\varkappa,\nu)$ vanishes for $2\nu < |\varkappa|$] to obtain

$$\int_0^\infty d\rho \left[F(\rho) - \beta_P - \rho^{-\frac{1}{2}}\beta_T\right] = -\gamma_F \; . \tag{93}$$

Note that, although $F(\rho)$ vanishes for $\rho < 1$, the integral in (93) must extend down to $\rho = 0$ since the Regge subtraction do not vanish for $0 < \rho < 1$. Since (92) is a global relation, we shall postpone a phenomenological discussion of it until Section I.C.1.

The free field functions (29)-(32) again provide simple illustrations of these considerations. We have

$$\nu T_f(\varkappa,\nu) \xrightarrow[R]{} -\frac{1}{2}\frac{\varkappa}{\nu} \; , \tag{94}$$

and so T_f possesses a $J = 0$ fixed pole, but of course does not contain the Pomeron or tensor trajectories. The residue $-\varkappa\gamma_F(\varkappa) = -\frac{1}{2}\varkappa$ is linear in $\varkappa$ and real. The fixed pole is not present in the absorptive part W_f which decreases faster than any inverse power in the R limit. Note also that (92) and (93) are satisfied.

So far in this section we have only dealt with electron-proton scattering. There are also electron-neutron data, however, and, although its accuracy is poorer, we will record here the results of a fit of the form (64) :

$$F^{(n)}(\rho) \xrightarrow[\rho\to\infty]{} \beta_P^{(n)} + \rho^{-\frac{1}{2}}\beta_T^{(n)} \equiv F_R^{(n)}(\rho) \; . \tag{95}$$

Here a superscript (n) is used to denote the neutron functions. An analysis of the type described above yields [10)]

$$\beta_P^{(n)} \simeq 0.19 \quad , \qquad \beta_T^{(n)} \simeq 0.47 , \tag{96}$$

with the same μ^2 as in (69). Comparison of (96) and (69) shows a striking difference between β_P and $\beta_P^{(n)}$. This discrepancy is surprising since the Pomeron is supposed to have zero isospin so that $\beta_P^{(n)}$ should be the same as β_P. Although this discrepancy may be simply the result of poor neutron data, it is interesting to speculate on what would happen if β_P and $\beta_P^{(n)}$ were really as different as is indicated by (69) and (96). A possible explanation of this difference could be that there is a $J=1$ fixed pole with non-zero isospin contributing. We suggested a mechanism of this sort several years ago, based on a new universality principle [32),33)]. We predicted from this that the double helicity flip scaling function $F_B^{ab}(\rho)$ for the process $B(p)+J^a(q) \to B(p)+J^b(q)$ [where J^a, $a=1-8$, are the SU(3) currents and B is a member of the $\frac{1}{2}^+$ baryon octet] has the highly symmetric asymptotic behaviour

$$F_B^{ab}(\rho) \longrightarrow \frac{1}{\pi} d^{abc} D_B^c \; \equiv \; W_B^{ab} , \tag{97}$$

where $D_B^c \equiv \langle B(p)|S^c|B(p)\rangle$, with $S^c(x)$ the scalar current in U(12), is a good SU(3) nonet described by $F \simeq 3/7$, $D \simeq -1/7$, $D^0 \simeq \frac{1}{2}(\frac{3}{2})^{\frac{1}{2}}$. For electro-production off protons and neutrons, (97) gives

$$W_P^{QQ} = \frac{1}{9\pi}\left(6F + 2D + 12\left(\tfrac{2}{3}\right)^{1/2} D^0\right) \simeq 0.29 \tag{98}$$

and

$$W_N^{QQ} = \frac{1}{9\pi}\left(-4D + 12\left(\tfrac{2}{3}\right)^{1/2} D^0\right) \simeq 0.23 . \tag{99}$$

The experimental results are $\beta_P^{(p)} \cong 0.29$ and $\beta_P^{(n)} \cong 0.19$, in agreement with (98) and (99) within the experimental errors.

B.2 Resonance averaging

Duality is perhaps the most important new concept to be introduced in particle physics in recent years, in spite of the fact that its precise meaning, beyond finite energy sum rules [34)], remains unclear. It is sufficient for our present purposes to take duality to be the semi-local averaging, over several resonance widths, of the absorptive part of an amplitude by the low energy extrapolation of its high energy behaviour. Particularly successful in hadron physics is the two-component duality framework [35], in which the ordinary Regge exchange extrapolation averages the resonances and the Pomeron exchange extrapolation averages the background. For semi-hadronic processes [36], the situation is, on the one hand, greatly complicated by the occurrence of non-Regge behaviours and, on the other hand, is greatly enriched by the occurrence of off-shell phenomena such as scaling.

To obtain a first glimpse of the resonance averaging phenomena in electroproduction, consider again the scaling curve $F(\rho)$ in Fig. 5. Consider a fixed (not too large) value of $\varkappa$ in the scaling region ($|\varkappa| \geq$ 1 GeV^2 if the ρ' variable is used). Then, for sufficiently small ρ, the corresponding value $-\frac{1}{2}\varkappa\rho$ of ν will be too small to be in the scaling region or, equivalently, the corresponding value $-\varkappa(\rho-1)+1$ of s will be too small to be out of the resonance region. The behaviour of $W(\varkappa,\nu) =$ $= W(\varkappa,-\frac{1}{2}\varkappa\rho)$ for these ρ values will therefore not be the same as that of $F(\rho)$ since $F(\rho)$ must be obtained from the large $\varkappa$ <u>and</u> large ν behaviour of $W(\varkappa,\nu)$. Rather, $W(\varkappa,-\frac{1}{2}\varkappa\rho)$ will exhibit bumps corresponding to the many s channel resonances present. The striking empirical observation is that this resonant behaviour of $W(\varkappa,-\frac{1}{2}\varkappa\rho)$ is strongly correlated with the smooth scaling curve $F(\rho)$. $F(\rho)$ is observed to average the resonance peaks of $W(\varkappa,-\frac{1}{2}\varkappa\rho)$ in a semi-local way. This is illustrated in Fig. 7. The same type of behaviour is obtained for other values of $\varkappa$. As $|\varkappa|$ is increased, a given resonance slides along the scaling curve to threshold $(\rho=1)$ and seems to decrease in magnitude at about the same rate as the scaling curve itself is decreasing.

The first detailed study of the duality properties of the electroproduction data was undertaken by Bloom and Gilman [37),38)]. To derive their sum rule, take a fixed $\varkappa$ in the scaling region [39] [so that $\beta_P(\varkappa) \sim \beta_P$, $\beta_T(\varkappa) \sim |\varkappa|^{\frac{1}{2}}\beta_T$, and $\gamma_F(\varkappa) \sim \gamma_F$] and subtract (93) from (92). Then replace the upper limit of the ν integral by some large N to obtain

$$\int_0^{2N|\varkappa|^{-1}} d\rho\left[W(\varkappa,-\tfrac{1}{2}\varkappa\rho)-F(\rho)\right] \cong 0\,. \qquad (100)$$

This relation becomes an identity for $|\varkappa| \to \infty$ for any $N = N(\varkappa)$. For large ρ (i.e., for ν in the scaling region or $s \equiv \varkappa+2\nu+1$ above the resonance region), the integrand approaches zero by scaling. For smaller ρ (s in the resonance region), $F(\rho)$ was observed to average the resonance peaks of

$W(\varkappa, -\frac{1}{2}\varkappa\rho)$ in a semi-local way [37]. The relation (100) of course guarantees that $F(\rho)$ will provide a global average of $W(\varkappa, -\frac{1}{2}\varkappa\rho)$. The observed semi-local averaging is much stronger and does not follow simply from (100). The observed averaging means that (100) is valid even for smaller N. The mysterious principle responsible for this is referred to as "duality".

This apparent manifestation of duality is quite different from strong-interaction duality. For purely hadronic reactions, the one or two leading Regge pole exchanges provide a good semi-local average to the low energy amplitude. The low energy extrapolations of the Pomeron and tensor Regge exchange contributions to W, on the contrary, look nothing like the correct low energy behaviour of W. For the scaling data, for example, the Pomeron is constant and the tensors increase like $\rho^{-\frac{1}{2}}$ as ρ decreases to 1, whereas the scaling curve vanishes like $(\rho-1)^3$ for small $\rho \sim 1$.

From a purely phenomenological point of view, it would be difficult to guess that it is the scaling extrapolation $F(\rho)$ and not the Regge extrapolation

$$R(\varkappa, \nu) \equiv \beta_P(\varkappa) + (2\nu)^{-\frac{1}{2}}\beta_T(\varkappa) \tag{101}$$

which averages W. In fact, we have the commutative diagram (65) so that both $F(\rho)$ and $R(\varkappa, -\frac{1}{2}\varkappa\rho)$ are approximately equal to $W(\varkappa, -\frac{1}{2}\varkappa\rho)$ for large ρ ($\varkappa$ in scaling region), but only $F(\rho)$ remains correlated for smaller ρ. In fact,

$$\left|W(\varkappa, -\tfrac{1}{2}\varkappa\rho) - R(\varkappa, -\tfrac{1}{2}\varkappa\rho)\right| \xrightarrow[\rho\to\infty]{} 0, \tag{102}$$

whereas

$$\left|W(\varkappa, -\tfrac{1}{2}\varkappa\rho) - F(\rho)\right| \xrightarrow[\rho\to\infty]{} \varepsilon_\varkappa , \tag{103}$$

with $\varepsilon_\varkappa$ small but non-zero for finite $\varkappa$. Note the contrast with purely hadronic duality, where only the R limit is possible, and where R and W are correlated.

It is of course the presence of the fixed pole in (92) and (93) which ruins the correlation between W and R. Let us ask if, nevertheless, some aspect of (two-component) duality is not in fact present in (92) or (93) [40]. [For notational simplicity, we will usually work with the scaling limit, Eq. (93). The analogous equations for finite $\varkappa$ will be obvious.] To this end, we decompose [41] $F(\rho)$ into its resonance contribution $F^R(\rho)$ and its background contribution $F^B(\rho)$ and write

$$F_P(\rho) = \beta_P \,, \qquad F_T(\rho) = \rho^{-\frac{1}{2}}\beta_T \,. \tag{104}$$

Then (93) becomes

$$\int_0^\Lambda d\rho \left[F^R(\rho) + F^B(\rho) - F_P(\rho) - F_T(\rho)\right] = -\gamma_F \,, \tag{105}$$

or

$$\int_0^\Lambda d\rho \left[F^R(\rho) + F^B(\rho) - F_P(\rho) - F_T(\rho)\right] \cong -\gamma_F \tag{106}$$

for large Λ. We now ask whether (106) can be decomposed into two non-trivial independent equations. A possible answer is suggested if one defines a function $F_F(\rho)$ which satisfies

$$\int_0^\Lambda d\rho \, F_F(\rho) \cong -\gamma_F \tag{107}$$

and write (106) as

$$\int_0^\Lambda d\rho \left[F^R(\rho) + F^B(\rho) - F_P(\rho) - F_T(\rho) - F_F(\rho)\right] \cong 0 \,. \tag{108}$$

We propose the following decomposition of (108) [40] :

$$\int_0^\Lambda d\rho \left[F^R(\rho) - F_T(\rho)\right] \cong 0 \,, \tag{109}$$

$$\int_0^\Lambda d\rho \left[F^B(\rho) - F_P(\rho) - F_F(\rho)\right] \cong 0 \,. \tag{110}$$

We are thus following purely hadronic duality in correlating the resonances with the tensor exchanges, but the background is now correlated with the sum of the Pomeron and fixed-pole exchanges. These correlations provide for a universal treatment of purely hadronic and semi-hadronic processes, since γ_F is zero for on-shell scattering.

The idea is, roughly speaking, to take $F_F(\rho)$ to behave like $\beta_F \rho^{-1}$ in the observed ρ region, say $1 < \rho < 10$, [$\beta_F < 0$ is to be chosen consistently with (107)] although such a form should not survive in the Regge region.

The viability of (109) and (110) is then illustrated if Figs. 8 and 9. Figure 8 shows how $F_T(\rho) \equiv \rho^{-\frac{1}{2}}\beta_T$ averages the resonances $F^R(\rho)$ [42] and Fig. 9 shows how the sum $F_P(\rho)+F_T(\rho)+F_F(\rho) \sim \beta_P+\rho^{-\frac{1}{2}}\beta_T+\rho^{-1}\beta_F$ reproduces the gross features of $F(\rho)$.

In order to clarify the role played by $F_F(\rho)$, we follow the usual derivation of finite energy sum rules [34], and cast (93), with cut-off $\Lambda \geq \Lambda_0 = 16$, into the approximate statement

$$\frac{1}{\Lambda}\int_0^\Lambda d\rho F(\rho) \cong \beta_P + 2\beta_T\Lambda^{-\frac{1}{2}} - \gamma_F\Lambda^{-1} . \tag{111}$$

Just as the usual finite energy sum rules suggest that the integrand at low energy is approximated by the simplest functional form that can yield correctly the right-hand side, so here we make the <u>ansatz</u> that $F(\rho)$ for $1 < \rho \lesssim \Lambda_0$ behaves on the average like

$$\tilde{F}(\rho) = \beta_P + \beta_T\rho^{-\frac{1}{2}} + F_F(\rho), \tag{112}$$

where $F_F(\rho)$ represents a <u>simulated</u> contribution from the fixed pole. The authentic contribution from a fixed pole in the real part of the form ρ^{-1} to $F(\rho)$ is of course proportional to $\delta(\rho)$, which does not survive in the Regge régime. As long as the finite energy sum rule (111) is maintained, one is free, however, to modify the functional form that is supposed to interpolate the actual small ρ behaviour of $F(\rho)$ according to the suggestive form of (111). In order not to wreck (111), $F_F(\rho)$ must satisfy the equality

$$\int_0^\Lambda d\rho F_F(\rho) = -\gamma_F , \tag{113}$$

for all $\Lambda > \Lambda_0$. It is also desirable to impose the condition that $\tilde{F}(\rho)$ should vanish identically below the threshold $\rho = 1$. The simplest possibility for $F_F(\rho)$ is thus [40]

$$F_F(\rho) = \begin{cases} -(\beta_P + \beta_T\rho^{-\frac{1}{2}}), & \text{for } 0<\rho<1 ; \\ \dfrac{c}{\rho}\dfrac{1}{\ln\Lambda_0}, & \text{for } 1<\rho<\Lambda_0 ; \\ 0, & \text{otherwise} . \end{cases} \tag{114}$$

The constant C is determined by the condition of correct fixed pole residue to be $C = -\gamma_F + \beta_P + 2\beta_T$. Using the values of γ_F, β_P, and β_T in Section I.B.1, this function is plotted in Fig. 10. $\tilde{F}(\rho)$ is seen to provide an acceptable average to $F(\rho)$ [43]. Better averages, incorporating constraints of locality, are given in Ref. 40).

We conclude this section by noting that the unification of $F_F(\rho)$ and $F_P(\rho)$ in (110) suggests that the Pomeron is a fixed pole. This possibility has been suggested many times in spite of possible unitarity difficulties, and is consistent with the rather flat Pomeron slope usually observed. It is further supported by the discussion of neutron electroproduction given at the end of Section B.1.

B.3 Threshold behaviour

It follows from (19) that the best description of $F(\rho)$ near threshold is given by

$$F(\rho) \xrightarrow[\rho \to 1]{} a_1(\rho-1)^3 . \tag{115}$$

There have been many attempts to correlate this threshold behaviour with the large $\mathcal{H}$ behaviour of the nucleon resonance excitation form factors $G_n(\mathcal{H})$ [37),44)]. [Here $G_0(\mathcal{H})$ is the elastic form factor, $G_1(\mathcal{H})$ is the $N-N^*$ transition form factor, etc.] Such a correlation is expected mathematically since a given resonance contribution is at a fixed mass^2 $s = \mathcal{H} + 2\nu + 1 = |\mathcal{H}|(\rho - 1) + 1$ and so moves to $\rho \sim 1$ for large $|\mathcal{H}|$. It is also expected empirically because as $|\mathcal{H}|$ increases the resonances are observed to not disappear relative to some background but to slide down the scaling curve while maintaining an approximately fixed fractional contribution [37)].

We parametrize the excitation form factors by

$$G_n(\mathcal{H}) \sim c_n \left(1 - \frac{\mathcal{H}}{\Lambda_n}\right)^{-d_n} . \tag{116}$$

The usually obtained relation of (116) to the threshold behaviour $(\rho - 1)^p$ of $F(\rho)$ is

$$2d_n = p + 1 . \tag{117}$$

This relation was first obtained in parton models [44)] and later in resonance saturation models [37),45)] if a universal fall-off rate (d_n = const.) is <u>assumed</u>. Equation (117) is in good agreement with the data ($d = 2$, $p = 3$).

To see quantitatively how resonances can build up a part of the scaling curve, it is convenient to work with narrow resonance models [45] [perhaps dual [46]]. In a narrow resonance model, the contribution W^R of the resonances to W is exhibited as an explicit sum; e.g.,

$$W^R(\varkappa,\nu) = \nu\sum_n W_n(\varkappa)\,\delta(\varkappa+2\nu+1-s_n)\,, \tag{118}$$

where s_n is the mass-squared of the n^{th} resonance, and

$$W_n(\varkappa) = g_n^2\left[G_n(\varkappa)\right]^2. \tag{119}$$

If (118) scales,

$$W^R(\varkappa,\nu) \xrightarrow[A]{} F^R(\rho)\,, \tag{120}$$

and has the threshold behaviour

$$F^R(\rho) \xrightarrow[\rho\to 1]{} \eta^R(\rho-1)^P\,, \tag{121}$$

then the relation (118) is usually obtained. The sum in (118) is usually converted into a continuous integral over s, with specified level densities, coupling parameters $g(s)$, and form factor constants $d(s)$, $r(s)$:

$$\begin{aligned} W^R(\varkappa,\nu) &\longrightarrow \nu\int ds\; G(s,\varkappa)\,\delta(\varkappa+2\nu+1-s) \\ &= \nu\, G(\varkappa+2\nu+1,\,\varkappa)\,. \end{aligned} \tag{122}$$

It is now convenient to define the F limit [40] in which $|\varkappa|$ and ν become large with fixed mass $s=\varkappa+2\nu+1$. We have

$$W^R(\varkappa,\nu) \xrightarrow[F]{} H(s)\,(-\varkappa)^{-\gamma(s)}\,,\qquad \gamma(s) = 2d(s)-1\,. \tag{123}$$

The proposed duality [40] between the resonances and tensor exchanges requires that

$$W^R(\varkappa,\nu) \xrightarrow[R]{} (2\nu)^{-1/2}\beta_T(\varkappa)\,, \tag{124}$$

and

$$F^R(\rho) \xrightarrow[\rho\to\infty]{} \rho^{-1/2}\beta_T\,. \tag{125}$$

We can show that the above constraints on W^R are consistent by constructing a simple explicit model function $\tilde{W}^R$ [40]. We take the factorized form

$$\tilde{W}^R(\varkappa,\nu) = 2\varkappa\nu A_1(\varkappa)A_2(\varkappa+2\nu)A_3(2\nu), \tag{126}$$

and the asymptotic behaviours

$$A_i(\tau) \xrightarrow[\tau\to\infty]{} c_i\tau^{\alpha_i}, \qquad i = 1-3. \tag{127}$$

The above constraints uniquely specify the α_i : $\alpha_1 = -\frac{1}{2}$. $\alpha_2 = \gamma$, $\alpha_3 = -\frac{3}{2} - \gamma$. Thus

$$\begin{aligned}\tilde{W}^R(\varkappa,\nu) &\xrightarrow[R]{} \varkappa A_1(\varkappa)(2\nu)^{-1/2} \equiv \tilde{\beta}_T(\varkappa)(2\nu)^{-1/2} \\ &\xrightarrow[A]{} \beta_T \rho^{-1/2}\left(\tfrac{1}{\rho}-1\right)^{\gamma} \equiv \tilde{F}^R(\rho) \\ &\xrightarrow[F]{} A_2(s)\varkappa^{1/2}(\varkappa-s)^{-\frac{1}{2}-\gamma},\end{aligned} \tag{128}$$

as required. This example has a fixed pole, but it is simple to remedy this. An example is [40]

$$\beta_T \rho^{-9/2}(\rho-1)^3(\rho+7) . \tag{129}$$

As a further consequence of locality, we can give a model independent derivation of the relation (117) and of related results [40]. We assume the behaviour

$$W(\varkappa, \tfrac{1}{2}(s-\varkappa)) \xrightarrow[F]{} H(s)(-\varkappa)^{-\gamma(s)} \tag{130}$$

in the F limit. Thus we are assuming that resonances dominate the F limit and that the form factor of the mass squared = s resonance is $(-\varkappa)^{-\frac{1}{2}[\gamma(s)+1]}$ for large $|\varkappa|$. Referring to (44), we see that the F limit of W is controlled by the behaviour of $\sigma(a,b)$ for $b\to 1$, since $b = (a-\varkappa)/2\nu \xrightarrow[F]{} 1+\varkappa^{-1}(s-a)+ \ldots$. We assume the behaviour [47]

$$\sigma(a,b) \xrightarrow[b\to 1]{} \tau(a)(1-b)^{\delta}, \tag{131}$$

and obtain [48]

$$W(\varkappa,\nu) \xrightarrow[F]{} -\tfrac{1}{2}(-\varkappa)^{1-\delta}\int_0^\infty da\, \tau(a)(s-a)^{\delta}, \tag{132}$$

and so $\gamma(s) = \delta - 1$, independent of s. Our assumptions thus imply a universal fall-off rate for excitation form factors, and we have not even assumed non-trivial scaling. Scaling gives

$$H(s) \xrightarrow[s\to\infty]{} s^{\delta-1}\,\tfrac{1}{2}\delta\int_0^\infty da\, \tau(a)\,a\,, \tag{133}$$

and so $[G_s(-\varkappa)]^2 \sim (-s/\varkappa)^{\delta}$ for $-\varkappa \gg s \gg 1$. This result is consistent with experiment and had been previously suggested [49]. Finally, (53) now gives

$$F(\omega) \xrightarrow[\omega\to 1]{} (1-\omega)^{\delta-1}\,\tfrac{1}{2}\delta\int_0^\infty da\, \tau(a)\,a\,, \tag{134}$$

which is the desired result. Its experimental verification is yet further support for locality. What we have done here is to establish, in a large class of models, the commutativity of the large s behaviour of the F limit and the small $(1-\omega)$ behaviour of the A limit [40]. This is strictly analogous to the commutativity of the large $\varkappa$ behaviour of the R limit and the small ω behaviour of the A limit discussed in Section B.1.

C. Global properties

C.1 Fixed pole

The first global relation for electroproduction we shall consider is the expression (92) for the $J=0$ fixed pole residue $-\varkappa\gamma_F(\varkappa)$. The finiteness of the $(\varkappa=0)$ real photon amplitude guarantees the existence of $\gamma_F(0) \equiv \delta_F$ (it may possibly vanish) and scaling requires the existence of $\gamma_F(\infty) \equiv \gamma_F$. It is therefore tempting to speculate that

$$\gamma_F(\varkappa) \overset{?}{=} \text{constant} = \gamma_F = \delta_F\,. \tag{135}$$

This would follow in particular if, as has been suggested, all fixed pole residues are polynomials. A consequence of (135) would be the absence of a $J=0$ fixed pole in photoproduction amplitudes. [The (particle P) photoproduction fixed pole Regge residues are the residues of the (P) single particle poles in the photon mass variable. See Fig. 11a and 11b. The double particle pole residue (Fig. 11c) is a purely hadronic amplitude and therefore, by unitarity, cannot contribute to the fixed pole residue.] If (135) were correct, $\gamma_F(\varkappa)$ would have no discontinuities and so would not contribute to photoproduction. Conversely, if there are no fixed poles in photoproduction amplitudes, it of course follows that $\gamma_F(\varkappa)$ is a polynomial [50].

There is a number of reasons for expecting the validity of (135) :

a) The best known (and only really theoretically well founded) fixed pole, the $J = 1$ right signature fixed pole in [SU(3) antisymmetric] current-hadron scattering whose existence is guaranteed by current algebra [29], has a residue which is independent of the current masses (i.e., is constant in $\varkappa$).

b) More generally, any integral fixed pole arising solely from a light cone singularity [51] will have a polynomial residue [52].

c) There are, in fact, indications from studies of Feynman diagrams [53] and from potential scattering [54] that fixed poles are not present in photoproduction.

d) The representation (90) shows that, whereas it is easy to obtain a constant contribution to $\gamma_F(\varkappa)$ [from $\sigma(a,b)$], a non-polynomial contribution only arises if the special term $\sigma_1(a)|b|$ satisfying the special condition (83) is present, and this contingency seems somewhat contrived.

In spite of all this, we will now see that the validity of (135) is almost definitely excluded by the present data. To evaluate γ_F from (93), we use the explicit fit (19) for $1 \leq \rho \leq 7$, the Regge behaviour (64) for $\rho \geq 16 = \rho_0$, and a straight line extrapolation in between [10]. This gives [55]

$$\gamma_F \simeq +2 . \qquad (136)$$

The positive sign here is already indicated in Fig. 6 which shows that the integrand (93) is everywhere negative. The value of $\gamma_F(0) = \delta_F$ has been determined from the (real) Compton scattering by several groups and the result is [25),26]

$$\delta_F \simeq -1 . \qquad (137)$$

The value $\delta_F = -1$ is precisely the Born contribution [56] to the fixed pole and so (137) says that the Born term is the only surviving contribution. Equations (136) and (137) clearly contradict (135) and so we are led to the conclusion that the fixed pole residue is not a constant [57] (or any polynomial) and that there are fixed poles in photoproduction.

Let us now ask how things must change if this conclusion is to be avoided. The sign and magnitude of (137) is quite definite if one insists on keeping $\alpha_T = \frac{1}{2}$, or for hadrons. If this value is given up, then the data are consistent with δ_F as small as zero [26]. It seems, of course, very unlikely that $\alpha_T \not\simeq \frac{1}{2}$ and so we must maintain (137) [58]. Concerning γ_F, it has been shown that the present scaling data $(\rho < 7)$ do not rule out a parametrization which gives $\gamma_F \simeq -1$ so that the fixed pole residue is indeed constant [59]. An example is

$$(0.12) + (0.462)\rho^{-1/2} + (4.02)\rho^{-3/2} . \qquad (138)$$

However, the fit (138) is extremely alarming. It has a non-leading piece $\rho^{-\frac{3}{2}}$ with a residue which is ~40 times bigger than that of the Pomeron ! If such a thing occurred, then asymptotic statements could never be made or tested in physics. The value of, say, β_P could certainly be made as one likes from any (finite) set of data if non-leading terms with sufficiently large coefficients are introduced. For example, a term $(50)\rho^{-5/2}$ cannot be excluded but would appreciably change the residues in (138). Furthermore, a fit of the form (138) would be in gross disagreement with the non-scaling data (which, unlike the scaling data, exists in the Regge region) if parametrizations of the form (66) are used. We conclude therefore that if a Regge type fit at present energies makes any sense at all, the difference between (136) and (137) us a real one and so $\gamma_F(\varkappa) \neq$ const. [60].

The conclusion that there is a fixed pole in some photoproduction amplitude is, of course, an immediate consequence of $\mathrm{Im}\,\gamma_F(\varkappa) \neq 0$. The precocity phenomenon and light cone controlled mass dispersion relation techniques [61] imply the much more specific assertion that there is a fixed pole in the ρ photoproduction amplitude [62]. The magnitude is predicted to be

$$\beta_\rho \simeq 2\gamma_\rho \frac{M^2}{M^2 - m_\rho^2} (\delta_F - \gamma_F) , \qquad (139)$$

where $\gamma_\rho \simeq 2.5$ is the usual $\gamma - \rho$ junction and $M^2 \simeq 2$ GeV^2. The prediction is therefore $\beta_\rho \simeq -21$. The prediction should be checked experimentally in the near future.

We would like to conclude by noting some further support for the non-polynomial character of the fixed pole residue. Testa [63] has derived the interesting relation (in our notation)

$$\beta_P + 2\beta_T \simeq \gamma_F \quad , \tag{140}$$

based on an assumed pole dominance in the complex angular momentum plane and a spinor structure for $SU(3) \times SU(3)$ currents. The numbers (136) and (69) give $\sim$+2 for the right side and $\sim$+1 for the left side. We consider this agreement to be satisfactory in view of the approximate nature of (140). Equation (140) is badly violated if γ_F is negative.

C.2 Causality

The next global quantity to be considered is the integral

$$I \equiv \int_1^\infty \frac{d\rho}{\rho}\left[\bar{F}(\rho) - \beta_P\right] = \int_0^1 \frac{d\omega}{\omega}\left[\bar{F}(\omega) - \beta_P\right] .$$

Are there any theoretical expectations for the value of I ? We will first show that I vanishes as a consequence of assumptions, mainly causality, introduced above [64]. The important assumptions are :

a) causality - the validity of (47);

b) scaling - the validity of (52);

c) Regge behaviour of $\sigma(a,b)$ - the validity of (60).

We will afterwards discuss what happens if these assumptions are weakened.

Let us suppose that a crossing even amplitude $\bar{W}(\varkappa, \nu)$ satisfies locality

$$\bar{W}(\varkappa,\nu) = \varkappa\nu \int_0^\infty da \int_{-1}^1 db\, \bar{\sigma}(a,b)\, \delta(\varkappa + 2b\nu - a)\, \varepsilon(\nu + b) , \tag{141}$$

satisfies scaling, $\bar{W}(\varkappa, \nu) \xrightarrow[A]{} \bar{F}(\omega)$, and has the Regge behaviour

$$\bar{W}(\varkappa,\nu) \xrightarrow[R]{} \bar{\beta}(\varkappa)\, \nu^{-\varepsilon} , \qquad \varepsilon > 0 ; \tag{142}$$

then,

$$\int_0^\infty \frac{d\nu}{\nu} \overline{W}(\varkappa, \nu) = 0, \tag{143}$$

and so

$$\int_0^1 \frac{d\omega}{\omega} F(\omega) = 0. \tag{144}$$

The proof is immediate :

$$\int_0^\infty \frac{d\nu}{\nu} \overline{W}(\varkappa,\nu) = \varkappa \int_0^\infty da \int_0^1 db\, \overline{\sigma}(a,b) \frac{1}{2b} = 0. \tag{145}$$

The existence of the b integral in (145) is crucial and follows from (142), which gives $\overline{\sigma}(a,b) \xrightarrow[b\to 0]{} b^{\varepsilon}$. The vanishing of (145) follows from the consequence $\int_0^\infty da\, \overline{\sigma}(a,b) = 0$ of scaling. If scaling were not assumed, but $\sigma(a,b)$ remains integrable, (145) would have the form (const)$\cdot\varkappa$ and would still be useful. Note that (143) is <u>not</u> a superconvergence relation, since $\overline{W}(\varkappa,\nu)\nu^{-1}$ is odd in ν .

In order to apply these results to (44), we must first subtract off the Pomeron contribution. The subtraction <u>must</u> be effected in a local way. To this end, we define the local Pomeron contribution [64)]

$$W_{P,\lambda}(\varkappa,\nu) \equiv \varkappa\nu \int_0^\infty da \int_{-\infty}^\infty db\, \sigma_P(a)(\ln b)\lambda(b)\delta(\varkappa+2b\nu-a)\varepsilon(\nu+b), \tag{146}$$

where $\lambda(b)$ is any function which satisfies

$$\lambda(b) \xrightarrow[b\to 0]{} 1 + O(b^{1+\varepsilon_0}), \quad \lambda(b) \xrightarrow[b\to\infty]{} b^{-\varepsilon_1}, \quad \varepsilon_{0,1} > 0. \tag{147}$$

We have $W_{P,\lambda}(\varkappa,\nu) \underset{R}{\rightarrow} \beta_P(\varkappa) + O(1/\nu)$ and so $\overline{W} \equiv W - W_{P,\lambda}$ satisfies the conditions (141) and (142) and we have

$$\int_0^\infty \frac{d\nu}{\nu}\left[W(\varkappa,\nu) - W_{P,\lambda}(\varkappa,\nu)\right] = 0. \tag{148}$$

The relation (148) has the pleasant features of finite energy sum rules which enable one to correlate large and small energy nehaviour. Equation (148) is particularly effective when used in conjunction with specific fits such as (66).

We will only be concerned here with the scaling limit of (148). It follows from (146) and (60)-(63) that

$$W_{P,\lambda}(x,\nu) \xrightarrow[A]{} \beta_P \Lambda(\omega) \equiv \beta_P\left[\lambda(\omega) + \omega\lambda'(\omega)\ln\omega\right] = \beta_P\,\omega\frac{\partial}{\partial\omega}\left[\lambda(\omega)\ln\omega\right], \tag{149}$$

and so (148) gives

$$\int_0^\infty \frac{d\omega}{\omega}\left[F(\omega) - \beta_P\Lambda(\omega)\right], \tag{150}$$

where $F(\omega)$ vanishes for $\omega > 1$. Taking $\lambda(\omega) = \theta(1-\omega)$ gives the interesting and useful result

$$\int_0^1 \frac{d\omega}{\omega}\left[F(\omega) - \beta_P\right] = 0. \tag{151}$$

To obtain the scaling limit sum rule (151) more simply, we use (53) and (54) to write

$$\int_0^1 \frac{d\omega}{\omega}\left[F(\omega) - \beta_P\right] = \int_0^1 d\omega \frac{\partial}{\partial\omega}\left[-\frac{1}{2}\int_0^\infty da\,\sigma(a,\omega)a - \beta_P\ln\omega\right]. \tag{152}$$

This expression vanishes because the bracket vanishes at the end point $\omega = 1$ by (131) and at the end point $\omega = 0$ by (60) and (63). [The assumption $\sigma(a,1) = 0$ needed here is implicit in the first derivation in our use of the discontinuous $\lambda(\omega) = \theta(1-\omega)$.]

Our result (151), although simple in appearance, is in fact very restrictive. Since $F(\omega) \geq 0$, an immediate consequence is that β_P cannot vanish unless $F(\omega)$ vanishes everywhere. Thus non-trivial scaling requires a Pomeron ! To compare it with experiment, we use the fit (69) to the small ω $(< 1/16)$ region, the fit (19) to the large ω region $(\omega > 1/7)$, and a linear connection for $1/16 < \omega < 1/7$. The result is [64)]

$$\int_0^1 \frac{d\omega}{\omega}\left[F(\omega) - \beta_P\right]_{expt} \cong 0.02. \tag{153}$$

The sum rule (151) is thus in striking agreement with the data. To emphasize the non-trivial nature of (151) and also to get an idea of how small (153) should be to be significant, we note that not a single model for $F(\omega)$ yet exhibited (of which we are aware) satisfies (151).

The models are classified into :

a) parton models [65];

b) resonance dominance models [45];

c) dual models [46];

d) partial sums of field theoretic perturbation series [66] and

e) none of the above [67].

The dual models c) tend to give 1 for the proton locality integral (151), while a), c) and d) agree on a value of 1/3 for the proton-neutron difference. All of them fail to satisfy the Pomeron subtracted sum rule (151).

In the free field case [Eqs. (29)-(32)], we have $W_f(\varkappa, \nu) \underset{R}{\rightarrow} 0$ (faster than any power) and so $\beta_{Pf} = 0$. Since

$$\int_0^1 \frac{d\omega}{\omega} F_f(\omega) = 1 , \tag{154}$$

the sum rule (151) is strongly violated in this free field theory. The largeness of (154) compared to zero again indicates the significance of the experimental result (153). It is of interest to ask which of our assumptions is violated in the free field case. Since

$$\sigma_f(a,b) = \delta'(a)\,\Theta(1-|b|) \xrightarrow[b\to 0]{} \delta'(a) \neq 0 , \tag{155}$$

it is assumption c), the validity of (60), which is violated. [For the scaling limit derivation (152), the condition $\sigma_f(a,1) = 0$ is also violated, corresponding to the (unphysical) constant form factors in the free field theory. This, however, is of no importance for the present discussion.]

It follows from Eq. (86) that the existence of a constant term $\sigma_0(a)$ in $\sigma(a,b)$ for $b = 0$ is the only way in which (60) can be violated if one has the correct Regge and scaling behaviour for W. The contribution of such a term is easily incorporated into our analysis. We will see that it changes (151), but in a pathological way. To see this, assume (86) and define

$$\begin{aligned} W^{(0)}(\varkappa, \nu) &= \varkappa\nu \int_0^\infty da \int_{-1}^1 db\, \sigma_0(a)\, \delta(\varkappa + 2b\nu - a) \\ &= \frac{\varkappa}{2} \int_0^{\varkappa+2\nu} da\, \sigma_0(a) \quad = \frac{\varkappa}{2} \int_0^{2\nu(1-\omega)} da\, \sigma_0(a) , \end{aligned} \tag{156}$$

the absorptive part of (75) for $\alpha = 0$. Because $\sigma_0(a)$ is assumed to decrease fast for large a, we see that $W^{(0)}(\varkappa,\nu)$ vanishes fast in both the R and A limits :

$$W^{(0)}(\varkappa,\nu) \xrightarrow[A]{} 0 . \tag{157}$$

We can repeat our analysis above provided $W^{(0)}$, in addition to W_P, is subtracted out :

$$\int_0^\infty \frac{d\nu}{\nu}\left[W(\varkappa,\nu) - W^{(0)}(\varkappa,\nu) - W_P(\varkappa,\nu)\right] = \varkappa\int_0^\infty \int_0^1 db\left[\sigma(a,b) - \sigma_0(a) - \sigma_P(a)\ln b\right]b^{-1} \tag{158}$$
$$= 0 .$$

It may seem that, in view of (157), the A limit of (158) gives us back our old result (151). This is not the case, however, because the A limit of the (existing) integral

$$\int_0^\infty \frac{d\nu}{\nu} W^{(0)}(\varkappa,\nu) = \frac{\varkappa}{2}\int_0^\infty da\,\sigma_0(a)\ln(\varkappa - a) \tag{159}$$

<u>cannot</u> be taken inside the integration. Indeed, the integral of the (vanishing) A limit of $W^{(0)}$ is zero, whereas the A limit of the integral gives

$$\int_0^\infty \frac{d\nu}{\nu} W^{(0)}(\varkappa,\nu) \xrightarrow[A]{} -\frac{1}{2}\int_0^\infty da\,\sigma_0(a)a . \tag{160}$$

The correct sum rule is therefore

$$\int_0^\infty \frac{d\nu}{\nu}\left[W(\varkappa,\nu) - W_P(\varkappa,\nu)\right] = \frac{\varkappa}{2}\int_0^\infty da\,\sigma_0(a)\ln(\varkappa - a), \tag{161}$$

and, in the A limit,

$$\int_0^1 \frac{d\omega}{\omega}\left[F(\omega) - \beta_P\right] = -\frac{1}{2}\int_0^\infty da\,\sigma_0(a)a . \tag{162}$$

The result (162) also follows immediately from (152) and (86). Equation (162) is satisfied by the free field functions. Note that the right-hand side is, according to (90), a (constant) piece of $\gamma_F(\varkappa)$.

The constant piece $\sigma_o(a)$ of $\sigma(a,b)$ is not needed for anything and our result (151) follows in the simplest situation when it is not present. The experimental result (153) seems to tell us that this simplest possibility is the one obeyed in nature [68].

C.3 Area

The final global quantity we shall consider is the area integral

$$L = 2\int_1^\infty \frac{d\rho}{\rho^2} F(\rho) = 2\int_0^1 d\omega F(\omega) . \tag{163}$$

L is the area under the scaling curve $F(\omega)$ between $\omega = -1$ and $\omega = +1$. In any canonical model with spin $\frac{1}{2}$ charged fields, it satisfies

$$(\delta_{ij} - \hat{p}_i \hat{p}_j) L = -\lim_{p_0\to\infty} \frac{1}{p_0^2} \int d^4x \langle p| [J_i(x), J_j(0)] |p\rangle \epsilon(x_0) . \tag{164}$$

The numerical value of L has been estimated theoretically in Ref. 69) by exploiting the canonical field equations and commutation relations in the gluon model [70] together with a Reggeized theory of symmetry breaking. The result can be stated as

$$L_B^{ab} \equiv 2\int_1^\infty \frac{d\rho}{\rho^2} F_B^{ab}(\rho) = \frac{1}{4} d^{abc} E_B^c . \tag{165}$$

Here $F_B^{ab}(\rho)$ is the double helicity flip scaling function for $B(p)+J^a(q) \to$ $\to B(p)+J^b(q)$, where $(J^a : a = 1-8)$ are the SU(3) currents and B is a member of the $\frac{1}{2}^+$ baryon octet, and $d^{abc}E_B^c$ is the coefficient of $p_i p_j$ in the equal time commutator

$$\int d^3x \langle B(p)| [J_i(0,\underline{x}), J_j^b(0)] |B(p)\rangle .$$

E_B^c is a good SU(3) nonet described by $f \simeq 1/7$, $d \simeq -1/3$, $E^0 \simeq \frac{1}{2}\sqrt{2/3}$. For electroproduction $(J^a = J^b = J^Q = J^3 + (1/\sqrt{3})\, J^8)$ off protons $(B = p)$ and neutrons $(B = n)$, (165) gives

$$L_p^{QQ} = \frac{1}{9}(6f + 2d) + \left(\frac{2}{3}\right)^{3/2} E^0 \simeq 0.31 \tag{166}$$

and

$$L_n^{QQ} = -\frac{4}{9} d + \left(\frac{2}{3}\right)^{3/2} E^0 \simeq 0.24 . \tag{167}$$

Numerically, the evaluation of (165) proceeds according to the decomposition $[F(\rho) \equiv F^{QQ}_{P,n}(\rho)]$ [10)]

$$2\int_1^\infty d\rho\, F(\rho)\rho^{-2} \cong 2\int_1^7 d\rho\, \tilde{F}(\rho)\rho^{-2} + 2\int_7^{16} d\rho\, \hat{F}(\rho)\rho^{-2} + 2\int_{16}^\infty d\rho\, F_R(\rho)\rho^{-2}, \tag{168}$$

where

$$\hat{F}(\rho) = \tilde{F}(7) + \left[F_R(16) - \tilde{F}(7)\right]\frac{\rho-7}{16-7} . \tag{169}$$

[Numerically, we find the unexpected result that $\tilde{F}(7) = F_R(16)$ to three decimal places for both p and n.] The results are [10)]

$$L_p^{exp} \cong 0.23 + 0.06 + 0.04 = 0.33 \tag{170}$$

and

$$L_n^{exp} \cong 0.14 + 0.05 + 0.03 = 0.22 . \tag{171}$$

In view of the experimental uncertainties, the agreement with (166) and (167) is quite good.

II. LIGHT CONE PHYSICS

In this Chapter we proceed from the purely phenomenological analysis of Chapter I, and attempt to understand the observed behaviour in electroproduction in terms of some underlying dynamical formalism. The formalism we will develop here is the theory of operator product expansions (OPE's) near the light cone (LC) with canonical singularities [71]. Because of the numerous exciting reviews [1)-5)] of the fundamentals of the subject, we will be sketchy in parts and concentrate mainly on some recent developments.

A. Electroproduction and the light cone

A.1 Light cone dominance

To obtain a first glimpse of the LC, we consider again the integral representation (I.40) for $W_2(\varkappa, \nu)$. The Fourier transformed equation reads

$$\hat{W}_2(x^2, x\cdot p) = \Box \int da \int db\, \sigma(a,b)\, \Delta(x; a+b^2)\, e^{-ib x\cdot p} \quad . \tag{1}$$

The behaviour of $\Delta(x;\mu^2)$ near the LC $(x^2 \to 0)$ is [72]

$$\Delta(x;\mu^2) \longrightarrow \frac{1}{2\pi}\,\varepsilon(x_0)\left[\delta(x^2) - \frac{1}{4}\mu^2\,\Theta(x^2) + O(x^2)\right] \quad . \tag{2}$$

Because of (I.52), the $\delta(x^2)$ term in (2) does not contribute in (1) and so the leading behaviour of (1) near the LC is

$$\hat{W}_2(x^2, x\cdot p) \longrightarrow \Box \int da \int db\, \sigma(a,b) \left(\frac{-1}{4\pi}\right) a\, \Theta(x^2)\, e^{-ibx\cdot p} . \tag{3}$$

Performing the indicated differentiations in (3), we obtain [17]

$$\hat{W}_2(x^2, x\cdot p) \longrightarrow \delta(x^2)\, \hat{f}_2(x\cdot p)\, \varepsilon(x_0) \;, \tag{4}$$

where

$$\hat{f}_2(\lambda) = \frac{1}{2\pi}\int da \int db\, \sigma(a,b)\, a\,(1 - ib\lambda)\, e^{-ib\lambda} \tag{5}$$

$$= -\frac{1}{2\pi}\int da \int db\, \sigma'(a,b)\, a\, e^{-ib\lambda} , \tag{6}$$

where we used $\sigma(a,1)=0$ [Eq. (I.131)] in obtaining (6) from (5) by integration by parts. Equation (4) exhibits the form of the leading LC singularity of $\hat{W}_2$. We note that if the scaling requirement (I.52) were not imposed, the leading LC singularity would be $\delta'(x^2)$ instead of $\delta(x^2)$.

The coefficient (6) of the leading LC singularity is readily recognizable from the representation (I.53) for $F(\omega)$. We have [17]

$$\int d\lambda\, e^{i\lambda\omega}\hat{f}_2(\lambda) = 2F(\omega) \tag{7}$$

or

$$\hat{f}_2(\lambda) = \frac{1}{\pi}\int d\omega\, e^{-i\lambda\omega}F(\omega)\,. \tag{8}$$

It follows that the leading LC singularity of $\hat{W}_2$ uniquely determines the A limit of W_2; i.e., the scaling function, In fact, if we substitute in

$$W_2(\varkappa,\nu) = \int d^4x\, e^{iq\cdot x}\,\hat{W}_2(x^2, x\cdot p) \tag{9}$$

the leading LC singularity (4) and use (8) and the relation

$$\int d^4x\, e^{iQ\cdot x}\delta(x^2)\varepsilon(x_0) \doteq \delta(Q^2)\,\varepsilon(Q_0) \tag{10}$$

with $Q=q-\eta p$, we obtain

$$2\int d\eta\, F(\eta)\,\delta(\varkappa-2\eta\nu+\eta^2)\,\varepsilon(Q_0) \tag{11}$$

which approaches $1/\nu\; F(\omega)$ in the A limit. It follows that the leading LC singularity of $\hat{W}_2$ dominates the integral in (9) in the A limit [17].

The LC dominance of the A limit can be understood much more generally [17]. Consider an arbitrary scalar absorptive part

$$W(\varkappa,\nu) = \int d^4x\, e^{iq\cdot x}\langle p|[j(x),j(0)]|p\rangle\,. \tag{12}$$

Choose the frame [recall $p=(1,\underset{\sim}{0})$]

$$q = (\nu, 0, 0, \sqrt{\nu^2-\varkappa}\,), \tag{13}$$

and note that in the A limit

$$\sqrt{\nu^2-\varkappa} \xrightarrow[A]{} \nu + \omega + O(\nu^{-1}) \tag{14}$$

so that

$$q\cdot x \xrightarrow[A]{} (x_0 - x_3)\nu + x_3\omega . \tag{15}$$

Now, since $\nu \to \infty$, the exponential in (12) will be highly oscillatory unless $x_0 - x_3 \sim 1/\nu$. It will also be highly oscillatory if x_3 becomes much larger than ω^{-1}. [These statements ignore the region where the two terms in (15) cancel. See Ref. 5) for arguments which dispose of this region.] Therefore, the region of configuration space which is expected to be important in the A limit satisfies $(\underline{x}^2 = x_1^2 + x_2^2)$

$$x^2 = (x_0 - x_3)(x_0 + x_3) - \underline{x}^2 \sim \frac{1}{\nu}\frac{2}{\omega} - \underline{x}^2 . \tag{16}$$

Since the integrand in (12) vanishes by causality for $x^2 < 0$, we are left with

$$x^2 \sim \frac{1}{|\varkappa|} \longrightarrow 0 \tag{17}$$

as the dominating region in x space in the A limit.

The above argument is of course not rigorous. It is only meant to be suggestive. The reader is referred to Ref. 5) for a discussion of some weaknesses in the argument and for attempts to improve it. At the present time, however, LC dominance is only a plausibility. Because it seems both experimentally correct and theoretically interesting, we will henceforth assume LC dominance of the A limit and its generalization.

Given the validity of the above phase arguments, it may appear that causality is necessary to obtain LC dominance. This requirement is, however, only apparent in the above discussion because of our use of the single Lorentz frame in which (13) is valid. By exploiting the full Lorentz invariance, phase arguments can be given which establish LC dominance also for non-causal amplitudes [61),71)].

By working similarly with the representation (I.41) for W_1, we obtain a leading LC behaviour of the form [17)]

$$\hat{W}_1(x^2, x\cdot p) \longrightarrow \delta'(x^2)\,\varepsilon(x_0)\,\hat{f}_1(x\cdot p) , \tag{18}$$

and the representations (I.38) give [17)]

$$\hat{V}_2(x^2, x\cdot p) \longrightarrow \theta(x^2)\,\varepsilon(x_0)\,K_2(x\cdot p), \tag{19}$$

$$\hat{V}_1(x^2, x\cdot p) \longrightarrow \delta(x^2)\,\varepsilon(x_0)\,K_1(x\cdot p). \tag{20}$$

Substitution into (I.34) gives the leading LC behaviour of the matrix element $\langle p|[J_\mu(x), J_\nu(0)]|p\rangle$ [73]. The details can be found in Ref. 17).

A.2 Consequences

In this section we shall recall several consequences of the LC dominance of the A limit. The first is a new derivation of the connection between the A and R limits, the second is a simple interpretation of the ρ' variable, and the third is a configuration space picture of high energy scattering.

To begin, we consider again the scalar amplitude (12). The leading LC singularity structure

$$\langle p|[J(x), J(0)]|p\rangle \xrightarrow[x^2 \to 0]{} \delta(x^2)\,\varepsilon(x\cdot p)\,f(x\cdot p) \tag{21}$$

implies the scaling law

$$2\nu W(\varkappa,\nu) \xrightarrow[A]{} F(\omega) \tag{22}$$

with

$$F(\omega) = i\int d\lambda\, e^{-i\lambda\omega} f(\lambda). \tag{23}$$

Given the Regge-like behaviour

$$F(\omega) \xrightarrow[\omega\to 0]{} \beta\,\omega^{-\alpha-1}, \tag{24}$$

it follows from (23) that [17]

$$f(\lambda) \xrightarrow[\lambda\to\infty]{} i\beta g\,\lambda^{\alpha}, \tag{25}$$

where

$$g^{-1} = \left(\sin\frac{\pi\alpha}{2}\right)\Gamma(\alpha+1) . \tag{26}$$

Thus the behaviour of $f(\lambda)$ for large λ is determined by the Regge behaviour of W :

$$W(\varkappa,\nu) \xrightarrow[R]{} \beta(\varkappa)(2\nu)^{\alpha} . \tag{27}$$

These observations can actually be used to rederive the connection between (24) and (27) [17]. The Regge behaviour (27) should hold provided $\nu \gg |\varkappa|$. If we also take $|\varkappa| \gg 1$, then the phase arguments of Section II.A.1 again give LC dominances [cf., Eq. (17)] and so we have

$$W \underset{\nu\gg|\varkappa|\gg 1}{\sim} \frac{i}{2\nu}\int d\lambda\, e^{i\lambda\varkappa/2\nu} f(\lambda) . \tag{28}$$

Comparison with (27) then requires (25) and

$$\beta(\varkappa) \xrightarrow[\varkappa\to\infty]{} (-\varkappa)^{-\alpha-1}\beta , \tag{29}$$

The behaviour (25) in turn applied in (23) then implies (24) and so the commutativity is again established.

It is useful to compare the above analysis with the analysis based on integral representations [17]. For the present scalar case, we have

$$W(\varkappa,\nu) = \int_0^\infty da \int_{-1}^{1} db\, \sigma(a,b)\,\delta(\varkappa+2b\nu-a) . \tag{30}$$

We assume as before that $\sigma(a,b)$ vanishes rapidly for large a. The behaviour

$$\sigma(a,b) \xrightarrow[b\to 0]{} \sigma(a)|b|^{-\alpha-1} \tag{31}$$

gives (27) with

$$\beta(\varkappa) = \int da\, \sigma(a)(a-\varkappa)^{-\alpha-1} \tag{32}$$

so that (29) is satisfied with

$$\beta = \int da\, \sigma(a) . \tag{33}$$

The scaling behaviour (22) is also obtained, with

$$F(\omega) = \int da\, \sigma(a,\omega) \tag{34}$$

and so (24) follows from (31) and (33). Finally, the Fourier transform of (30) gives the leading LC behaviour (21) with

$$f(\lambda) = \frac{-i}{4\pi}\int da\, db\, \sigma(a,b) e^{-ib\lambda} \,, \tag{35}$$

from which (25) immediately follows if use is made of (31).

We proceed to discuss the non-leading contributions to the A limit (22) [74),75)]. In the canonical framework, we have

$$2\nu W(\varkappa,\nu) \xrightarrow[A]{} F(\omega) + \frac{1}{\nu}F_1(\omega) + \frac{1}{\nu^2}F_2(\omega) + \dots \,. \tag{36}$$

In general, the first non-leading term $\nu^{-1}F_1(\omega)$ receives contributions from two distinct sources - the leading contribution of the first non-leading LC singularity $[\theta(x^2)\,\varepsilon(x_0)f_1(x\cdot p)]$ and the first non-leading contributions of the leading LC singularity. The latter contribution is easily computable from the form

$$W_L \equiv \int d^4x\, e^{iq\cdot x}\, \delta(x^2)\,\varepsilon(x_0)\, f(x\cdot p) \tag{37}$$

of the leading LC contribution to W. Using (23), (36) gives

$$W_L \xrightarrow[A]{} (2\nu + 2\omega)^{-1} F(\omega_L) + O(\tfrac{1}{\nu^2}) \tag{38}$$

where

$$\omega_L = \omega - \frac{\omega^2}{2\nu} \tag{39}$$

is the contributing root of $\varkappa - 2b\nu + b^2 = 0$ to order $1/\nu$ [76)]. The variable ω_L which has emerged from the leading LC contribution is effectively the same in the relevant threshold region $\omega \sim 1$ as the variable

$$\omega' = \frac{1}{\rho'} = \omega - \frac{\omega}{2\nu} + O(\tfrac{1}{\nu^2}) \tag{40}$$

which provided the early scaling. The first source of contributions to $\nu^{-1}F_1(a)$ is thus empirically very small and the relevance of the variable (40) means that the leading LC singularity dominates already at $|\mathcal{H}| = 1$ GeV^2.

Equation (38) gives

$$2\nu W_L(\mathcal{H},\nu) \xrightarrow[A]{} F(\omega) - \frac{\omega}{\nu}F(\omega) - \frac{\omega^2}{2\nu}F'(\omega) . \qquad (41)$$

For the physical case of vector currents, the corresponding statement is [74]

$$\left[\nu W_2(\mathcal{H},\nu)\right]_L \xrightarrow[A]{} \left(1 - \frac{3\omega}{2\nu}\right)^{-1} F_2\left(\omega - \frac{\omega^2}{2\nu}\right) + O(\nu^{-2}) . \qquad (42)$$

We see that the leading LC contribution is not quite the simple replacement $\omega \to \omega'$. Nevertheless, the form (42) fits the data as well as the simpler $F(\omega')$. The natural appearance of empirically accurate corrections to scaling is further testament to the advantages of a configuration space approach.

A consequence of the behaviour (25) is that the leading LC singularity (21) determines the $t = 0$ intercept α of the leading Regge trajectory. The LC thus provides a configuration space picture of high energy behaviour at $t = 0$. Regge theory also describes what happens away from $t = 0$ and these constraints can be similarly imposed [77]. It is found that the coefficient of the leading LC singularity determines the entire trajectory function $\alpha(t)$ [77),78].

A.3 Model

In this section we shall show how the so-called "non-perturbative parton model" [79] illustrates the scaling and LC behaviour discussed above. This model includes as special cases the usual naïve parton model [80] and the multiperipheral model [81]. For simplicity, we will here consider a scalar version of the model. The generalization to vector electromagnetic currents, composed of either scalar ($\varphi^{\dagger}\overleftrightarrow{\partial}_\mu\varphi$) or spinor ($\bar{\psi}\gamma_\mu\psi$) fields, is straightforward [79]. We shall refer to the basic fields $\varphi(x)$ in the theory as quarks even though they have zero spin.

Two basic assumptions are made in the formulation of the model :

1) the currents have the form

$$j(x) = :\varphi(x)\varphi(x): \quad , \qquad (43)$$

with the quark fields $\varphi(x)$ obeying canonical equal time commutation relations

$$\delta(x_0)[\dot{\varphi}(x), \varphi(0)] = -i\,\delta^4(x) , \tag{44}$$

2) hadron-quark scattering amplitudes decrease rapidly as the (off-shell) quark mass becomes large.

Neither of these assumptions is true in any renormalizable [82] field theory in perturbation theory [83].

An immediate consequence of the first assumption is that the exact propagator function

$$D_F(k^2) \equiv \int d^4x\, e^{ik\cdot x} \langle 0|T[\varphi(x)\varphi(0)]|0\rangle \tag{45}$$

has the behaviour

$$D_F(k^2) \xrightarrow[k^2 \to \infty]{} \frac{1}{k^2} . \tag{46}$$

The exact rate of decrease of the hadron-quark amplitudes need not be specified - as long as the decrease is fast enough.

Given (43), the exact current-hadron amplitude

$$T(\mu,\nu) = \int d^4x\, e^{iq\cdot x} \langle p|T[j(x)j(0)]|p\rangle \tag{47}$$

(Fig. 12a) has the form shown in Fig. 12b, where the four upper solid lines are the eaxct quark propagators (45) and the two lower lines represent the hadron of momentum p. The amplitude of Fig. 12b is the sum of the contribution (Fig. 12c) of the disconnected (2 hadron)-(4 qurak) amplitude and the contribution (Gif. 12d) of the connected (2 hadron)-(4 quark) amplitude. It can be shown that, as a consequence of the above assumptions, the dominant term in the A limit comes entirely from the disconnected contribution of Fig. 12c [79]. We will not derive this result here but only note an intuitive explanation. The large momentum q carried in by the external current must go into the quark lines attached to the current. As the mass of any of the quark lines in Fig. 12d becomes large, however, the value of the contribution decreases rapidly ny assumption 2). Thus the contribution of Fig. 12b vanishes rapidly in the A limit. In Fig. 12c, on the contrary, the quark propagator connecting the two currents only decreases as $(k^2)^{-1}$ for large mass k^2 [Eq. (46)] and so the large momentum q can go through this line without causing the contribution to decrease rapidly.

The assumptions further imply that the contribution of Fig. 12c scales correctly [79] and so the model satisfies the scaling law (22) with the scaling function $F(\omega)$ given as an explicit function of the imaginary part of the quark-hadron scattering amplitude $T(s',k^2)$, $s' \equiv (p-k)^2$ (Fig. 13) :

$$F(\omega) \sim \omega^2 \int du\, dv\; \mathrm{Im}\, T\left((\omega-1)(u-1)+v,\; \omega u + v\right). \tag{48}$$

In a similar way, the expected scaling laws (I.18) and (I.26) are satisfied in the case of vector currents.

The contribution of Fig. 12c is precisely the one which provides the leading LC singularity. It can be written as [84]

$$\int d^4x\, e^{iq\cdot x}\, D_F(x) \langle p | T[\varphi(x)\varphi(0)] | p\rangle, \tag{49}$$

in terms of the x space quark propagator $D_F(x) = \langle 0|T[\varphi(x)\varphi(0)]|0\rangle$ and the x space quark-hadron scattering amplitude shown in Fig. 13 :

$$\langle p | T[\varphi(x)\varphi(0)] | p\rangle \equiv f(x\cdot p, x^2). \tag{50}$$

Equation (49) has the leading LC singularity $\sim(1/x^2) f(x\cdot p, 0)$ which is the matrix element of the leading term in the LC expansion

$$j(x)\, j(0) \xrightarrow[x^2 \to 0]{} \Delta_+(x) : \varphi(x)\varphi(0) : \;. \tag{51}$$

We note that in this model the Regge-like behaviour of the scaling function $F(\omega) \sim \int d\lambda\, e^{i\lambda\omega} f(\lambda, 0)$ for small ω corresponds to the ordinary Regge behaviour of the quark-hadron amplitude $T(s',k^2)$ for $s' \to \infty$ at fixed k^2 [79),85)].

In spite of its nice features, it must be emphasized that the model is only illustrative and cannot be taken as a serious candidate for an exact description of nature. One of the reasons for this is that the scaling behaviour of the model comes about <u>entirely</u> because of the large $(\sim\nu)$ separation in momentum space between the produced particles (intermediate states) in the upper quark line in Fig. 12c and the produced particles (intermediate states) in the lower quark-hadron amplitude. All of the incident current energy goes into producing a spray of fast (energy $\sim\nu$) particles with quark quantum numbers, even if the propagator (45) has no pole so that quarks themselves are not produced. This cannot be got around - the separation sets in as soon as scaling does. The presumed empirical absence of this effect unfortunately means that the mechanism responsible for the scaling properties of the model is not the mechanism at work in nature.

B. Models for light cone expansions

B.1 Operator product expansions and scale invariance

In renormalized perturbation theory [86], in soluble field theoretic models [87], and axiomatically [88),89], operator product expansions describe the behaviour of products $A(x)B(0)$ of local field operators at short distances. These expansions have the form

$$A(x)\,B(0) \xrightarrow[x\to 0]{} \sum_{i=0}^{N} F_i(x)\,O_i(0)\,, \tag{52}$$

where $O_1,\dots,O_N$ is a finite set of local field operators and the $F_i(x)$ are functions with singularities for $x\to 0$.

In a scale invariant theory, the degree of the singularities of the $F_i(x)$ is simply given in terms of the dimensions of the field operators. In a scale invariant theory [90], there exists a one-parameter group $U(s)$ of unitary transformations such that each local field $\chi(x)$ satisfies [91]

$$U(s)\,\chi(x)\,U^{-1}(s) = s^d \chi(sx) \tag{53}$$

for some real number d. The field χ is then said to have (dynamical) dimension d and we write $\dim\chi = d$. Applying this scale transformation to Eq. (52), we obtain

$$F_i(x) \sim \left(\frac{1}{x}\right)^{d_A + d_B - d_i}\,, \tag{54}$$

where $\dim A = d_A$, $\dim B = d_B$, $\dim O_i = d_i$.

The generator D of scale transformations, defined by

$$U(s) = e^{-i(\ln s)D}\,, \tag{55}$$

satisfies

$$i[D, \chi(x)] = (d + x\cdot\partial)\,\chi(x)\,. \tag{56}$$

In canonical theories, D is given formally as the charge

$$D = \int d^3x\, D_0(x) \tag{57}$$

of the dilatation current

$$D_\mu(x) = x^\nu \Theta_{\mu\nu}(x) , \tag{58}$$

where $\Theta_{\mu\nu}$ is the stress energy tensor. We also have the commutation relations

$$i[D, P^\mu] = P^\mu \tag{59}$$

with the generator P^μ of space time translations.

The theories of interest to us will not necessarily be scale invariant, but will have (or essentially have) a scale invariant limit. The leading short distance singularities will be mass independent and will therefore be determined by dimensional analysis in the scale invariant limit.

B.2 Free field results

Consider a free scalar field $\phi(x)$. It satisfies

$$(\Box + m^2)\varphi(x) = 0 , \quad [\varphi(x), \varphi(y)] = -i\Delta(x-y; m^2) I , \tag{60}$$

where we have explicitly indicated the unit operator by I to emphasize that the commutator is a c number. As is well known, the Wick product

$$j(x) \equiv :\varphi(x)\varphi(x): = \lim_{\xi \to 0} [\varphi(x+\xi)\varphi(x) - \Delta_+(\xi; m^2)] \tag{61}$$

exists as a finite local field operator.

In our free field theory, we can define non-local Wick products such as $:\phi(x)\phi(y):$ by the usual method of putting all creation operators on the left. When this is done, the short distance limits such as $\lim_{x \to y} :\phi(x)\phi(y): = j(x)$ will all exist. Wick's theorem then states that

$$\varphi(x)\varphi(y) = :\varphi(x)\varphi(y): + \Delta_+(x-y; m^2) . \tag{62}$$

Another example is

$$\begin{aligned} j(x)j(y) = {} & 2\Delta_+(x-y; m^2)\Delta_+(x-y; m^2) + 4\Delta_+(x-y; m^2) :\varphi(x)\varphi(y): \\ & + :\varphi(x)\varphi(x)\varphi(y)\varphi(y): . \end{aligned} \tag{63}$$

Equation (61) can be rewritten as

$$\varphi(x)\varphi(0) \xrightarrow[x\to 0]{} \Delta_+(x) I + j(0) \tag{64}$$

$$\sim \frac{-1}{4\pi^2}\frac{1}{x^2} I + j(0) \tag{65}$$

Here and elsewhere x^2 means $x^2 - i\varepsilon x_0$. This has the form (52). The nature of the expansion (65) can be described in terms of the above "dimensionality" concept, with $\dim I = 0$, and $\dim \phi = 1$. Also $\dim j = 2$ and $\dim \partial_\alpha \phi = 2$.

The behaviour of the product of any two local fields in the theory can be determined in a similar way. One simply expands in terms of all other local fields with dimensions small enough to give singularities. It follows, for example, from (63) that

$$j(x)j(0) \xrightarrow[x\to 0]{} c_0\left(\frac{1}{x^2}\right)^2 I + c_1\left(\frac{1}{x^2}\right) j(0) + c_2\left(\frac{1}{x^2}\right) x^\alpha :\varphi\partial_\alpha\varphi: + c_3 :jj: \,. \tag{66}$$

The description of the behaviour of $j(x)j(y)$ near the light cone $(x-y)^2 \to 0$, rather than at short distance $(x-y)^\mu \to 0$, is somewhat more complicated. This can be seen from Eq. (66). For $x^\mu \to 0$, $(1/x^2)$ is a power more singular than $(1/x^2)x^\alpha$. Near the light cone, however, since $x^2 \to 0$ but $x^\alpha \not\to 0$, each function has the same singularity. In fact, it easily follows from (63) that, if the LC behaviour is expressed in terms of local operators, an infinite number of such operators occur to carry the $(1/x^2)$ singularity. The result has the form [71]

$$j(x)j(y) \xrightarrow[LC]{} c_1\left(\frac{1}{\xi^2}\right)^2 I + \frac{1}{\xi^2}\sum_{n=0}^{\infty} \xi^{\alpha_1}\cdots\xi^{\alpha_n} O^{(n)}_{\alpha_1\cdots\alpha_n}(\eta) \tag{67}$$

where we have introduced the variables

$$\xi = \tfrac{1}{2}(x-y)\,, \qquad \eta = \tfrac{1}{2}(x+y)\,, \tag{68}$$

and $\xrightarrow[LC]{}$ means $\xrightarrow[\xi^2\to 0]{}$. Here $\dim O^{(n)} = n+2$ so that each term in the sum has dimension two and carries a LC singularity $1/\xi^2$.

In the present free field model, (67) can be verified directly from (63) [92]:

$$j(x)j(y) \xrightarrow[LC]{} \frac{1}{4}\left[\Delta_+(\xi)\right]^2 + \Delta_+(\xi) :\varphi(x)\varphi(y): \;, \tag{69}$$

or

$$j(x)j(y) \xrightarrow[LC]{} \frac{1}{4}\left[\Delta_+(\xi)\right]^2 + \Delta_+(\xi)\sum_m \frac{1}{m!}\xi^{\alpha_1}\cdots\xi^{\alpha_m} :\varphi(\eta)\overleftrightarrow{\partial}_{\alpha_1}\cdots\overleftrightarrow{\partial}_{\alpha_m}\varphi(\eta): \;. \tag{70}$$

B.3 Perturbation theory and Thirring model

Let us now ask what happens to the simple free field LC expansion (67) if interactions are turned on. If the interaction is a renormalizable one such as $\lambda\varphi^4$, then, in each order of perturbation theory, the form of (67) is only altered by logarithmic factors and one obtains [71),88)]

$$j(x)j(0) \xrightarrow[x^2\to 0]{} d_c\left(\frac{1}{x^2}\right)^2\left(\ln x^2\right)^{a_c} + \frac{1}{x^2}\sum_n d_{in}\left(\ln x^2\right)^{a_{in}} x_{\alpha_1}\cdots x_{\alpha_n} O_i^{\alpha_1\cdots\alpha_n}(0) \,. \tag{71}$$

Here a_c and a_{in} are integers which depend, in general, on the order of perturbation theory and $O_i^{\alpha_1\cdots\alpha_n}(x)$, $(i = 1,2,\ldots,I_n,\;\; n = 1,2,\ldots)$ are suitably defined local fields [88)]. Now $j(x)$ is not given simply by (61) but requires a more complicated definition. We see that in any finite order of a renormalizable perturbation theory, because of the occurrence of logarithmic factors, the renormalized fields do not have well-defined dynamical dimensions. Nevertheless, the short distance behaviour of any Wightman function is, apart from logarithmic factors, the same as it would be if the fields did have canonical dynamical dimensions. Put differently, the short distance behaviour is determined, apart from log's, by the naïve dimensions of the fields. In particular, the nature of short distance expansions and of LC expansions are so determined. Similar results hold for all OPE's of all currents in all renormalizable theories [86)-91)].

The interesting question associated with the occurrence of the logarithms is the nature of their sum over all orders of perturbation theory. The log's could, in principle, sum up to a power according to

$$1 + \alpha\ln z + \frac{1}{2}\alpha^2\ln^2 z + \cdots = e^{\alpha\ln z} = z^\alpha \;, \tag{72}$$

and thus reinstate the scale invariance of the theory at the LC. This is precisely what happens in the Thirring model [93]. The Thirring model [94] is a two-dimensional (one space and one time) relativistic field theory involving a massless Dirac field $\psi(x)$ (two components) coupled to itself by the current-current interaction $\lambda j^{\mu}(x) j_{\mu}(x)$ with $j = \bar{\psi}\gamma_{\mu}\psi$. The exact solution is scale invariant [93),95] with

$$\dim \psi = \frac{1}{2} + \frac{\lambda^2}{4\pi^2}\left(1 - \frac{\lambda^2}{4\pi^2}\right)^{-1} \tag{73}$$

and

$$\dim :\bar{\psi}(1 \pm \gamma^5)\psi: = \left(1 - \frac{\lambda}{2\pi}\right)\left(1 + \frac{\lambda}{2\pi}\right)^{-1} . \tag{74}$$

The dimensions of the fields and currents are seen to be coupling constant dependent and consequently so are the degrees of singularities in OPE's [93),96]. The dimensions take on their free field values $(\dim\psi = \frac{1}{2}\dim\bar{\psi}\psi = 1)$ only in the free field limit $\lambda = 0$.

It is unknown whether or not a similar power behaviour obtains in conventional four-dimensional non-trivial theories. Leading logarithmic summations of ladder graphs scale with anomalous dimensions [97], but this is not the case for leading logarithmic summations of all graphs [98),99]. It is possible that scale invariance comes about via the existence of a Gell-Mann - Low eigenvalue [100),99], and conformally invariant theories of this type have been constructed [101], but it seems difficult to understand Bjorken scaling in this framework.

C. Canonical formalism

C.1 Lessons from SLAC

The results discussed in Section II.B.3 make it clear that the dimensions of fields are dynamical quantities which cannot, in general, be determined *a priori*. In order to determine what are the dimensions of the local fields (presumably) encountered in nature, one must turn to experiment. The most precise information about dimensions comes from the SLAC-MIT data. We will show here how these data strongly support the idea that nature's fields have canonical dimensions [71].

In Section II.A.1, we have exhibited the LC singularities for the (spin averaged) nucleon matrix elements of the product $J_\mu(x)J_\nu(0)$, which were equivalent to the scaling laws (I.18) and (I.26) experimentally verified in the SLAC-MIT electroproduction experiments. The important thing to notice here is that these LC singularities are of integral power. The degrees of the singularities are, in fact, precisely those given by canonical dimensionality.

One cannot, however, conclude from the SLAC proton data that canonical dimensions are to be expected in general. The reason is that conserved quantities (with non-vanishing charges) have canonical dimensions and only such (or related) quantities might be seen at SLAC. The electromagnetic currents $J_\mu(x)$ have their canonical dimension 3. In the expansion of $J_\mu(x)J_\nu(0)$ there can occur another conserved field - the stress energy tensor $\Theta_{\mu\nu}(x)$ of (canonical) dimension 4 [e.g., $P_\mu = \int d^3x \Theta_{0\mu}(x)$ has dimension 1]. Thus

$$J_\mu(x)J_\nu(0) \xrightarrow[LC]{} \frac{c}{x^2}\Theta_{\mu\nu}(0) + \cdots , \tag{75}$$

and the leading LC singularities might well be canonical even if (most) fields in nature do not have canonical dimension.

The above explanation of how the Bjorken scaling laws could be valid in a non-canonical world does not, hwoever, really work. The reason is that the electron-neutron data also satisfy the scaling laws, and the neutron scaling function is definitely different from the proton-scaling function. This <u>cannot</u> be explained by (75) because $\Theta_{\mu\nu}$ has $I=0$ and so gives equal contribution to protons and neutrons. There must therefore be <u>another</u> infinite string of local fields in (75) with canonical dimensions, but with $I=1$. There is, however, no known conserved (or partially conserved, which would do as well) $I=1$ tensor. It therefore seems that there is at least one (and hence, by causality, an infinite number) of non-partially conserved local fields with canonical dimension. This suggests that all fields in nature have canonical dimensions.

C.2 Canonical light cone expansions

Consider again the scalar currents $j(x) \equiv :\phi(x)\phi(x):$ in $\lambda\phi^4$ theory. Motivated by the empirical results described in the previous section, we assume canonical (i.e., free field) dimensions for all currents. It then follows that the LC behaviour is (67). The specific form of the O's can be obtained from consideration of the short distance $(x_\mu \to 0)$ expansions of all products $\partial_{\alpha_1},\ldots,\partial_{\alpha_n} j(x)j(0)$ [71] or, alternatively, from the highest spin contributions to the equal time commutators (ETC) $[\partial_{\alpha_1},\ldots,\partial_{\alpha_n} j(x), j(0)]\times \delta(x_0)$. These commutators can be formally evaluated by using the canonical commutation relations for the fields $\phi(x)$ and the equation of motion $(\Box + m^2)\phi(x) = \lambda :\phi(x)^3:$ to eliminate higher time derivatives. The results for the leading LC singularity are obviously independent of the interaction term, since, for a given dimension, the leading LC singularity is carried by fields with the most Lorentz indices and these come from the kinetic term $\phi\partial_\alpha\partial^\alpha\phi$ [e.g., the interaction contribution $:\phi\phi:\lambda:\phi\phi:$ cannot carry a LC singularity, whereas the free contribution $:\phi\,\partial_\alpha\partial_\beta\phi:$ of the same dimension (four) carries a LC singularity $x^\alpha x^\beta/x^2$] and so the free field expansion remains formally valid. This free field expansion follows simply from Wick's theorem to be given by Eq. (70). Expansions of the type (70), which uniquely follow from the assumptions of canonical commutators and field equations, will be referred to as canonical LC expansions. They have been extensively used in Refs. 102)-104).

For actual physical applications, we will use the canonical gluon model, in which the quark fields Ψ interact via a neutral vector meson B_μ coupled to the baryon number. The interesting currents are the vector, axial vector, scalar and pseudoscalar ones

$$V_\mu^a = \frac{1}{2}:\bar\Psi\gamma_\mu\lambda^a\Psi: \qquad (76.a)$$

$$A_\mu^a = \frac{1}{2}:\bar\Psi\gamma_\mu\gamma_5\lambda^a\Psi: \qquad (76.b)$$

$$S^a = \frac{1}{2}:\bar\Psi\lambda^a\Psi: \qquad (76.c)$$

$$P^a = \frac{1}{2}:\bar\Psi\gamma_5\lambda^a\Psi: \qquad (76.d)$$

of dimension three. The free field expansions, for example

$$V^0(x)V^0(y) \xrightarrow[\xi^2\to 0]{} \frac{\partial}{\partial\xi_\alpha}\Delta_+(\xi)\Big[g_{\mu\alpha}O_\nu^{[-]}(\xi,\eta) + g_{\nu\alpha}O_\mu^{[-]}(\xi,\eta) - g_{\mu\nu}O_\alpha^{[-]}(\xi,\eta) + i\,\varepsilon_{\mu\nu\alpha\beta}O_{5\beta}^{[+]}(\xi,\eta)\Big], \qquad (77)$$

$$O_\mu^{[\pm]}(\xi,\eta) = \sum_{ij}\frac{1}{i!\,j!}\xi^{\alpha_1}\cdots\xi^{\alpha_i}\xi^{\beta_1}\cdots\xi^{\beta_j}:\bar\Psi(\eta)\gamma_\mu\overleftarrow{\partial}_{\alpha_1}\cdots\overleftarrow{\partial}_{\alpha_i}\overrightarrow{\partial}_{\beta_1}\cdots\overrightarrow{\partial}_{\beta_j}\Psi(\eta): \pm (x\leftrightarrow y), \qquad (78)$$

are now, however, altered by the interaction term, since, for example, $:\bar{\Psi}\partial_\alpha\partial_\beta\Psi:$ and $g^2:\bar{\Psi}B_\alpha B_\beta\Psi:$ can carry the same (leading) LC singularity. The effect of the interaction can, however, be simply accounted for by invoking the invariance of the theory under the gauge transformation [71]

$$\Psi(x) \longrightarrow e^{ig\Lambda(x)}\Psi(x), \qquad B_\mu(x) \longrightarrow B_\mu(x) + \partial_\mu\Lambda(x), \tag{79}$$

$$(\Box + \mu^2)\Lambda(x) = 0 .$$

The result [71] is simply to replace the derivatives ∂_ν in (78) by the gauge invariant derivatives

$$\Delta_\nu = \partial_\nu - igB_\nu . \tag{80}$$

The resulting expansions are then the unique ones which follow from the assumptions of canonical commutators and field equations.

In the free quark model, the bilocal form of the LC commutation relations has been given by Fritzsch and Gell-Mann, who showed that many of the parton model results follow from the structure of the expansions [105].

Let us next address ourselves to the problem of incorporating the constraints of current conservation and partial current conservation on LC operator product expansions (LCOPE's) [106]. To illustrate the problem, consider the LC behaviour of the product of a conserved current J_μ and another local operator K :

$$J_\mu(x)K(y) \xrightarrow[\xi^2\to 0]{} \sum_i E_i(\xi)\, O_{\mu i}(\xi,\eta) . \tag{81}$$

In all cases of interest, the corresponding LCOPE for the T product is [61),71),103]

$$T[J_\mu(x)K(y)] \xrightarrow[\xi^2\to 0]{} \sum_i \tilde{E}_i(\xi)\, O_{\mu i}(\xi,\eta) , \tag{82}$$

where $\tilde{E}_i(\xi)$ is obtained from $E_i(\xi)$ by replacing $\xi^2 - i\varepsilon\xi_0$ by $\xi^2 - i\varepsilon$.

Strictly speaking, current conservation $\partial^\mu J_\mu = 0$ only requires that the coefficient of the leading LC singularity of

$$\left(\frac{\partial}{\partial x}\right)^\mu \left[\sum_i E_i(\xi)\, O_{\mu i}(\xi,\eta)\right]$$

vanishes. It is, however, extremely convenient [61),71),103)] to satisfy the exact conservation condition

$$\left(\frac{\partial}{\partial x}\right)^{\mu}\left[\sum_{i} E_i(\xi)\, O_{\mu i}(\xi,\eta)\right] = 0 . \tag{83}$$

Suppose further that the ETC

$$\left[J_0(x), K(y)\right]\delta(x_0-y_0) = L(x)\,\delta^4(x-y) \tag{84}$$

does not vanish. Then one has the operator Ward identity

$$\left(\frac{\partial}{\partial x}\right)^{\mu} T\left[J_\mu(x)K(y)\right] = L(x)\,\delta^4(x-y), \tag{85}$$

and it is again very convenient to have (82) satisfy this constraint exactly :

$$\left(\frac{\partial}{\partial x}\right)^{\mu}\left[\sum_{i} \tilde{E}_i(\xi)\, O_{\mu i}(\xi,\eta)\right] = L(x)\,\delta^4(x-y) . \tag{86}$$

We will show how this and the related constraints of partial current conservation (PCC) can be accomplished.

For the case of Abelian (ET commuting) currents such as the electromagnetic current

$$J_\mu^{\gamma} = V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8 , \tag{87}$$

the manifestly conserved form of the LCOPE was given in Refs. 61) and 71). It is

$$\begin{aligned} J_\mu^{\gamma}(x)\, J_\nu^{\gamma}(y) \xrightarrow[\xi^2\to 0]{} & \left(\partial_\mu^x \partial_\nu^y - g_{\mu\nu}\, \partial^x\cdot\partial^y\right) E_0(\xi^2) \\ & + \left(\partial_\mu^x \partial_\nu^y - g_{\mu\nu}\, \partial^x\cdot\partial^y\right)\left[E_1(\xi^2)\, O_1(x;y)\right] \\ & + \left(g_{\mu\nu}\partial_\alpha^x\partial_\beta^y - g_{\mu\alpha}\partial_\nu^x\partial_\beta^y - g_{\beta\nu}\partial_\alpha^x\partial_\mu^y + g_{\beta\nu}g_{\alpha\mu}\,\partial^x\cdot\partial^y\right)\left[E_2(\xi^2)\, O_2^{\alpha\beta}(x;y)\right] \end{aligned} \tag{88}$$

In (88), the explicit current conservation is achieved by including suitably many non-leading LC singularities. As we will see, the corresponding conserved form for non-Abelian currents is rather more involved. It contains infinitely many non-leading LC singularities.

To illustrate the procedure used in Ref. 106) for the non-Abelian case, we consider the typical LCOPE

$$V_\mu^a(x)\, S^b(y) \xrightarrow[\xi^2\to 0]{} [\partial^\nu \Delta_+(\xi)]\, \mathcal{A}^{ab}(x;y) + \ldots , \tag{89}$$

or

$$[V_\mu^a(x),\, S^b(y)] \longrightarrow [\partial^\nu \Delta(\xi)]\, \mathcal{A}^{ab}(x;y) + \ldots , \tag{90}$$

where we have only kept the bilocal operator relevant to our immediate discussion. Note that

$$\mathcal{A}^{ab}(x;x) = f^{abc}\, S^c(x) . \tag{91}$$

We also have

$$T[V_\mu^a(x)\, S^b(y)] \longrightarrow [\partial^\nu \Delta_F(\xi)]\, \mathcal{A}^{ab}(x;y) + \ldots . \tag{92}$$

Now

$$\partial_x^\mu\, T[V_\mu^a(x)\, S^b(y)] = -i f^{abc}\, S^c(x)\, \delta^4(\xi) , \tag{93}$$

and our problem is to add sufficiently many non-leading terms to the above LCOPE's to satisfy this operator Ward identity. To accomplish this, we introduce a projection operator $\mathcal{D}_\mu((\partial/\partial x),\, \xi^2 - i\varepsilon\xi_0)$ which satisfies the differential operator conditions

$$\left(\frac{\partial}{\partial x}\right)^\mu \mathcal{D}_\mu\left(\frac{\partial}{\partial x},\, \xi^2 - i\varepsilon\xi_0\right) = 0 , \tag{94}$$

$$\left(\frac{\partial}{\partial x}\right)^\mu \mathcal{D}_\mu\left(\frac{\partial}{\partial x},\, \xi^2 - i\varepsilon\right) = -i\, \delta^4(\xi) , \tag{95}$$

and

$$\mathcal{D}_\mu\left(\frac{\partial}{\partial x},\, \xi^2\right) \xrightarrow[\xi^2\to 0]{} \partial_\mu \Delta_+(\xi) . \tag{96}$$

Our procedure is then to use

$$V_\mu^a(x)\, S^b(y) \longrightarrow \mathcal{D}_\mu\left(\frac{\partial}{\partial x},\, \xi^2 - i\varepsilon\xi_0\right) \mathcal{A}^{ab}(x;y) + \ldots \tag{97}$$

and

$$T[V_\mu^a(x)S^b(y)] \longrightarrow \mathcal{D}_\mu\left(\frac{\partial}{\partial x}, \xi^2 - i\varepsilon\right)\mathcal{S}^{ab}(x;y) + \ldots, \tag{98}$$

which satisfies (93) by construction.

The expression derived for the projection operator

$$\mathcal{D}_{+\mu}(\partial,\xi) \equiv \mathcal{D}_\mu\left(\frac{\partial}{\partial x}, \xi^2 - i\varepsilon\xi_0\right) \tag{99}$$

in Ref. 106) is simply

$$\mathcal{D}_{+\mu}(\partial,\xi) = [\partial_\mu \Delta_+(\xi,\Box)] + \Delta_+(\xi,\Box)\partial_\mu . \tag{100}$$

The corresponding time-ordered projection operator

$$\mathcal{D}_{F\mu}(\partial,\xi) \equiv \mathcal{D}_\mu\left(\frac{\partial}{\partial x}, \xi^2 - i\varepsilon\right) \tag{101}$$

is, of course,

$$\mathcal{D}_{F\mu}(\partial,\xi) = [\partial_\mu \Delta_F(\xi,\Box)] - \Delta_F(\xi,\Box)\partial_\mu . \tag{102}$$

These expressions clearly satisfy (94) and (95).

A similar procedure can be followed to ensure current conservation for the product $V_\mu^a(x)V_\mu^b(y)$ of two vector currents in each of the vector indices [106]. We obtain expressions of the form

$$V_\mu^a(x)V_\nu^b(y) \xrightarrow[\xi^2\to 0]{} \mathcal{D}_{+\mu\nu}(\partial^x,\partial^y,\xi)\, i f^{abc}\mathcal{V}^c(x;y) + \ldots . \tag{103}$$

We emphasize the need for an infinity of non-leading terms here. The expression (103) is manifestly conserved in the forward direction [106] [i.e., for diagonal matrix elements $\langle p|\ldots|p\rangle$] and this <u>cannot</u> be achieved in a local way in the non-Abelian case [107]. This means that no finite number of terms will suffice [108].

Given the conserved form (103) of the LC expansion, it is a simple matter to use the LC algebra to derive relations among the deep inelastic neutrino-nucleon structure functions [106),109]. When neutrino data are available, they will provide crucial tests of the above canonical formalism.

To conclude this section on the canonical formalism, we note that it is not consistent [106),110)]. Precisely the same assumptions used above lead to a contradiction with naïve scale invariance. To see this, consider, for example, $\lambda\phi^4$ theory. Some canonical results are [111)]

$$[\partial_0^3\varphi(x),\varphi(0)]_T\,\delta(x_0) = 3i\lambda j(0)\,\delta^4(x)\,, \tag{104}$$

$$[\partial_0 j(x), j(0)]_T\,\delta(x_0) = 2i\, j(0)\,\delta^4(x)\,, \tag{105}$$

where

$$j(x) = :\varphi(x)\varphi(x): \tag{106}$$

$$= \lim_{\xi\to 0}\left[\varphi(x+\xi)\varphi(x) - \langle 0|\varphi(\xi)\varphi(0)|0\rangle\right. . \tag{107}$$

These expressions are, however, mutually inconsistent since (104) implies that a term of the form $\lambda(\ln x^2)j(0)$ is present in the short distance expansion of $\phi(x)\phi(0)$ so that (107) does not exist. Thus, contrary to what is frequently said, canonical computation does <u>not</u> imply the existence (apart from c numbers) of field products at the same point.

We see that canonical evaluation cannot be carried too far without running into conflict with naïve scale invariance. The canonical result (104) implies the short distance expansion

$$\varphi(x)\varphi(0) \xrightarrow[x\to 0]{} \Delta_+(x)I + [1+3\lambda\Gamma_+(x)]\,j(0) + \cdots\,, \tag{108}$$

$$\Gamma_+(x) \propto \ln(-x^2+i\epsilon x_0)\,. \tag{109}$$

This means that the local scalar current of dimension two cannot be identified with (107) and that there is a manifest inconsistency between canonical evaluation and naïve scale invariance.

These difficulties with the naïve canonical formalism persist at the current-current level. The naïvely expected canonical result (69) is incorrect because the bilocal operator $:\phi(x)\phi(0):$ has singularities on the LC.

The behaviour of the gluon model is similar. There we find, for example [87),112),106),110)]

$$\bar{\Psi}(x)\gamma_\mu\lambda^a\Psi(0) \xrightarrow[x\to 0]{} \delta^{a0}\left[\partial_\mu\Delta_+(x)\right] e^{-ig\int_0^x d\xi\cdot B(\xi)} + \dots$$
$$+ g^2\Gamma_+(x) J_\mu^a(0) + \dots \quad . \tag{110}$$

We see that in order to define J^0_μ, we have to subtract from $\bar{\Psi}\gamma_\mu\lambda^0\Psi$ a number of operator terms with singular coefficients and divide by a logarithmically singular factor. To define J^a_μ for $a = 1-8$, only the division is necessary. We see here again that formal canonical manipulation does <u>not</u> imply the existence of the local product of the fields.

It is clear from the above that, in order to have a consistent canonical formalism and maintain the results obtained earlier in this section, a more careful statement of the rules of the game must be given. An approach to this problem will be outlined in the following section.

C.3 Reducible scale invariance

We will show here that it is possible to avoid the inconsistencies discussed above while maintaining canonical field ETCR's and exact scale invariance. The idea is to employ reducible representations of the dilatation group $\mathcal{D}$, as formulated by Dell'Antonio [113)]. The analysis is taken from Ref. 110).

Our approach is most simply illustrated in a scalar field theory. We postulate the field equation

$$\Box\varphi(x) = gJ(x), \tag{111}$$

the ETCR's

$$[\dot{\varphi}(x), \varphi(0)]\delta(x_0) = -i\delta^4(x), \quad etc., \tag{112}$$

$$[\dot{J}(x), \varphi(0)]\delta(x_0) = j(0)\delta^4(x), \tag{113}$$

and exact dilatation invariance, as generated by $U(s)$. Here $j(x)$ is <u>defined</u> by (113) and $J(x)$ will be precisely defined below. For simplicity we make also the minimality assumption that only the minimum number of fields of each dimension necessary for consistency are present. It follows from (112) that

$$U(s)\varphi(x)U^{-1}(s) = s\,\varphi(sx), \tag{114}$$

and φ can be taken as the only field of dimension 1 so that it transforms irreducibly. It next follows from (111) that

$$U(s)J(x)U^{-1}(s) = s^3 J(sx), \tag{115}$$

and then from (113) that

$$U(s)j(x)U^{-1}(s) = s^2 j(sx). \tag{116}$$

Another consequence of (111)-(113) is

$$[\dddot{\varphi}(x), \varphi(0)]\,\delta(x_0) = g\,j(0)\,\delta^4(x). \tag{117}$$

From (112) and (117) follows the SD expansion

$$\varphi(x)\varphi(0) \xrightarrow[x\to 0]{} \Delta_+(x)I + \bar{g}(\ln x^2)\,j(0) + \bar{j}(0), \tag{118}$$

where $\bar{g} = -ig/4\pi^2$ and $\bar{j}(x)$ is an as yet unspecified local current. Scale invariance (and minimality), however, require that

$$\bar{j}(x) = j(x) + k(x), \tag{119}$$

where $k(x)$ transforms under dilatations accroding to

$$U(s)k(x)U^{-1}(s) = s^2\left[\bar{g}(\ln s^2)j(0) + k(0)\right]. \tag{120}$$

The fields $j(x)$ and $k(x)$ thus form a basis for the two-dimensional reducible representation of $\mathcal{D}$ and the SD expansion

$$\varphi(x)\varphi(0) \longrightarrow \Delta_+(x)I + \left[1+\bar{g}(\ln x^2)\right]j(0) + k(0) \tag{121}$$

is consistent with scale invariance. Equation (121) provides explicit expressions for j and k in terms of φ :

$$j(0) = \lim_{x\to 0}\left\{\frac{\varphi(x)\varphi(0) - \Delta_+(x)}{1+\bar{g}(\ln x^2)}\right\}, \tag{122}$$

$$k(0) = \lim_{x \to 0} \left\{ \varphi(x)\varphi(0) - \Delta_+(x) - \left[1 + \bar{g}(\ln x^2)\right] j(0) \right\} . \tag{123}$$

It is to be emphasized that (122) and (123) are fundamentally different in that $j(0)$ is defined from $\varphi(0)\varphi(0)$ by removing a divergence multiplicatively whereas $k(0)$ is defined by an additive renormalization. Note the non-perturbative character of the multiplicative renormalization in (122), which is additive to any finite order in g.

We can now proceed to consider other SD expansions. The scale invariance puts strong restrictions on their form. An important example is [110)]

$$j(x)j(0) \longrightarrow \frac{1}{x^2}\left[a - b \ln x^2\right] j(0) + \frac{1}{x^2} b k(0) + \ldots , \tag{124}$$

with a and b arbitrary constants [they occur also in the jk and kk short distance expansions (SDE)]. It follows from (124) [plus the related results for $\partial_0^n j(x)j(0)$] that the structure function

$$W(q^2, \nu) \equiv \int d^4x\, e^{iq\cdot x} \langle p | [j(x), j(0)] | p \rangle \tag{125}$$

will exhibit at most logarithmic deviations from the scaling behaviour

$$\nu W(q^2, \nu) \xrightarrow[A]{} F(\omega) . \tag{126}$$

Although this is gratifying, we would like to go further and obtain exact scaling. This requires the <u>further</u> assumption that $b = 0$ (etc.), in which case the ETC

$$[j(x), j(0)]\, \delta(x_0) = 4\pi^2 a i\, \delta^4(x)\, j(0) \tag{127}$$

exists.

The structure of Eq. (124) offers a very appealing way to understand the origin of this exact scaling [110)]. Let us suppose that $j(x)$ is a physical and measurable current (e.g., it couples to leptons) but that $k(x)$ is unphysical and unmeasurable. This means that $k(0)$ cannot occur in the $j(x)j(0)$ SDE and so, from (124), we must have $b = 0$. The condition $b = 0$ is, in turn, precisely the condition for the existence of the ETC (127) and for scaling (126). Thus, if we decouple k from (124) in this way, we obtain purely canonical results such as (127). The $[j,k]$ and $[k,k]$ ETC's might not exist, but that is unimportant since the log's have been decoupled from relations among observables. What we are proposing is to allow only the minimum number of log's necessary for the consistency of the theory to be present.

The non-observability of $k(x)$ suggests that an extra symmetry principle is operating. An interesting possibility for this is invariance under the field shift transformations [110]

$$R: \varphi(x) \longrightarrow \varphi(x) + \sigma , \qquad \sigma = \text{const.} , \tag{128}$$

which we refer to as R invariance. R invariance precludes the measurability of $\varphi(x)$ first as gauge invariance in electrodynamics precludes the measurability of the potential $A_\mu(x)$ [114]. R symmetry has been previously used in particle physics in connection with pion low energy theorems [115]. It is a spontaneously broken symmetry and not unitarily implementable [116], but the localized charge integrals exist and that is sufficient for our purposes.

We will identify the observables in our theory as the R invariant local fields. This does what we want it to since (122) and (123) give :

$$\delta_R j(x) = 0 , \qquad \delta_R k(x) = 2\sigma\varphi(x) + \sigma^2 , \tag{129}$$

so that j is an observable and k is not. Furthermore, R symmetry applied to (124) leads to the desired result $b = 0$. Proceeding in this way, we obtain the scaling law (126) as a consequence of the combined symmetries of scale invariance and R invariance.

One must, of course, check that our postulates (111) - (113) and all of the OPE's are consistent with R invariance. A detailed investigation shows that this is the case, and that we can obtain a canonical structure for the algebra of observables [110]. An important condition is that $\delta_R J(x) = 0$ in order that the field equation (111) is R invariant. This requires that J be a member of an (at least) three-dimension reducible representation of $\mathcal{D}$. [J must be properly defined as a limit of, e.g., $j(x)\varphi(0)$ or $k(x)\varphi(0)$.] Note that in the usual irreducible scale invariant (ISI) canonical theories, R invariance is only possible in the trivial cases when the Lagrangian only depends on $\partial_\mu\varphi$. In reducible scale invariant (RSI) theories, on the contrary, because of the possibility of multiplicative renormalizations of the type in (122), R invariance allows for a much richer structure.

The algebraic structure of the combined dilatation and R group is specified by the commutation relation (CR)

$$[D, R] = iR \,, \tag{130}$$

valid if φ and $\dot{\varphi}$ are irreducible. A simple example is the free massless scalar field theory, where

$$\Box \varphi_f(x) = 0 \tag{131}$$

and

$$R_{f\mu}(x) = \partial_\mu \varphi_f(x) \tag{132}$$

is the local current whose charge, suitably defined [116] generates R transformations.

R invariance can also be applied to physically more interesting theories such as the gluon model. There again we find that R invariance gives scaling [110]. An interesting mathematical structure emerges from the combined requirements of scale invariance, gauge invariance, and R invariance.

One of the interesting features of our approach is the connection between exact scaling and R invariance. In ISI theories with canonical dimensions, scaling follows just from the dilatation symmetry whereas in RSI theories with canonical dimensions, the extra symmetry is required. As we have seen, the RSI theories have the advantage that they can be based on simple field equations and canonical ETCR's. It is, of course, impossible to conclude at present whether the scaling observed in nature is a consequence of the relevance of theories of this sort or whether it is due to some other mechanism.

The R invariant currents which we have encountered, such as (122), are obtained from field products by dividing out their singularities appropriately. Such currents are consequently smoother than the R variant currents and so the algebra of observables is a smooth subalgebra of the algebra of local operators. The scaling behaviour of the physical current-hadron amplitudes is a particular consequence of this smoothness. It would appear to be of some interest to explore further consequences. It is possible that this approach could substantiate the often expressed hope that nature is smoother than perturbation theory.

D. Massive lepton pair production

D.1 Preliminaries

In this section we shall consider the inclusive process shown in Fig. 14. Two particles, of momenta p and p', collide to produce an observed lepton pair of momentum q and an unobserved hadronic final state. We will work with scalar particles and currents until a comparison with experiment [protons into (spin 1) photons] is made later on. The amplitude for the process is a particular discontinuity [117] of the three-to-three diagram of Fig. 15 and is given by

$$W(\varkappa, s, \nu, \nu') = \int d^4x \, e^{iq\cdot x} \, {}_{in}\langle pp' | J(x) J(0) | pp' \rangle_{in} \quad . \tag{133}$$

The variables are

$$\varkappa = q^2 \ , \quad s = (p+p')^2 \ , \quad \nu = p\cdot q \ , \quad \nu' = p'\cdot q \ , \tag{134}$$

and we have taken $p^2 = p'^2 = 1$.

We consider now the behaviour of W in the generalization of the scaling A limit in which each of the four variables (134) becomes large with the three ratios fixed :

$$\text{A-limit} : \ \varkappa, s, \nu, \nu' \longrightarrow \infty \quad \text{with } \frac{\nu}{\varkappa}, \frac{\nu'}{\varkappa}, \frac{s}{\varkappa} \text{ fixed.} \tag{135}$$

From phase arguments of the type used in Section II.A.1, the region of configuration space which controls the behaviour of W in the A limit is again seen to be the LC [5),61),117),118)] :

$$|x^2| \lesssim \frac{1}{|\varkappa|} \quad . \tag{136}$$

In spite of (136), it does <u>not</u> follow in the present case, contrary to the situation for electroproduction, that the <u>leading</u> LC singularity dominates in the A limit. To see why, assume for the moment canonical singularities so that

$$\langle pp' | J(x) J(0) | pp' \rangle \xrightarrow[x^2 \to 0]{} \frac{1}{x^2} f(x\cdot p, x\cdot p', s) + g(x\cdot p, x\cdot p', s) + \ldots \tag{137}$$

where we have exhibited the leading and first non-leading contributions. Because of (136), the contribution of f will be a power of $\varkappa$ greater than the contribution of g. If, however, the large s behaviour of g is greater than that of f by a power (or more), then the contribution of g will be the same as (or greater than) that of f since $\varkappa/s$ is fixed in the A limit. E.g., if $f \sim s^{\alpha}$ but $g \sim s^{\alpha+h}$, then the contribution of f is $\sim 1/\varkappa\, s^{\alpha}$ and that of g is $\sim 1/\varkappa^2 s^{\alpha+h}$ which dominates if $h > 1$.

It thus becomes a dynamical question whether or not the leading LC singularity dominates. It turns out that in all presently known models [multiperipheral [119], parton [120], Feynman diagrams [121], non-perturbative parton [122]], the above possibilities for ruining <u>leading</u> LC dominance do <u>not</u> occur. [Actually, since in the parton model the only configurations considered <u>exclude</u> those which contribute to the <u>leading</u> LC singularity, there it is the <u>second leading</u> contribution which dominates.] This will be seen in Section D.2. More generally, any uniform bound on the large s behaviour at fixed $\varkappa$, such as that provided by Regge theory, is sufficient to ensure leading LC dominance. Strictly speaking, such a bound would only be relevant in the Regge limit $s \to \infty$ with $\varkappa$ fixed, but commutativity relations of the type discussed in the previous chapters make these bounds relevant in the A limit as well [123]. Furthermore, in the LC treatment the large $\varkappa$ and large s dependencies are effectively decoupled and only the behaviours of the five-point functions $\langle pp'|O_{\alpha_1,\dots,\alpha_n}(0)|pp'\rangle$ in the R limit are relevant, provided the sum over n is sufficiently well-convergent [117),123].

It follows that under the quite general circumstances described above, the leading LC singularity dominates the A limit and we have

$$W(\varkappa,s,\nu,\nu') \xrightarrow[A]{} W_A(\varkappa,s,\nu,\nu'), \tag{138}$$

where W_A is obtained from (133) by keeping only the leading LC singularity [$(1/x^2)f(x\cdot p, x\cdot p', s)$ in the canonical case].

The relevant Regge limits are :

pionization limit (P) : $\nu, \nu', s \to \infty$; $\frac{\nu\nu'}{s}$ fixed;

fragmentation limit (F) : $\nu, s \to \infty$; $\frac{\nu}{s}$, ν' fixed.

There is also the F' limit, which is the same as the F limit but with ν and ν' interchanged. The behaviour of (133) in these limits is

$$W \xrightarrow[P]{} s^{\alpha}\beta\left(\frac{\nu\nu'}{s\varkappa}, \varkappa\right) \tag{139}$$

and

$$W \xrightarrow[F]{} s^{\alpha}\beta(\tfrac{\nu}{s}, \nu', \varkappa) . \qquad (140)$$

The (hadronic) scaling properties exhibited here are consequences of Regge pole dominance of Fig. 15 in the Regge limits [124]. Commutativity relations [123], such as

$$\lim_{\substack{\varkappa\to\infty \\ \eta \text{ fixed}}} s^{\alpha}\beta(\eta,\varkappa) = \lim_{\substack{\frac{s}{\varkappa},\frac{\nu}{\varkappa},\frac{\nu'}{\varkappa}\to\infty \\ \eta \text{ fixed}}} W_A(\varkappa,s,\nu,\nu'), \qquad (141)$$

can be derived as in Section I.B.1 from the integral representation

$$W(\varkappa,s,\nu,\nu') = \int_0^\infty da \int_{-1}^{1} db \int_{-1}^{1} db' \, \sigma(a,b,b';s)\,\delta_+(\varkappa+2b\nu+2b'\nu'+bb's). \qquad (142)$$

The representation (142) is a straightforward generalization of the ones used in Chapter I.

D.2 Model calculations

In this section we will discuss the amplitude (133) in the non-perturbative parton model reviewed in Section II.A.3. The amplitude (Fig. 15) in the model has the form shown in Fig. 16. Particular disconnected contributions are shown in Fig. 16a and 16b. The diagrams (a) are exactly the parton model (annihilation) diagrams [120], as can be seen by inserting intermediate states in the blobs. They have been previously discussed in this model in Ref. 125) and in the multiperipheral model in Ref. 3). Our analysis of these diagrams will follow Wilson's [3] treatment. The (bremmstrahlung) diagrams (b) have not been previously treated in this model. They are claimed to give insignificant contributions compared to (a) diagrams in Refs. 120) and 125), but we will see that this depends on strong specific dynamical assumptions which are not necessary for the successes of the parton models in electroproduction. Likewise, there are no good reasons for neglecting omitted diagrams in Fig. 16.

For simplicity, and in order to compare with the parton model calculations, we assume that the LC expansion has the free field form :

$$j(x)j(0) \longrightarrow \Delta_+(x):\varphi(x)\varphi(0): \; + \; :j(x)j(0): \; + \; O(x^2), \tag{143}$$

where the bilocals $:\varphi(x)\varphi(0):$ and $:j(x)j(0):$ are analytic. Then the integrand in (133) has the LC behaviour (137).

We consider first the (a) diagrams. Their contribution factorizes in configuration space [3]:

$$\begin{aligned} \langle pp'|j(x)j(0)|pp'\rangle_a &= \langle p|\varphi(x)\varphi(0)|p\rangle\langle p'|\varphi(x)\varphi(0)|p'\rangle \\ &= f(x\cdot p, x^2)\, f(x\cdot p', x^2) \;. \end{aligned} \tag{144}$$

Here $f(x\cdot p, x^2)$ is the quark-hadron amplitude (50) [126] shown in Fig. 13. The Fourier transform of the product (144) gives precisely the (a) diagrams by convolution. Going to the LC, we get

$$f(x\cdot p, x^2) \xrightarrow[x^2\to 0]{} f(x\cdot p, 0) = f(x\cdot p) = \frac{1}{2\pi}\int_{-1}^{1} d\omega\, e^{-i\omega x\cdot p} F(\omega), \tag{145}$$

with $F(\omega)$ the current-hadron scaling function in the model [Eqs. (22) and (23)]. We see that the (a) diagrams contribute only the second leading singularity in (143). We also see that, since (144) is <u>independent of $s \sim 2p\cdot p'$</u>, the mechanism described after Eq. (137) for ruining (second) leading LC singularity dominance is <u>not</u> operating here and so the leading LC behaviour (145) <u>does</u> control the A limit.

Substitution of (144) and (145) into (133) gives [127),128)]

$$W_a(q) \xrightarrow[A]{} \int_{-1}^{1} d\omega \int_{-1}^{1} d\omega'\, F(\omega)F(\omega')\,\delta^4(q-\omega p-\omega' p') \,. \tag{146}$$

The cross-section $d\sigma/d\mathscr{M}$ is proportional to the amplitude

$$W_a(\mathscr{M}, s) \equiv s\int d^4q\, \delta(q^2-\mathscr{M})\, W_a(q) \,. \tag{147}$$

We obtain from (146)

$$W_a(\mathscr{M}, s) \xrightarrow[A]{} \mathcal{F}(\tau) \,, \qquad \tau \equiv \frac{\mathscr{M}}{s} \,, \tag{148}$$

where

$$\mathcal{F}(\tau) = \int d\omega\, d\omega' F(\omega) F(\omega') \delta(\omega\omega' - \tau). \tag{149}$$

This is precisely the parton model result [120),125)] for the scalar case under consideration.

We consider next the (b) diagrams. They immediately give in configuration space

$$\langle pp' | j(x) j(0) | pp' \rangle_b = D_+(x) \langle pp' | \varphi(x) \varphi(0) | pp' \rangle . \tag{150}$$

These diagrams thus contribute to the leading LC singularity in (143), which is easily seen to dominate their A limit. Their contribution to the A limit of (133) is

$$W_b(q) \xrightarrow[A]{} \int d^4k\, \delta((k-q)^2)\, \theta(k_0 - q_0)\, T(k^2, k\cdot p, k\cdot p'; s), \tag{151}$$

where $T(k^2, \nu, \nu'; s)$ is the (off-shell) quark-hadron-hadron forward amplitude (including the quark propagators). We introduce now the Sudakov variables [129),130)] by writing

$$k = \omega p + \omega' p' + k_\perp , \tag{152}$$

with $k_\perp \cdot p = k_\perp \cdot p' = 0$, and we obtain [131)]

$$W_b(q) \xrightarrow[A]{} s \int d\omega\, d\omega'\, d^2k_\perp\, \delta(\varkappa - 2\omega\nu - 2\omega'\nu' + \omega\omega' s)\, \theta(\omega\omega' - \tfrac{\varkappa}{s}) \cdot T(\omega\omega' s - k_\perp^2, \omega' \tfrac{s}{2}, \omega \tfrac{s}{2}; s). \tag{153}$$

To estimate (153), we assume the usual Regge behaviour of T, e.g., in the pionization limit

$$T \xrightarrow[P]{} s^\alpha \beta(\omega\omega' s, k_\perp^2). \tag{154}$$

Assuming as usual [130)] that T is biggest in this limit and that T decreases rapidly when the quark masses become large, (153) becomes

$$W_b(q) \xrightarrow[A]{} s^{1+\alpha} \int d\omega\, d\omega'\, B(\omega\omega' s)\, \delta(\varkappa - 2\omega\nu - 2\omega'\nu' + \omega\omega' s)\, \theta(\omega\omega' - \tfrac{\varkappa}{s}), \tag{155}$$

where

$$B(z) = \int d^2k_\perp \, \beta(z, k_\perp^2). \tag{156}$$

We will not attempt to evaluate (155) further now because it will be seen in the following section to be a special case of the general LC expression which we investigate in detail. We note only that to compare (146) and (155), or (148) and the behaviour of the corresponding $W_b(\varkappa, s)$ obtained from (155) [128], one must know the fall-off rate of $B(z)$ and this is a dynamical question. It is, in general, possible for W_b to be comparable to, or even to dominate, W_a. If we in fact accept the calculation of the fall-off of $W_a(q)$ in $q_\perp$ in Ref. 120), then $W_b(q)$ should certainly dominate away from $q_\perp = 0$.

Now, what about the remaining diagrams in Fig. 16 ? It is argued in Refs. 120) and 125) that they [as well as diagram (b)] are dominated in the A limit by diagram (a).[132]. This again depends, however, on specific and as yet untested extra strong dynamical assumptions. As an example, we consider the form factor corrections to diagram (a)[133]. That is, instead of the current coupling to a point in Fig. 16a, we allow a current-quark-quark form factor, say $[G(\varkappa)]^{\frac{1}{2}}$ to be present. This might be expected to be important since the quark rungs in diagram (a) are at bounded mass. Note the contrast with diagram (b) and with the electroproduction diagram in Fig. 12c. There the quark propagator connecting the currents has mass approaching infinity (as ν) in the A limit and so the form factor, say $G(q^2,(k+q)^2)$, is to be evaluated in the asymptotic limit $q^2 \to \infty$ with $q^2/(k+q)^2$ fixed and in this limit (in canonical theories) it approaches a constant. This result is, of course, implicit in the proof that the diagram of Fig. 12b dominates in the A limit. For the diagram of Fig. 16a, on the contrary, <u>both</u> quark lines attached to the currents have small masses by assumption 2) [after Eq. (44)] and so the form factors relevant are like on-shell hadron form factor, which decrease rapidly, perhaps exponentially, for large $\varkappa$. Calling $\overline{W}_a(\varkappa, s)$ the contribution of the (a) diagrams with complete quark-quark-current form factors, we have

$$\overline{W}_a(\varkappa, s) \xrightarrow[A]{} G(\varkappa)\, \mathcal{F}(\tau), \tag{157}$$

with $\mathcal{F}(\tau)$ given by (149) and $G(\varkappa)$, the square of the form factor, an unknown function of $\varkappa$, of possibly fast decrease. Actually, it is argued in Ref. 79) that $G(\varkappa) \xrightarrow[\varkappa \to \infty]{}$ const., but this depends on the assumption that the quark-quark scattering amplitude decreases fast as the (off-shell) quark mass becomes large. Unlike the corresponding assumption for quark-<u>hadron</u>

amplitudes, whose decrease in the quark mass is a consequence of the composite nature of the hadron [83], we see no reason for expecting the validity of this assumption for purely quark amplitudes. The assumption is, in fact, incorrect for the quark-quark propagator [Eq. (46)]. Thus, there is no compelling experimental (e.g., from the success of parton models in electroproduction) or theoretical (e.g., from the composite nature of hadrons) information on the behaviour of $G(\varkappa)$ in (157).

In view of the lack of control of these other diagrams, and also because of weaknesses of the model itself, such as those mentioned at the close of Section II.A.3, a model independent approach seems desirable. Some attempts [123),134)] in this direction will be outlined in the following section.

D.3 Light cone Regge analysis

The result of the LC multi-Regge analysis is [123)]

$$W_A \xrightarrow[A]{} s^{\alpha}(\ln s)\int_0^1 da \int_{-1}^{1} d\alpha \int_{-1}^{1} d\alpha'\, s^{1-a}\, \Psi(\alpha s^{a}, \alpha' s^{1-a}; a)$$
$$\cdot\, \delta(\varkappa - 2\alpha\nu - 2\alpha'\nu' + \alpha\alpha' s)\, \Theta(\alpha\alpha' - \tfrac{\varkappa}{s}) \equiv W_L'(s,\varkappa,\nu,\nu'), \tag{158}$$

where $\Psi(\beta, \beta'; a)$ is of fast decrease in its first two arguments and is independent of its third argument for $\varepsilon < a < 1-\varepsilon$. [Here $\varepsilon = \varepsilon(s)$ is such that $s^{\varepsilon(s)} \simeq 2\ \mathrm{GeV}^2$.] W_L can therefore be conveniently decomposed into the sum of a "pionization" piece $\tilde{W}$ coming from $\varepsilon < a < 1-\varepsilon$ and a "fragmentation" piece $\bar{W}$ coming from $0 \leq a \leq \varepsilon$ and $1-\varepsilon \leq a \leq 1$ [123)]:

$$W_L = \tilde{W} + \bar{W}, \tag{159}$$

with

$$\Psi(\beta, \beta'; a) \cong \tilde{\Psi}(\beta, \beta'), \qquad \varepsilon < a < 1-\varepsilon, \tag{160}$$

and

$$\Psi(\beta, \beta'; a) \cong \bar{\Psi}(\beta, \beta'), \qquad a < \varepsilon \text{ or } a > 1-\varepsilon. \tag{161}$$

Let us compare our model independent result (158) with the model calculation of Section II.D.2. The contribution (155) of the (b) diagram is immediately seen to have the form (158) with spectral function

$$\Psi_b(\beta,\beta',a) = B(\beta\beta') \tag{162}$$

since

$$\int_0^1 da\, s^{-a} = \frac{1}{\ln s}\left(1-\frac{1}{s}\right) \xrightarrow[s\to\infty]{} \frac{1}{\ln s} \quad . \tag{163}$$

Even the fast decrease of $B(z)$ for $z\to\infty$ is necessary in order to have a fast cut-off in transverse momentum of produced quarks in hadron-hadron collisions.

In order to compare with the contributions (146)-(149) of the (a) diagrams [135], we must first replace (154), which comes from a $1/x^2$ LC singularity [the first term in (143)], with the result of using a constant LC singularity [the second term in (143)] since, as we have seen, that in the leading LC behaviour in the (a) diagrams. The resulting amplitude W_ℓ has the behaviour

$$W_\ell(q) \xrightarrow[A]{} s^\alpha(\ln s)\int_0^1 da \int d\alpha\, d\alpha'\, s^{1-a}\,\Psi_\ell(\alpha s^a, \alpha' s^{1-a}; a)\,\delta^4(q-\alpha p-\alpha' p') \, . \tag{164}$$

Thus

$$W_\ell(x,s) \xrightarrow[A]{} s^\alpha(\ln s)\int da \int d\alpha\, d\alpha'\, s^{1-a}\,\Psi_\ell(\alpha s^a, \alpha' s^{1-a}; a)\,\delta(\omega\omega'-\tau) \, . \tag{165}$$

This appears to be quite different from (148)-(149). In fact, the (a) diagrams do not satisfy one of the main assumptions involved in the derivation [123] of (165), namely leading Regge behaviour for the five-point functions

$$\langle pp'| : j(\xi)\overleftrightarrow{\partial}_{\alpha_1}\cdots\overleftrightarrow{\partial}_{\alpha_m} j(\xi) : |pp'\rangle$$

encountered in the expression for the LC expansion (143) as a sum of local fields. It indeed follows from (144) that the five-point functions from the (a) diagrams have a large s behaviour, namely constant, which corresponds to a leading fixed singularity in the J plane - a Kronecker delta at $J=0$ [136]. Although there is nothing which precludes the presence of this Kronecker delta, its presence is perhaps somewhat surprising and unlikely. But, as we have stressed in the previous section, the remaining diagrams in Fig. 16 are not necessarily dominated by the (a) diagrams and so the possibility exists for them to cancel the Kronecker delta from Fig. 16a in such a way that the large s behaviour of the sum of all contributions is described by a Regge pole [132].

This would certainly be the simplest possibility from the complex J plane point of view - although clearly not from the diagrammatic point of view. It is, of course, also possible that such a cancellation does not occur and that the Kronecker delta remains. Then its contribution should be added on to (158).

The form factor corrections discussed in the previous section nicely illustrate how the above cancellation can occur - if they are such that the total form factor is of fast decrease. Equations (165) and (157) are indeed consistent in this case. To see this, take

$$\Psi_l(\beta,\beta';a) = \bar{G}(\beta\beta')\left[g(\beta)+g(\beta')\right], \tag{166}$$

with $\bar{G}$ and g to be specified. Substitution in (165) gives

$$W_l(\varkappa,s) \xrightarrow[A]{} s^{\alpha}(\ln s)\,\bar{G}(\varkappa)\int_C^1 da\int_\tau^1 d\alpha\, s^{1-a}\left[g(\alpha s^a)+g\left(\frac{\varkappa}{\alpha s^a}\right)\right]. \tag{167}$$

Changing integration variables from a to $u = \alpha s^a$ and taking $g(\beta)$ to be of fast decrease gives

$$W_l(\varkappa,s) \xrightarrow[A]{} s^{\alpha+1}\,\bar{G}(\varkappa)\,\mathcal{H}(\tau) \tag{168}$$

where

$$\mathcal{H}(\tau) = \int_\tau^1 d\alpha\,\alpha\int_\alpha^\infty \frac{du}{u^2}\,g(u). \tag{169}$$

Finally, choosing

$$\mathcal{H}(\tau) \equiv \tau^{\alpha+1}\mathcal{F}(\tau), \qquad \bar{G}(\varkappa) = \varkappa^{-\alpha-1}G(\varkappa), \tag{170}$$

(168) becomes

$$W_l(\varkappa,s) \xrightarrow[A]{} G(\varkappa)\,\mathcal{F}(\tau), \tag{171}$$

just as (157). The reason for the consistency of (157) and (145) is that diagrams (a) <u>with</u> form factors of fast decrease <u>do</u> satisfy the five-point function Regge behaviour.

It should be noted that in the above analysis the only place we use the fast decrease of $G(\varkappa)$ was for (166) to be consistent with the fast decrease properties of Ψ, which are consequences of, among other things, the Regge behaviour assumption [123]. If the fast decrease of Ψ is given up, then the above analysis shows that the LC result (165) is consistent with (157) for any $G(\varkappa)$, including the naïve parton model result (148) with $G(\varkappa)=1$. [Likewise, the (b) diagram result (155) can be obtained without $B(z)$ of fast decrease.] This is perhaps not surprising since the leading LC singularities do dominate in these models. Thus, our LC forms (158) and (164) encompass all possibilities if the fast decrease of Ψ is relaxed. Since we feel that the five-point Regge behaviour and consequent fast decrease of Ψ is the most elegant and likely possibility, we will proceed to discuss (158) with these restrictions on Ψ. The present data will be seen to be quite consistent with this.

Returning to (158), we note that since $\Psi(\beta,\beta')$ must be symmetric under interchange of β and β' ($\nu \rightleftarrows \nu'$ symmetry of W), we can write

$$\Psi(\beta,\beta') = \Phi\left(\frac{\beta+\beta'}{2}, \sqrt{\beta\beta'}\right) . \tag{172}$$

One integral in (158) can be done with the δ function and the others can be estimated if Φ is a sufficiently smooth function of its arguments [134]. The result for $\tilde{W}$ and $\bar{W}$ gives the expression

$$\frac{d\tilde{\sigma}}{d\varkappa} \sim \Phi(\sqrt{\varkappa}, \sqrt{\varkappa}) , \qquad \frac{d\bar{\sigma}}{d\varkappa} \sim \Phi(\varkappa, \sqrt{\varkappa}) , \tag{173}$$

for the contributions of the pionization region $(d\tilde{\sigma}/d\varkappa)$ and the fragmentation regions $(d\bar{\sigma}/d\varkappa)$ to the cross-section

$$\frac{d\sigma}{d\varkappa} = \frac{d\tilde{\sigma}}{d\varkappa} + \frac{d\bar{\sigma}}{d\varkappa} . \tag{174}$$

The interesting feature is the difference in the behaviour of the sum variable. The sum (174) can thus easily appear as the superposition of two different rapidly decreasing contributions, one $(d\tilde{\sigma}/d\varkappa)$ dominating at large $\varkappa$ and the other $(d\bar{\sigma}/d\varkappa)$ dominating at small $\varkappa$. The result of the combination of these two contributions can produce a shoulder, as seems to be present experimentally (see Fig. 17).

When spin is correctly included, the result is an expression of the form (158) with Ψ replaced by a sum over Ψ_i, $i=1-4$ [134]. To compare with experiment, we take the simplest possible phenomenological model for the Ψ_i's. We take [134]

$$\Psi_{\lambda}(\beta,\beta') \xrightarrow[P]{} P_{\lambda}' \, e^{-h(\beta+\beta')} \tag{175}$$

the pionization limit with β and β' both large, and

$$\Psi_{\lambda}(\beta,\beta') \xrightarrow[F]{} F_{\lambda}' \, e^{-k\beta} (1-\beta')^{\ell} \tag{176}$$

the fragmentation limit with β large and $\beta' \sim 1$. In (176) we have included a threshold factor analogous to the one in electroproduction. With these explicit forms, the integrals over α , α', and a can be performed in the A limit. The final result is the sum of a P contribution, with an unknown over-all coefficient P and an F contribution, with another unknown over-all coefficient F. So, in this simplest case, we obtain a representation in terms of the five free parameters P,F,h,k, and ℓ [134].

To compare with the experimental data, we must integrate over phase space, respecting the experimental cuts [138]. We obtain expressions for $d\sigma/d\mathcal{K}^{\frac{1}{2}}$, $d\sigma/d\cos\theta$, $d\sigma/dq_{\parallel}$, and $\sigma(E_p)$ to be compared with the experimental results [138]. Our procedure was to fix our five parameters by fitting the $d\sigma/d\mathcal{K}^{\frac{1}{2}}$ curve [134]. A typical hand fit, with

$$P = 1.67 \,, \quad F = 10^{4} \,, \quad h = 0.10 \,, \quad k = 2.0 \,, \quad \ell = 4 \,, \tag{177}$$

is shown in Fig. 17. The fit is seen to be quite good. The shoulder appears, as expected, from the interference of the two exponentials and the final rapid decrease of the curve is due to phase space. Using the same values (177), we obtain predictions for the other curves [134]. These are shown in Figs. 18, 19, and 20, together with the experimental data. The agreement is seen to be very good in all cases. It is possible to obtain still better fits using more sophisticated fitting methods, but this hardly seems warranted at present because our assumed forms (175) and (176) are only guesses and because of the crudity of the exerting data. More detailed comparisons and predictions for the future experiments will be given in Ref. 134).

Accepting at least the gross features of the present data, a few remarks about the significance of the fit (177) are in order. The small value $\sim 10^{-4}$ obtained for the ratio P/F suggests that the Pomeron-particle-Pomeron coupling at $t=0$ is very small and perhaps vanishes. This must be the case if the Pomeron is an isolated pole at $\alpha(0)=1$ [139]. The value $\ell = 4$ for the threshold power decrease is similar to the value $\ell = 3$ found in electro-production. We also obtain acceptable fits with $\ell = 3$. We finally note in Fig. 18 the smooth fall-off of $d\sigma/d\cos\Theta$. The behaviour is $\sim e^{-500(1-\cos\Theta)}$ and we fit this nicely. This should be compared to the behaviour $e^{-2000(1-\cos\Theta)}$ which one finds for hadronic single particle production in similar experimental conditions and which is the parton model prediction [120].

FOOTNOTES AND REFERENCES

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53) H.R. Rubinstein, G. Veneziano and M.A. Virasoro - Phys.Rev. 167, 1441 (1968).

54) R.F. Dashen and S.Y. Lee - Phys.Rev.Letters 22, 366 (1969).

55) For earlier estimates, see :
R. Rajaraman and G. Rajasekaran - Phys.Rev. D3, 266 (1971) ;
M. Elitzur - Phys.Rev. D3, 2166 (1971).

56) That is, the Thompson limit.

57) The same conclusion has been reached by :
Y. Matsumoto et al. - Phys.Letters 39B, 258 (1972).
As in Ref. 10), Regge fits were made in this reference for all q^2.

58) If $\alpha_T \not\leq \frac{1}{2}$ in Compton scattering, the significance of the Regge formalism would be dubious.

59) F. Close and J. Gunion - Phys.Rev. D4, 742 (1971).

60) J.M. Cornwall, D. Corrigan and R.E. Norton [Phys.Rev. D3, 536 (1970)] have assumed that $\gamma_F(\varkappa)$ is a polynomial and derived the sum rule $\zeta_F = \gamma_F$.
In the "field theoretic" parton model [Ref. 79)] a fixed $J = 0$ pole with polynomial residue was shown to possibly occur in the diagrams which dominate in the A limit. We note, however, that these same diagrams give rise to a non-polynomial residue if the fixed pole is present in the off-shell quark-proton scattering amplitude, where nothing prohibits its occurrence. Furthermore, the remaining diagrams can also give rise to a $J = 0$ fixed pole with a non-polynomial residue.
For recent discussions and attempts to achieve a theoretical understanding of the fixed pole, see :
K. Bitar - Phys.Rev. D5, 1498 (1972) ;
A. Zee - Phys.Rev. D5, 2829 (1972) ;
M. Bander - Phys.Rev. D5, 3274 (1972).

61) R.A. Brandt and G. Preparata - "Broken Scale Invariance and the Light Cone", M. Gell-Mann and K. Wilson, Eds., p. 43, Gordon and Breach (1971).

62) R.A. Brandt, W.-C. Ng, G. Preparata and P. Vinciarelli - Nuovo Cimento Letters 2, 937 (1971).

63) M. Testa - CERN Preprint TH. 1515 (1972), to be published. I thank the author for discussions on this paper.

64) R.A. Brandt and W.-C. Ng - N.Y.U. Technical Report No 13/72, to be published.

65) K. Gottfried - Phys.Rev.Letters 18, 1174 (1967) ;
J.D. Bjorken and E.A. Paschos - Phys.Rev. 185, 1975 (1969) ;
J. Kuti and V.F. Weisskopf - Phys.Rev. D4, 3418 (1971).

66) P.M. Fishbane and D.Z. Freedman - Phys.Rev. D5, 2582 (1972) ;
S.D. Drell and T.D. Lee - Phys.Rev. D5, 1738 (1972) ;
T.D. Lee - Columbia University Preprint (1972).

67) S.S. Shei and D.M. Tow - Phys.Rev. D4, 2056 (1971) ;
H. Kleinert - Freie Universität Berlin Preprint, unpublished ;
S. Orfanidis - Yale University Preprint 2726-620 (1972), unpublished

68) Mathematical counterexamples, with non-vanishing $\sigma_0(a)$, can of course be constructed. An example is given by A. suri, to be published. Perturbative counterexamples, with non-decreasing form factor, also exist in φ^3 theory.

69) R.A. Brandt and G. Preparata - Phys.Rev. D1, 2577 (1970).

70) In the gluon model, quarks interact via exchange of a neutral massive vector boson coupled to the quark baryon number current. Details of the model are given in Chapter II.

71) R.A. Brandt and G. Preparata - Nuclear Phys. B27, 541 (1971).

72) See, for example :
N.N. Bogoliubov and D.V. Shirkov - "Introduction to the Theory of Quantized Fields", Interscience, New York (1959).

73) An independent argument for LC dominance of the A limit was given by :
B.L. Ioffe - Phys.Letters 30B, 123 (1969).
An alternate deduction of the LC singularities at SLAC is given by :
H. Leutwyler and J. Stern - Nuclear Phys. B20, 77 (1970).

74) R.A. Brandt and G. Preparata - Phys.Rev. D4, 3522 (1971).

75) See also :
H. Schnitzer - Phys.Rev. D4, 1429 (1972).

76) The emergence of ω_L from the LC was also noted by :
D. Bhaumik and O.W. Greenberg - Phys.Rev. D4, 2048 (1972).

77) R.A. Brandt - Phys.Rev. D4, 444 (1971).

78) For a speculative application of this result, see :
R.A. Brandt and C.A. Orzalesi - Phys.Letters 34B, 641 (1971).

79) P.V. Landshoff, J.C. Polkinghorne and R. Short - Nuclear Phys. B28, 225 (1971).

80) R. Feynmann - Phys.Rev.Letters 23, 1415 (1969) ;
J.D. Bjorken and E. Paschos - Phys.Rev. 185, 1975 (1969) ;
S.D. Drell, D. Levy and T.M. Yan - Phys.Rev. D1, 1035 (1970), and references therein ;
For an interesting recent discussion, see :
W. Weisberger - Phys.Rev. D5, 2600 (1972).

81) For discussions of electroproduction in multiperipheral models, see :
S.S. Shei and D.M. Tow - Phys.Rev. D4, 2056 (1971) ;
R. Griffith - Phys.Rev. D4, 3162 (1971) ;
S. Nussinov and A. Rangwala - Phys.Rev. D5, 220 (1972).

82) As opposed to super-renormalizable.

83) The second assumption is presumably a consequence of the composite nature of the external hadrons. For recent investigations, see :
S.D. Drell and T.D. Lee - Phys.Rev. D5, 1738 (1972), and
C.H. Woo - to be published.
This assumption is, however, not sufficient to achieve scaling. The first assumption is also necessary and corresponds to the neglect of certain diagrams in the above papers.

84) The loop integration in Fig. 12e arises from (49) because the Fourier transform of the product is the convolution of the Fourier transforms of the factors.

85) C.A. Orzalesi- Ref. 52).

86) K. Wilson - unpublished ;
R.A. Brandt - Ann.Phys. 44, 221 (1967) ;
W. Zimmermann - Commun.Math.Phys. 6, 161 (1967).

87) R.A. Brandt - Ann.Phys.(N.Y.) 52, 122 (1969).

88) W. Zimmermann - "Lectures on Elementary Particles and Quantum Field Theory", M.I.T. Press, Cambridge (1971).

89) K.Wilson and W. Zimmermann - New York University Preprint 19/71, to be published ;
P. Otterson and W. Zimmermann - New York University Preprint 18/71, to be published.

90) We assume here that fields transform irreducibly under the dilatation group. More general possibilities will be considered in Section II.C.3.

91) K. Wilson - Phys.Rev. 129, 1499 (1969) ;
See also :
H.A. Kastrup - Nuclear Phys. 58, 561 (1964), and
G. Mack - Nuclear Phys. B5, 499 (1968).

92) Y. Frishman - Phys.Rev.Letters 25, 966 (1970).

93) K. Wilson - Phys.Rev. D2, 1473 (1970).

94) W. Thirring - Ann.Phys.(N.Y.) 3, 91 (1958).

95) J.H. Lowenstein and B. Schroer - Phys.Rev. D3, 1981 (1971).
See also :
H. Georgi - Phys.Rev. D2, 2908 (1970).

96) J.H. Lowenstein - Commun.Math.Phys. 16, 265 (1970).

97) S.J. Chang and P.M. Fishbane - Phys.Rev. D2, 1084 (1970) ;
P.M. Fishbane and J. Sullivan - Phys.Rev. D4, 2516 (1971).

98) V.N. Gribov and L.N. Lipatov - Phys.Letters 37B, 78 (1971), and to be published.

99) For an elegant derivation of these results using LC expansions, see :
N. Christ, B. Hasslacher and A. Mueller - Columbia Preprint CO-3067(2)-9 (1972).

100) M. Gell-Mann and F.E. Low - Phys.Rev. 95, 1300 (1954) ;
C. Callan - Phys.Rev. D2, 1541 (1970) ;
K. Symanzik - Commun.Math.Phys. 18, 227 (1970) ;
K. Symanzik - Commun.Math.Phys. 23, 49 (1971) ;
K. Wilson - Phys.Rev. D3, 1818 (1971) ;
C. Callan - Phys.Rev. D5, 3202 (1972) ;
G. Parisi - Phys.Letters 34B, 643 (1972).

101) A.M. Polyakov - Soviet Phys. JETP 28, 533 (1969) ;
G. Parisi and L. Peliti - Nuovo Cimento Letters 2, 627 (1971) ;
A.A. Migdal - Phys.Letters 37B, 98 (1971) ;
G. Mack and I. Todorov - ICTP Preprint IC/71/139 (1971) ;
G. Ferrara and G. Parisi - Frascati Preprint LNF-72/1 (1972).
For a review, see :
B. Schroer - Lectures given at the IVth Simposio Brasileiro De Fisica Teorica.

102) R.A. Brandt and G. Preparata - Phys.Rev. D1, 2577 (1970).

103) R.A. Brandt and G. Preparata - Phys.Rev.Letters 25, 1530 (1970) ; 26 1605 (1971).

104) D. Gross and S. Treiman - Phys.Rev. D4, 1059 (1971).

105) H. Fritzsch and M. Gell-Mann - "Broken Scale Invariance and the Light Cone", M. Gell-Mann and K. Wilson, Eds., p. 1, Gordon and Breach (1971).

106) R.A. Brandt and G. Preparata - New York University Technical Report 15/72, Nuclear Phys. in press.

107) J.W. Meyer and H. Suura - Phys.Rev. 166, 1366 (1967).

108) As shown in Ref. 106), the expression given in Ref. 104) for a manifestly covered LCOPE involving finitely many terms is incorrect. This has also been noted by :
W. Kummer - University of Pennsylvania Preprint.

109) For other LC discussions of neutrino reactions, see :
Y. Georgelin, J. Stern and J. Jersak - Nuclear Phys. B27, 493 (1971) ;
W.-C. Ng - Nuovo Cimento Letters 3, 503 (1972) ;
B.W. Lee and J.E. Mandula - Phys.Rev. D4, 3475 (1972) ;
D. Leven and D. Palmer - Phys.Rev. D4, 3716 (1972) ;
W.-C. Ng and P. Vinciarelli - Phys.Letters 39B, 219 (1972).

110) R.A. Brandt and W.-C. Ng - "A Theory of Canonical Scaling", Max-Planck Institute Preprint (1972).

111) $[A,B]_T \equiv [A,B] - \langle 0 | [A,B] | 0 \rangle$.

112) R.A. Brandt - Phys.Rev. 166, 1795 (1968).

113) G.F. Dell'Antonio - New York University Preprint
See also :
S. Ferrara and A.F. Grillo - Nuovo Cimento Letters 2, 177 (1971).

144) See Ref. 87) for a derivation of the restrictions of gauge invariance on OPE's in quantum electrodynamics.

115) G. Kramer and W.F. Palmer - Phys.Rev. 128, 1492 (1969), and references therein.

116) R.F. Streater - Proc.Roy.Soc.(London) A287, 510 (1967).

117) G. Altarelli, R.A. Brandt and G. Preparata - Phys.Rev.Letters 26 42 (1970).

118) It was stated in :
R. Jaffe - Phys.Letters 37B, 517 (1971),
that, on the contrary, all values of x^2 are important in the A limit in the parton model ; but this claim has been withdrawn (R. Jaffe, private communication).

119) See Ref. 3).

120) S.D. Drell and T.M. Yan - Phys.Rev.Letters 25, 316 (1970) ; Ann.Phys.(N.Y.) 66, 578 (1971).

121) H.C. Backer E.Y.C. Lu and E. Schrauner - to be published ;
H.C. Backer - to be published.

122) μ pair production in this model will be discussed in Section III.D.2.

123) R.A. Brandt and G. Preparata - Phys.Rev. D6, 619 (1972).

124) A.H. Mueller - Phys.Rev. D2, 2963 (1970).

125) P.V. Landshoff and J.C. Polkinghorne - Nuclear Phys. B33, 221 (1971).

126) The time-ordering is irrelevant at $x^2 = 0$.

127) We denote the contribution of the a and b diagrams to $W(\mathcal{H}, s, \nu, \nu')$ as $W_a(q)$ and $W_b(q)$.

128) Because of the singular nature $[\delta(q_\perp)]$ of the transverse momentum distribution in (146), the LC does not necessarily dominate this distribution - non-leading singularities can change $\delta(q_\perp)$ to a function of fast decrease. [I thank G. Parisi for discussions on this point.] The LC does dominate the integrated amplitude (147) and this is all we will consider in this case. For the b diagrams, both $W_b(q)$ and $W_b(\varkappa, s)$ are leading LC dominated.

129) V.V. Sudakov - Soviet Phys.JETP 3, 65 (1956).

130) V.N. Gribov - Soviet Phys.JETP 26, 414 (1968).

131) We assume that T has a fast fall-off in $k_\perp$.

132) An exception is the suggestion made in Ref. 125) that Pomeron exchange between the upper and lower blobs in Fig. 16a is not negligible. It appears, however, that this effect is small (P.V. Landshoff, and J.C. Polkinghorne, private communication and to be published). Also, the "wee" partons discussed in Ref. 120) are contained in the remaining diagrams. It is argued in Ref. 120) that order by order the wee partons are negligible, but that their sum may be important.

133) Al Mueller suggested to me that these corrections may be of importance here. I thank him for informative discussions.

134) R.A. Brandt, A. Kaufman and G. Preparata - to be published.

135) In Ref. 3), K. Wilson has compared the LC and parton approaches in the context of the multiperipheral model.

136) I thank Al Mueller for discussions on the nature of this singularity.

137) The inclusion in the remaining diagrams of Pomeron exchange and/or "wee" parton exchange [Ref. 132)] is particularly suggestive here.

138) J.H. Christenson et al. - Phys.Rev. Letters 25, 1523 (1970).

139) For a review, see :
F. Zachariasen - Physics Reports 2C (1971).

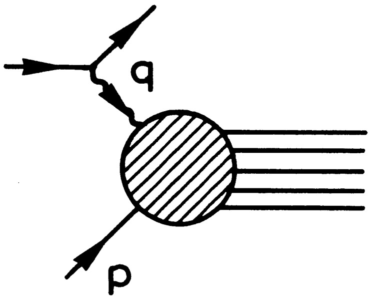


FIG. 1

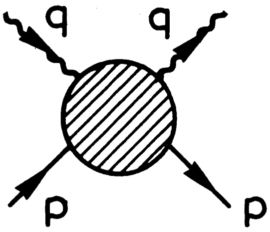


FIG. 2

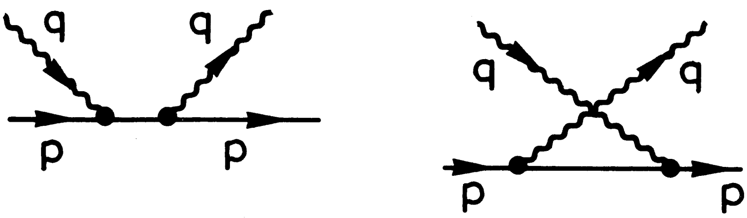


FIG. 3

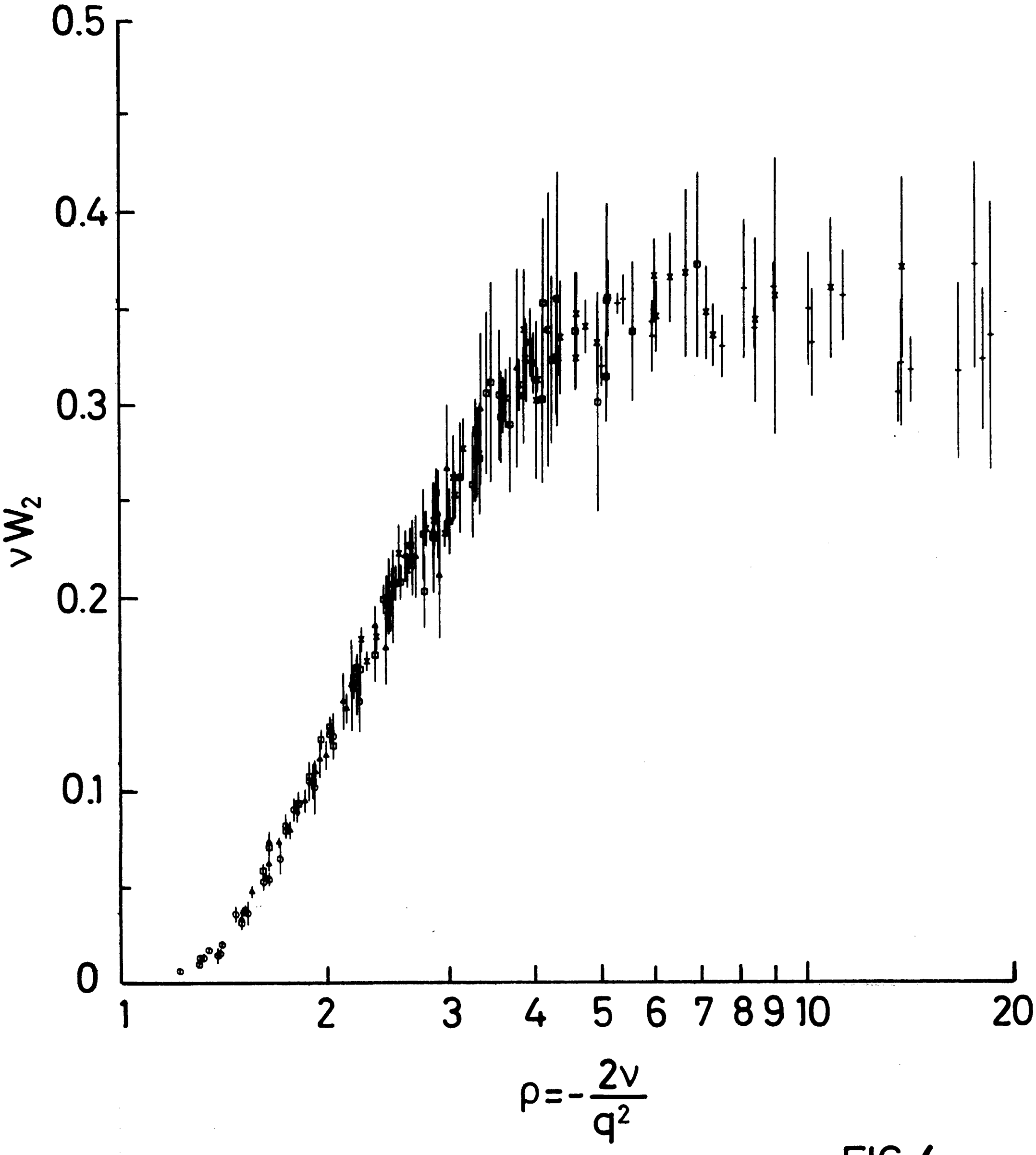


FIG.4

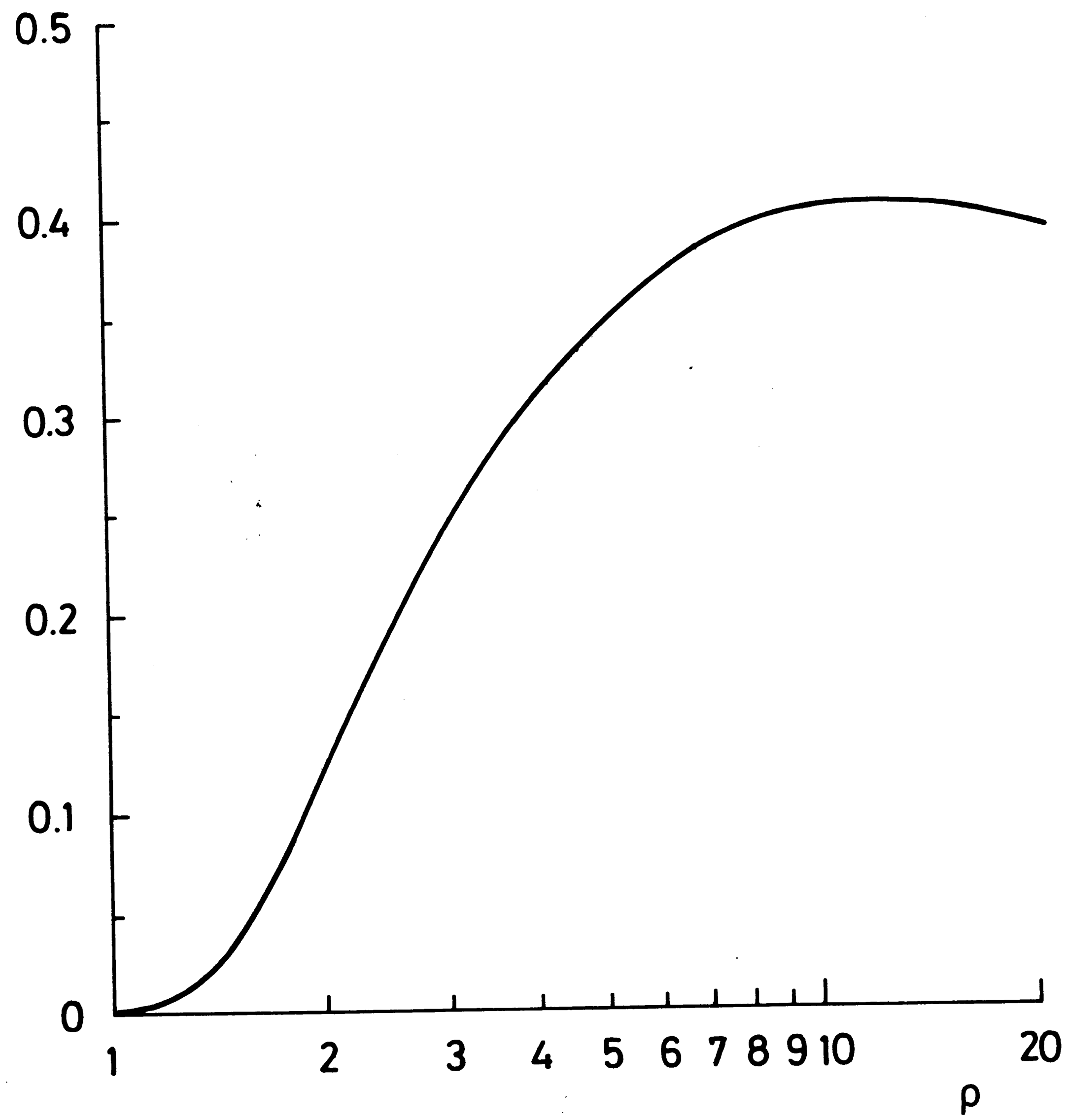
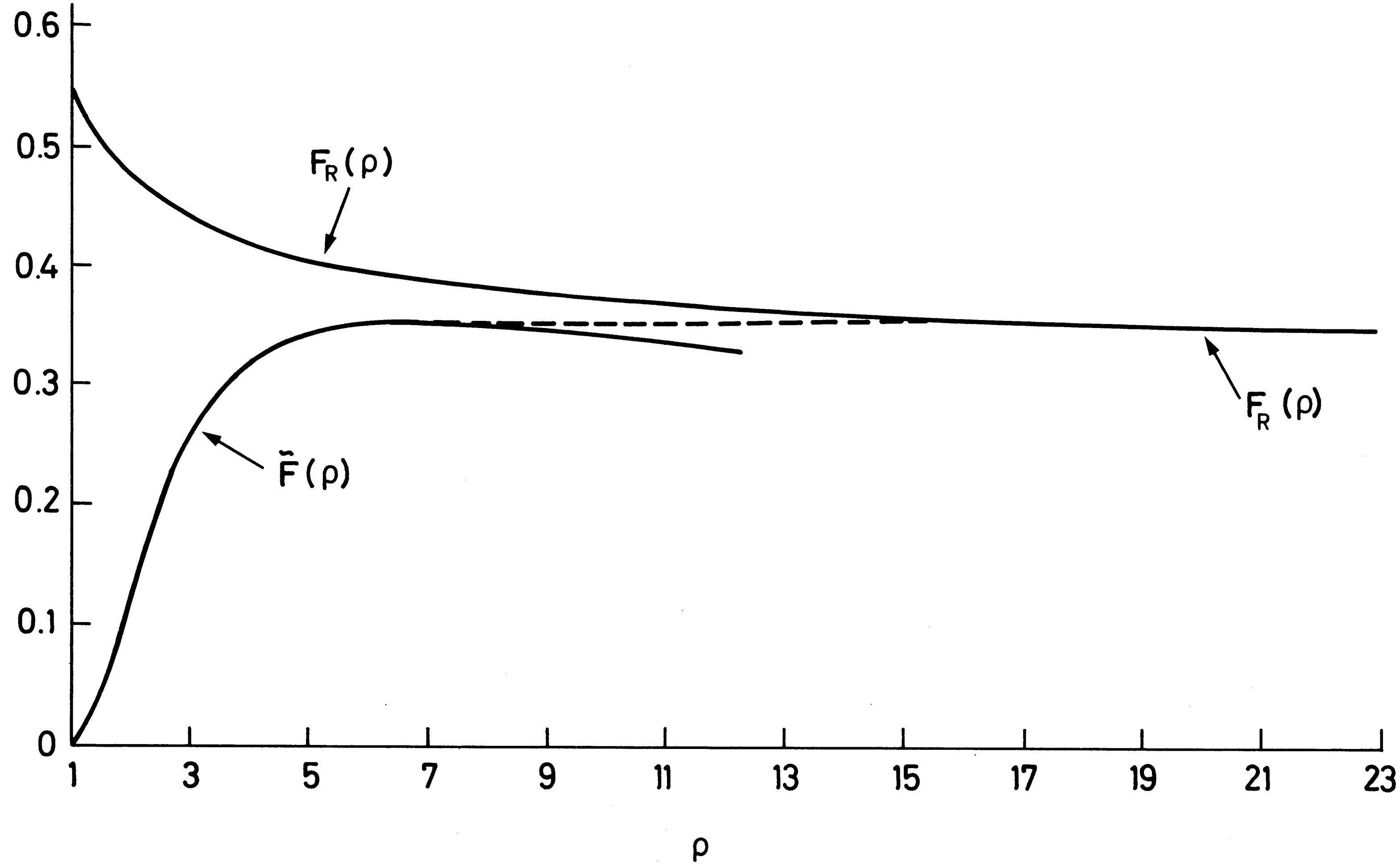


FIG.5



0.6
0.5
0.4
0.3
0.2
0.1
0
1
3
5
7
9
11
13
15
17
19
21
23
ρ
$F_R(\rho)$
$\tilde{F}(\rho)$
$F_R(\rho)$

FIG.6

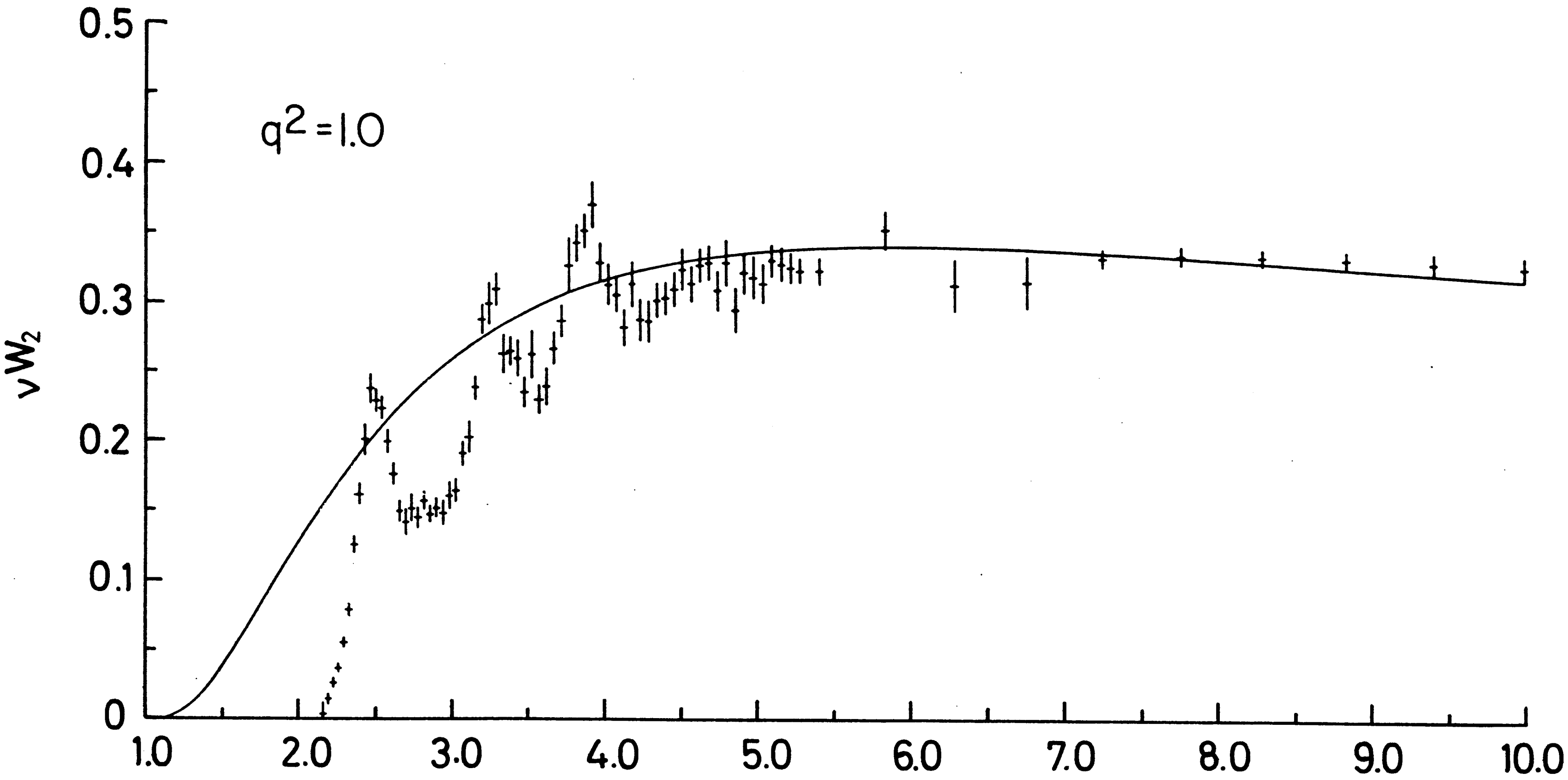


FIG. 7

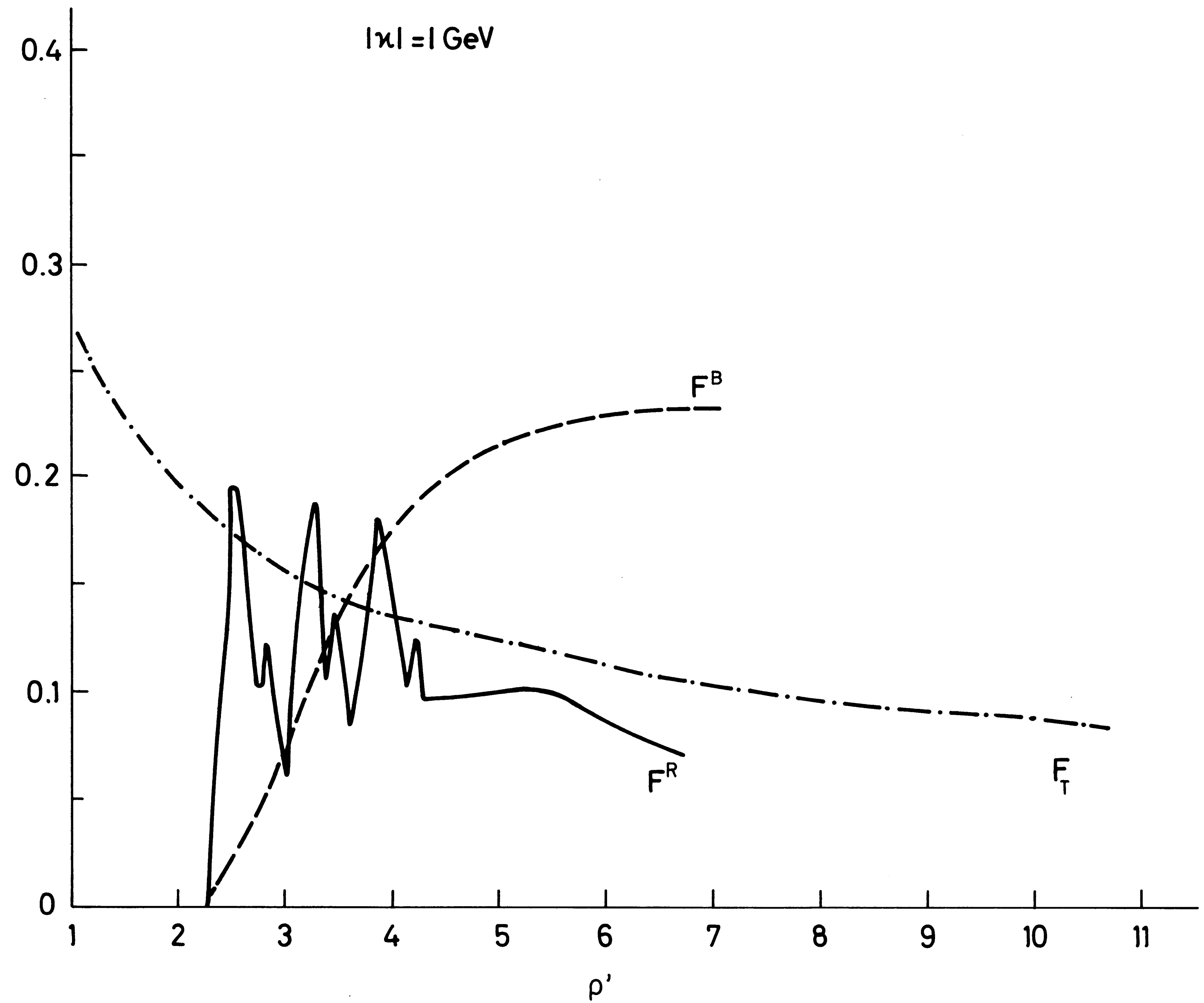


FIG. 8

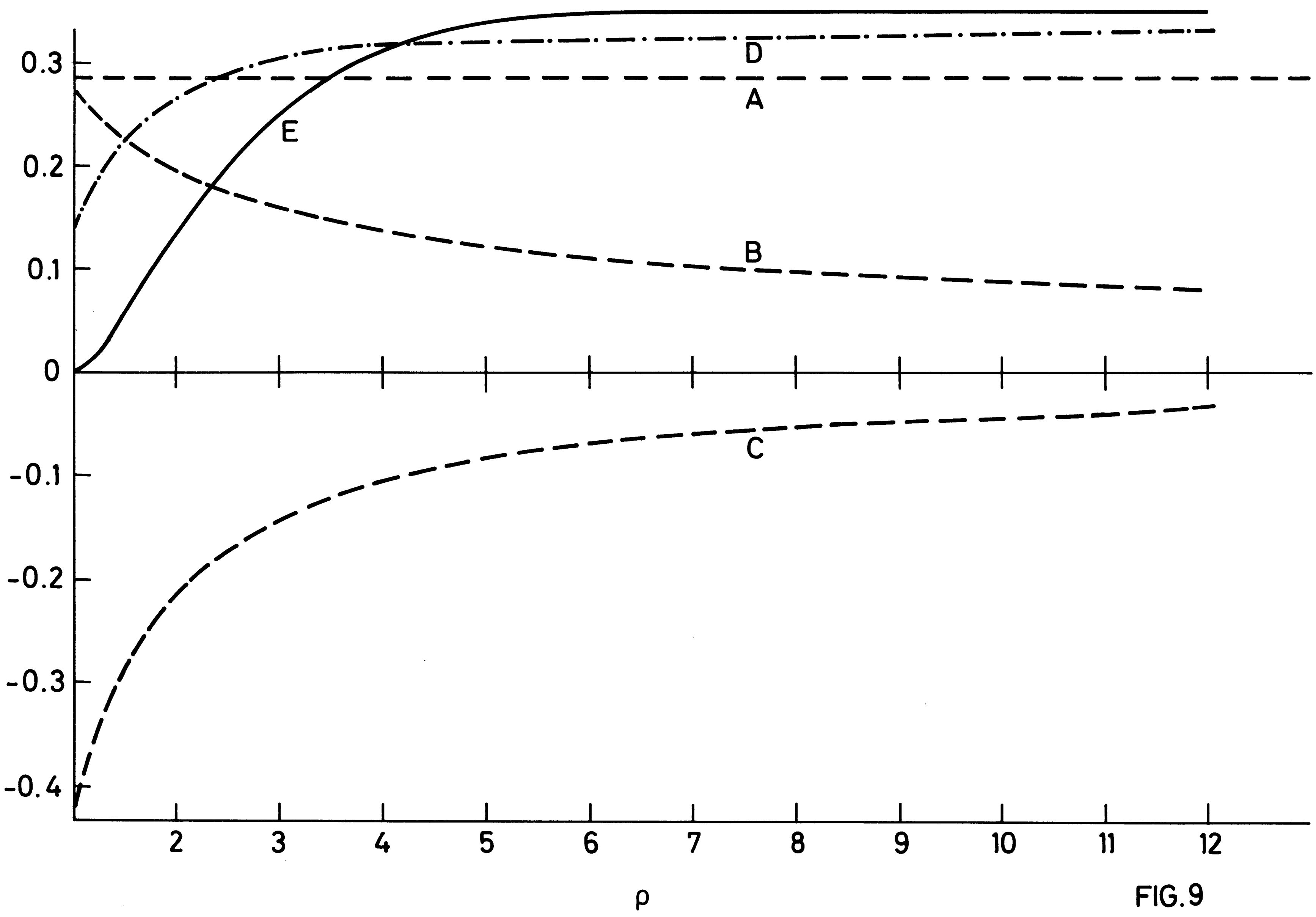


FIG.9

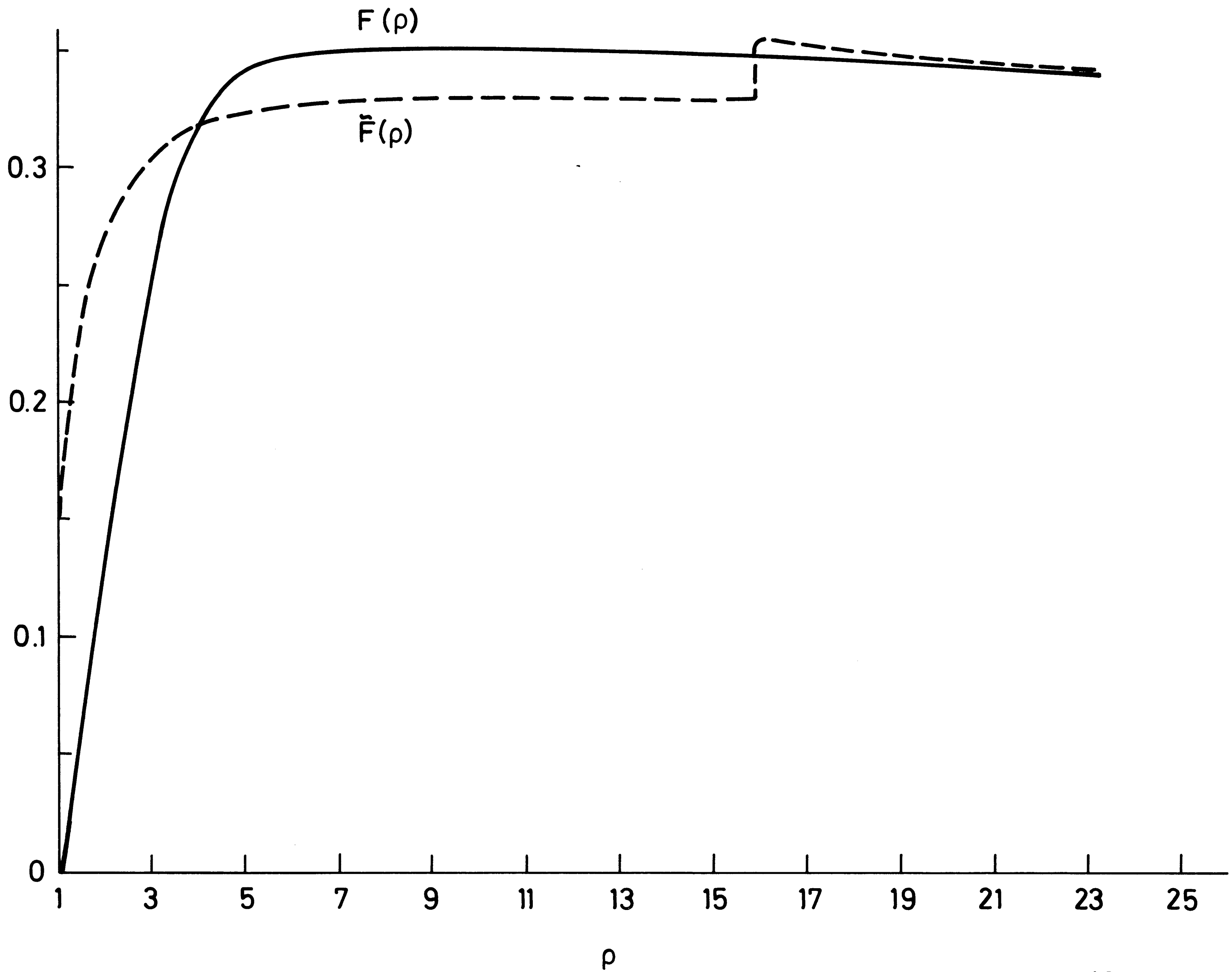


FIG.10

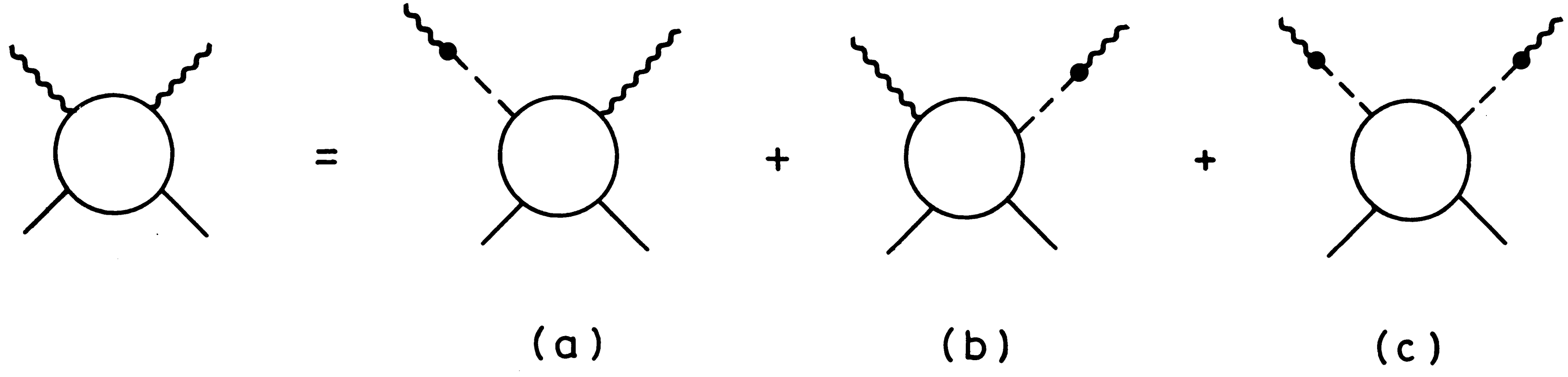


FIG. 11

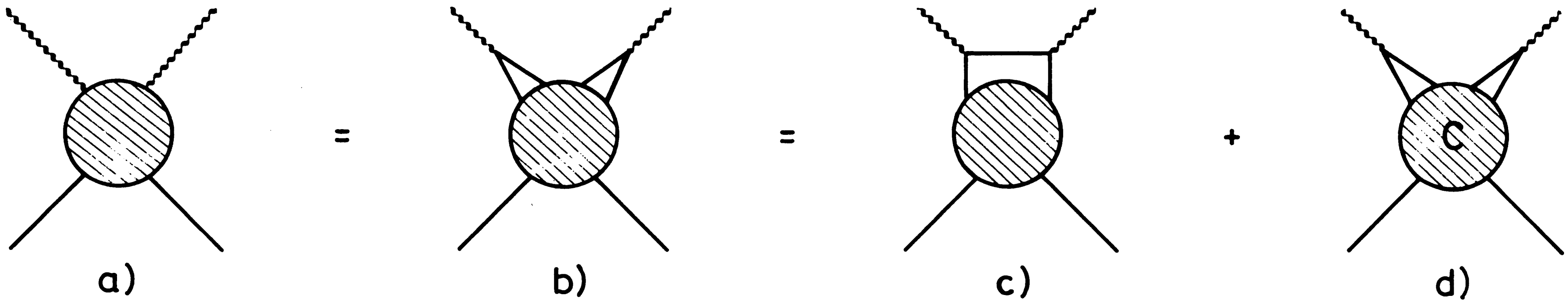


FIG. 12

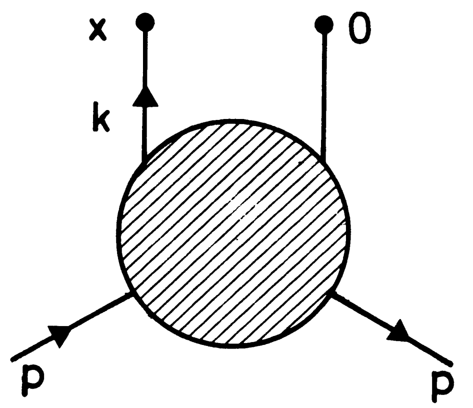


FIG.13

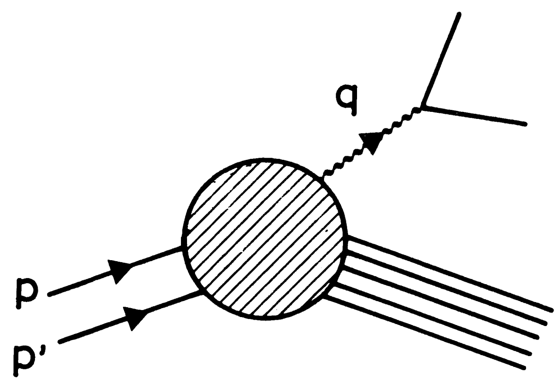


FIG.14

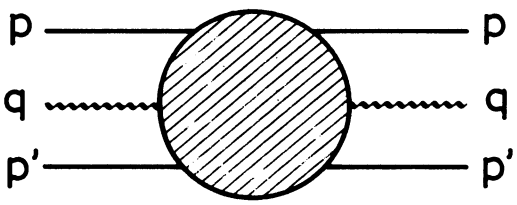


FIG.15

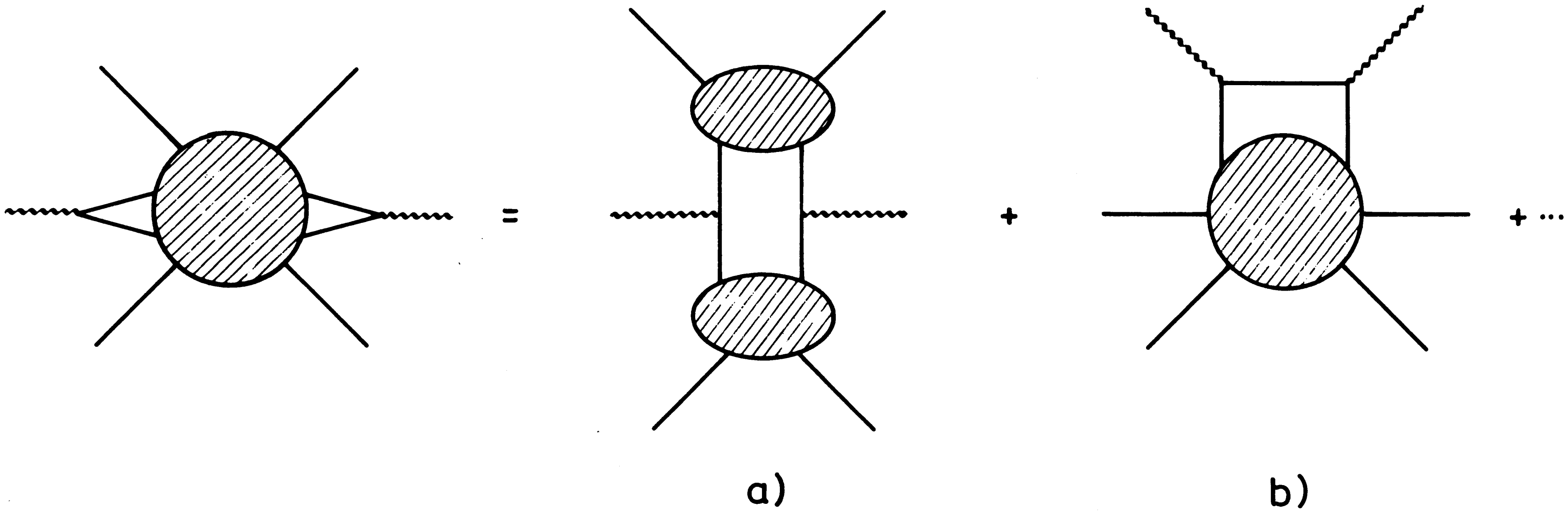
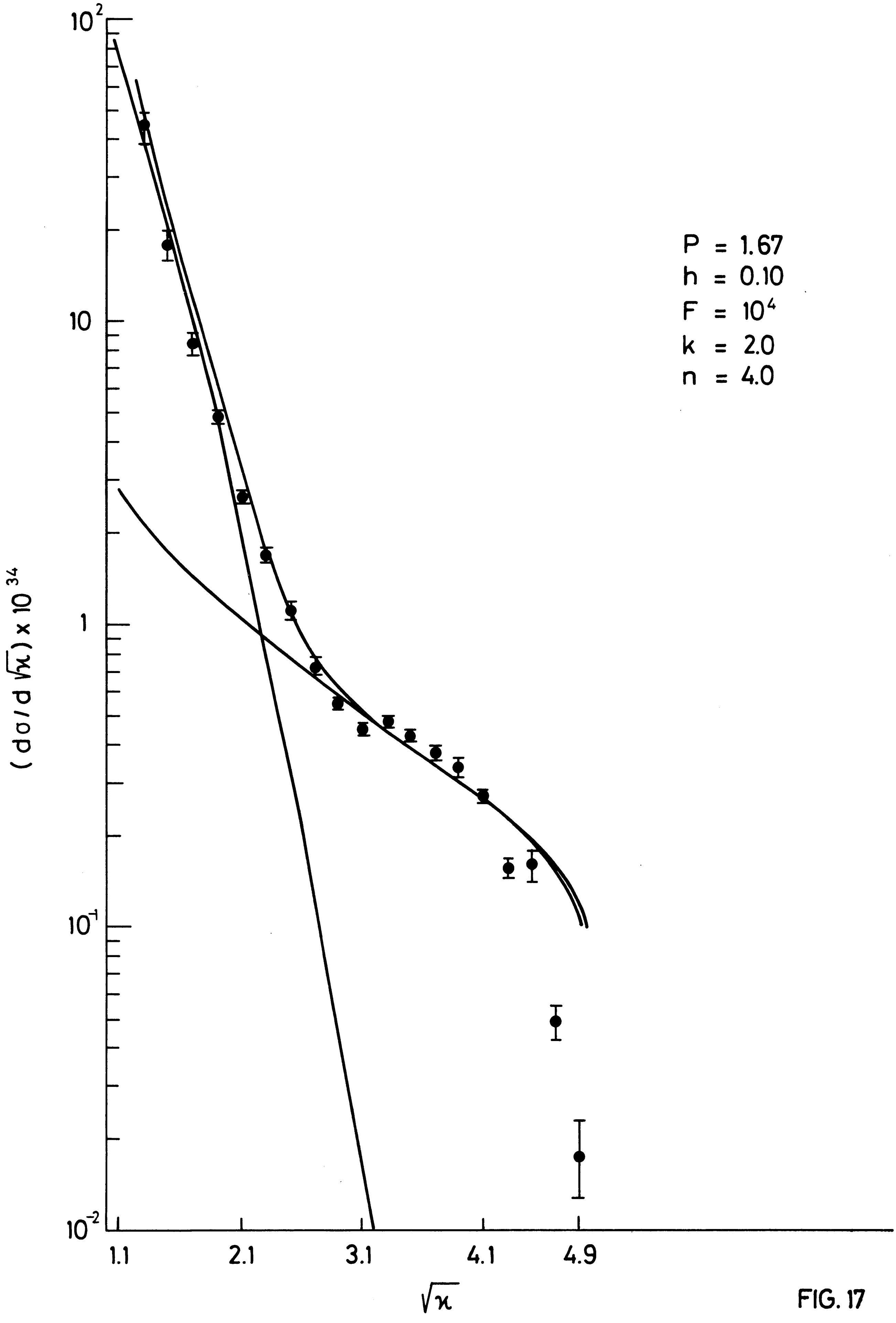
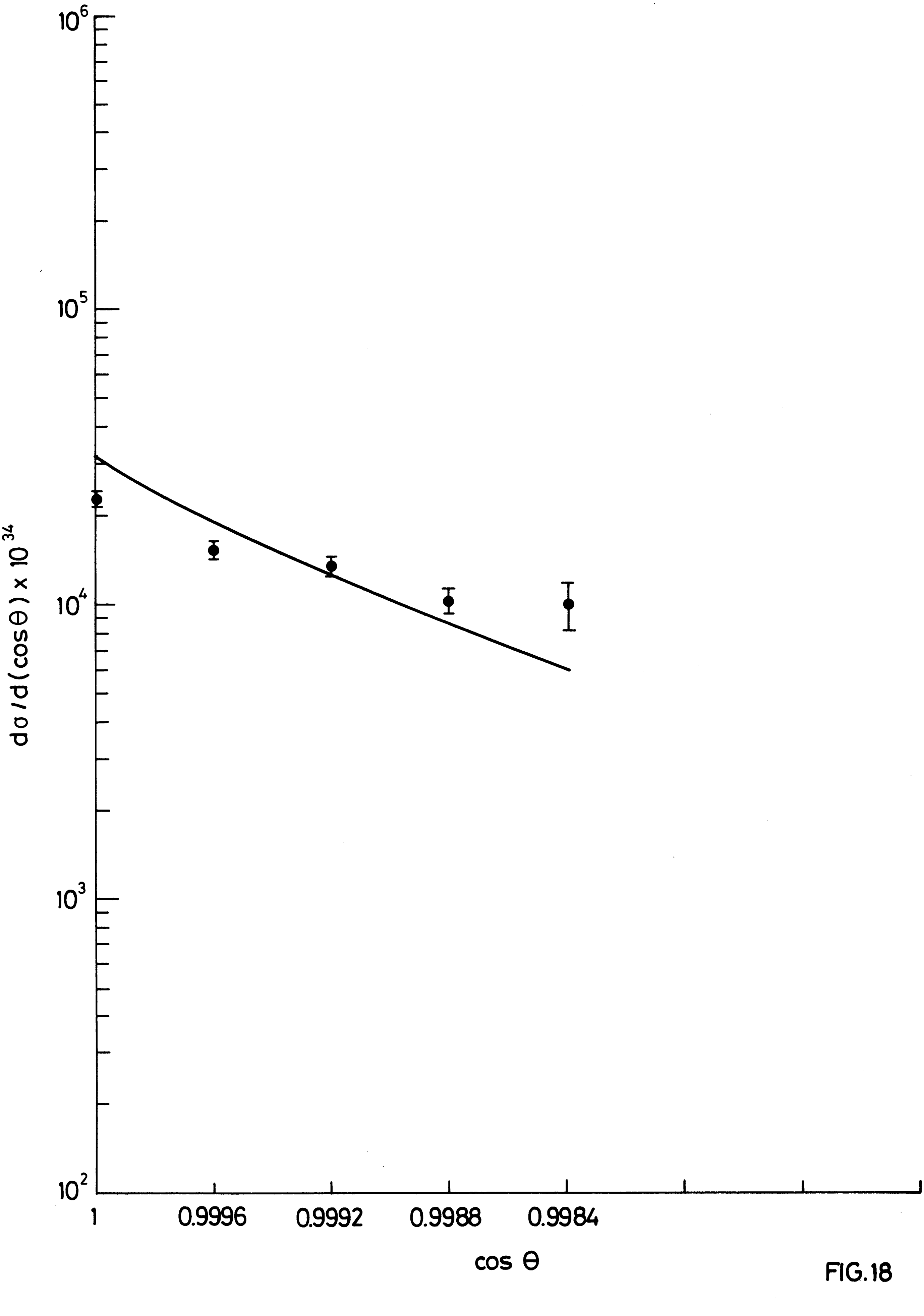


FIG.16



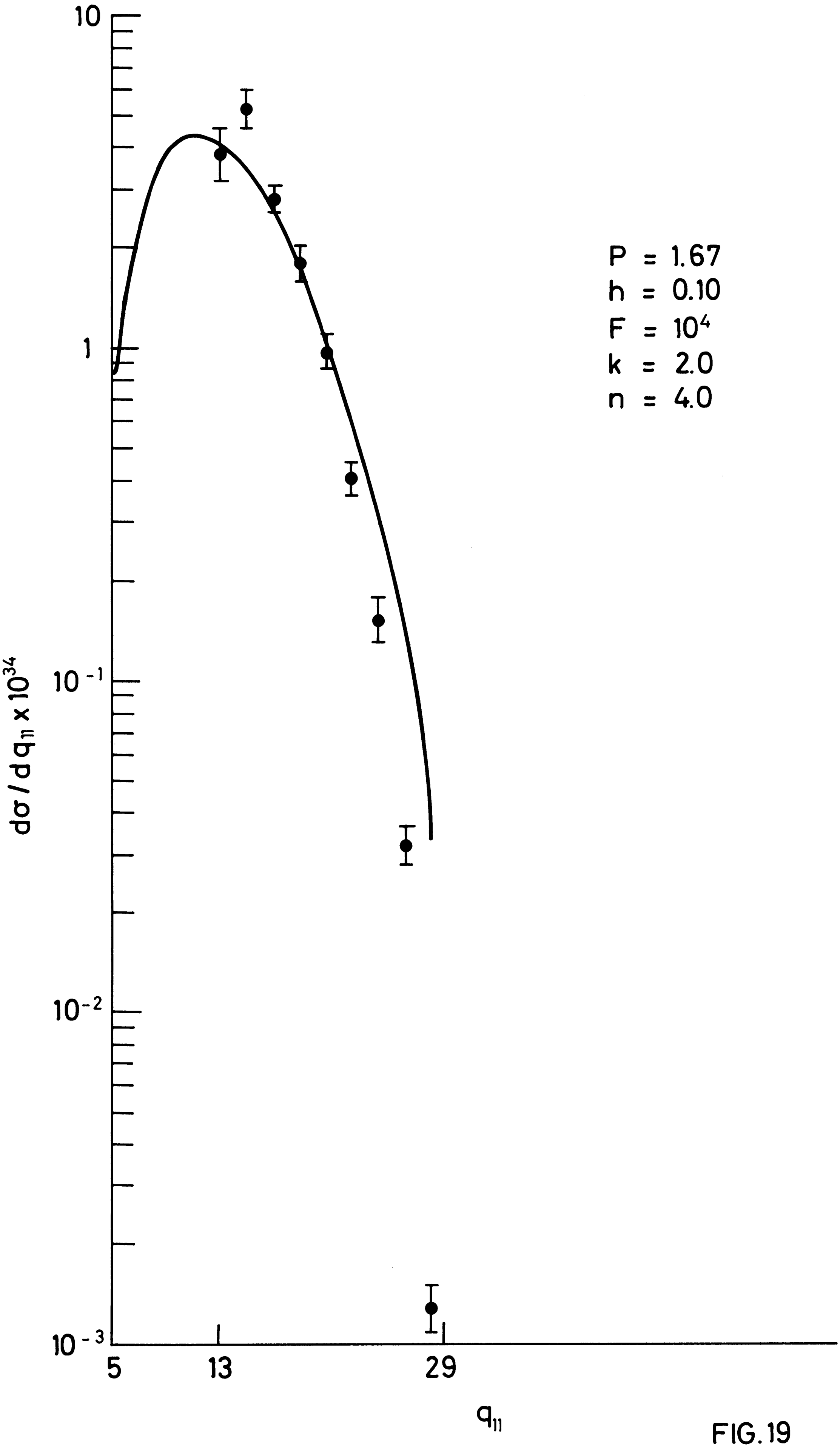
10^2
10
1
10^{-1}
10^{-2}
$(d\sigma/d\sqrt{x}) \times 10^{34}$
1.1
2.1
3.1
4.1
4.9
$\sqrt{x}$
P = 1.67
h = 0.10
F = 10^4
k = 2.0
n = 4.0

FIG. 17



10^6
10^5
10^4
10^3
10^2
$d\sigma/d(\cos\theta)\times10^{34}$
1
0.9996
0.9992
0.9988
0.9984
$\cos\theta$

FIG. 18



10
1
10^{-1}
10^{-2}
10^{-3}
$d\sigma / dq_{11} \times 10^{34}$
5
13
29
q_{11}
P = 1.67
h = 0.10
F = 10^4
k = 2.0
n = 4.0

FIG. 19

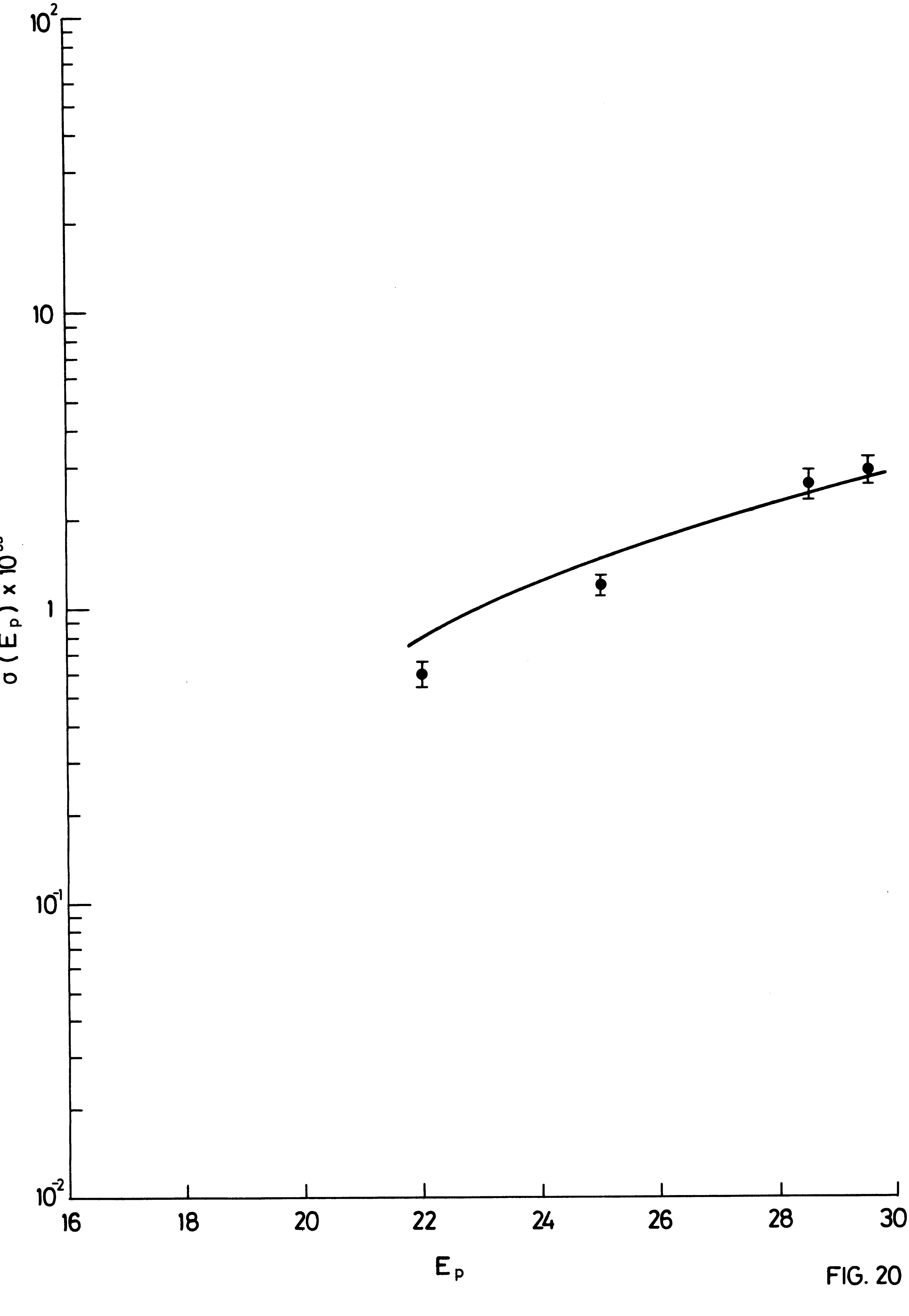


FIG. 20