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HOW WELL DOES νW_2 SCALE? +)

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A B S T R A C T

We investigate the breaking of scale-invariance of νW_2 in ω at $W=2$ in some detail for all values that have been experimentally separated from W_1, and introduce a scaling-breaking parameter B to give a quantitative discussion of the degree of scale-invariance of both structure functions. The scaling of νW_2 in ω' is demonstrated by an explicit parametrization.

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Even though the scaling of νW_2 in ω $(=2m\nu/q^2)$ in the deep inelastic region $(2\leq W,\ 1\leq q^2)$ has been widely accepted [1] as a rather good approximation of experimental data, it has nevertheless been recognized [2),3)] that some breaking of scaling is observable at $W\sim 2$, just beyond the resonance region. Confining our attention to values of νW_2 which have been experimentally separated [4)] from $2mW_1$, here we wish to study this deviation from scale-invariance in greater detail, and to give a quantitative description of its behaviour by means of a scaling breaking parameter B. We also confirm the observation [2),5)] that νW_2 scales much better in the other scale-invariant variable ω' $(=1+W^2/q^2)$ by giving it an explicit parametrization in this variable.

We showed before [6)] that $2mW_1/(1+q^2/\nu^2)$ is consistent with scaling in the region $2\leq W\leq 4$, $1.5\leq q^2\leq 11$, so that, through the relation

$$\nu W_2 = (1+R)\,2mW_1/[\omega(1+\frac{q^2}{\nu^2})] \tag{1}$$

and that fact that $R(=\sigma_s/\sigma_t)$ is well approximated by 0.18, it seems νW_2 should also scale in this region.

This, however, is not the case. As shown in Fig. 1, the points of νW_2 having $W=2$ clearly form a separate curve lying above the single curve of the other points. To confirm this, we carry out the curve-fitting test previously employed on $2mW_1$ [6)] by using the Moffat-Snell formula [1)] as a reasonable interpolation:

$$\nu W_2 \sim \left(\frac{\omega-1}{\omega-\omega_0}\right)^3\left(a\omega^{-\frac{1}{2}}+b\right) \tag{2}$$

It is found that, as expected, this formula can fit either the points of $W=2$ or all the other points in the same interval $1.2<\omega<3.1$ well ($\chi^2=0.6$ and 2.7 for three and seven degrees of freedom, respectively, in the two cases), but fails badly (with confidence limit <1%) if made to fit both sets of points at the same time.

A close examination of Fig. 1 shows that the scaling-breaking appears only in the region $2 \leq W \leq 2.5$, $1 \leq \omega \leq 3$, as has already been noticed [2),3)] for a much larger set of data points obtained under the assumption $R = 0.18$ or $R = 0$. This result does not contradict Eq. (1), because the percentage error of νW_2 is in general smaller than that of $2mW_1/(1+q^2/\nu^2)$ by a factor of two to three, so that the statistical consistency with scaling of the latter by no means implies that of the former.

We suggested [6)] that it seems desirable to have a quantitative indication of the "degree" of scaling instead of simply asking whether scaling holds or not. This can be done by the use of a scaling-breaking parameter $B(\omega, W)$ defined as the percentage slope of the structure functions $F_i(\omega, W)$ $(F_1 = 2mW_1,\ F_2 = \nu W_2)$ in W integrated over a segment of W at fixed ω :

$$B_i(\omega, W) = \int_W^{W+\Delta W} \frac{1}{F_i} \frac{\partial F_i}{\partial W} dW \qquad (i = 1, 2)$$

$$= \ln \frac{F_i(\omega, W+\Delta W)}{F_i(\omega, W)} . \tag{3}$$

B_i can be roughly determined by fitting the Moffat-Snell expansion [1)] $f_i(\omega, a, b)$ (we set $\omega_0 = 0$) individually to each set of points of F_i having the same W (which can be done with good confidence in all cases) and then use the resultant expressions in Eq. (3). To estimate the error in B we vary the parameters a and b from their optimal values to find the limit curve in the (a, b) plane where the confidence of the fit falls to 50%. The maximum deviation of $f_i(\omega, a, b)$ from its optimal value when evaluated on this curve is taken to be the error in f_i, from which the error in B follows.

The results are shown in Fig. 2, where we plot B_i $(i=1,2)$, for $\Delta W=0.5$ and $W=2$ and 2.5, as broad bands bounded by the 50% confidence limit curves. Several interesting features are evident. Since deviation from scaling means B_i significantly different from 0, we expect $|B_i|$ to decrease as W increases at fixed ω, which is observed. We also see that for $W=2$ the breaking of scaling in $2mW_1$ is quite large, and increases with ω, whereas it is much smaller in νW_2 and decreases to 0 at $\omega \sim 3$ [7]. As expected, at $W=2.5$, no significant scaling-breaking is observed in νW_2, but this does appear in $2mW_1$ to approximately the same extent as in νW_2 at $W=2$. To be absolutely sure of the non-scaling behaviour, we also go to the extreme and compute the 99% confidence limit of B_i at $W=2$ (dotted lines in Fig. 2). Even with such a stringent limitation it is seen that scaling-breaking is still clear and evident for both structure functions over considerable segments of ω, bearing out our earlier results from the curve-fitting tests.

Finally, we wish to report an extremely good fit ($\chi^2=13$ for 20 degrees of freedom) by Eq. (2) to all the data on νW_2 [4] if ω' instead of ω is used as the variable [8] and the optimal values $\omega_0=-0.24$, $a=2.444$, $b=-0.307$ are used (Fig. 3) This shows explicitly that, just like $2mW_1$, νW_2 also scales much better in ω' in the region of our concern. The negative value of b is rather embarrassing, since it implies νW_2 becomes negative at large ω, and, hence cannot have much physical significance. But the point beyond which this happens $(\omega' \gtrsim 60)$ is rather remote, and we certainly do not expect the fit can be extended to cover such a far-out region.

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REFERENCES AND FOOTNOTES

1) See for instance
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G. Domokos, S. Kovesi-Domokos and E. Schonberg, NYO-4076-12 (1970);
T.D. Lee, NYO-1932(2)-187 (1970);
O. Nachtman, Orsay preprint LPTHE 70/49 (1970).

2) E.D. Bloom, G. Buschhorn, R.L. Cottrell, D.H. Coward, H. DeStaebler, J. Dress, C.L. Jordan, G. Miller, L. Mo, H. Piel and R.E. Taylor, SLAC-PUB-796 (1970).

3) C.W. Gardiner and D.P. Majumdar, NYO-3399-236

4) In this paper we deal exclusively with the values of W_i listed in Table III of Ref. 2).

5) E.D. Bloom and F.J. Gilman, SLAC-PUB-779 (1970)
F.J. Gilman, SLAC-PUB-842 (1970)
C.H. Llewellyn Smith, SLAC-PUB-843 (1970)

6) Fong-Ching Chen, CERN Preprint TH.1285 (1971).

7) One also expects scaling to become better at smaller ω. $B_2(W = 2)$ seems to contradict this because B_2 is essentially a percentage difference in F_2, which is quite small at low ω. If we multiply B_2 by the mean value of F_2 at $W = 2$ and 2.5, then the corresponding band curves toward 0 at low ω as expected.

8) After such a substitution $(\omega'-1)^3$ can no longer be interpreted as a threshold factor. We are grateful to Professor J. Prentki for calling this to our attention.

FIGURE CAPTIONS

Figure 1 : $\nu W_2/\omega$ plotted against ω. The solid curves are fits by Eq. (2) to $W = 2$ and the other points, respectively. The dashed curve is a fit to all points in $1.3 < \omega < 3.1$.

Figure 2 : The scaling-breaking parameter B as a function of ω. The cross-hatched bands give the 50% confidence limit of B. The dashed lines give the 99% confidence limit in case of $W = 2$.

Figure 3 : νW_2 as a function of ω'. The smooth curve is a fit by Eq. (2) with ω' as the variable.

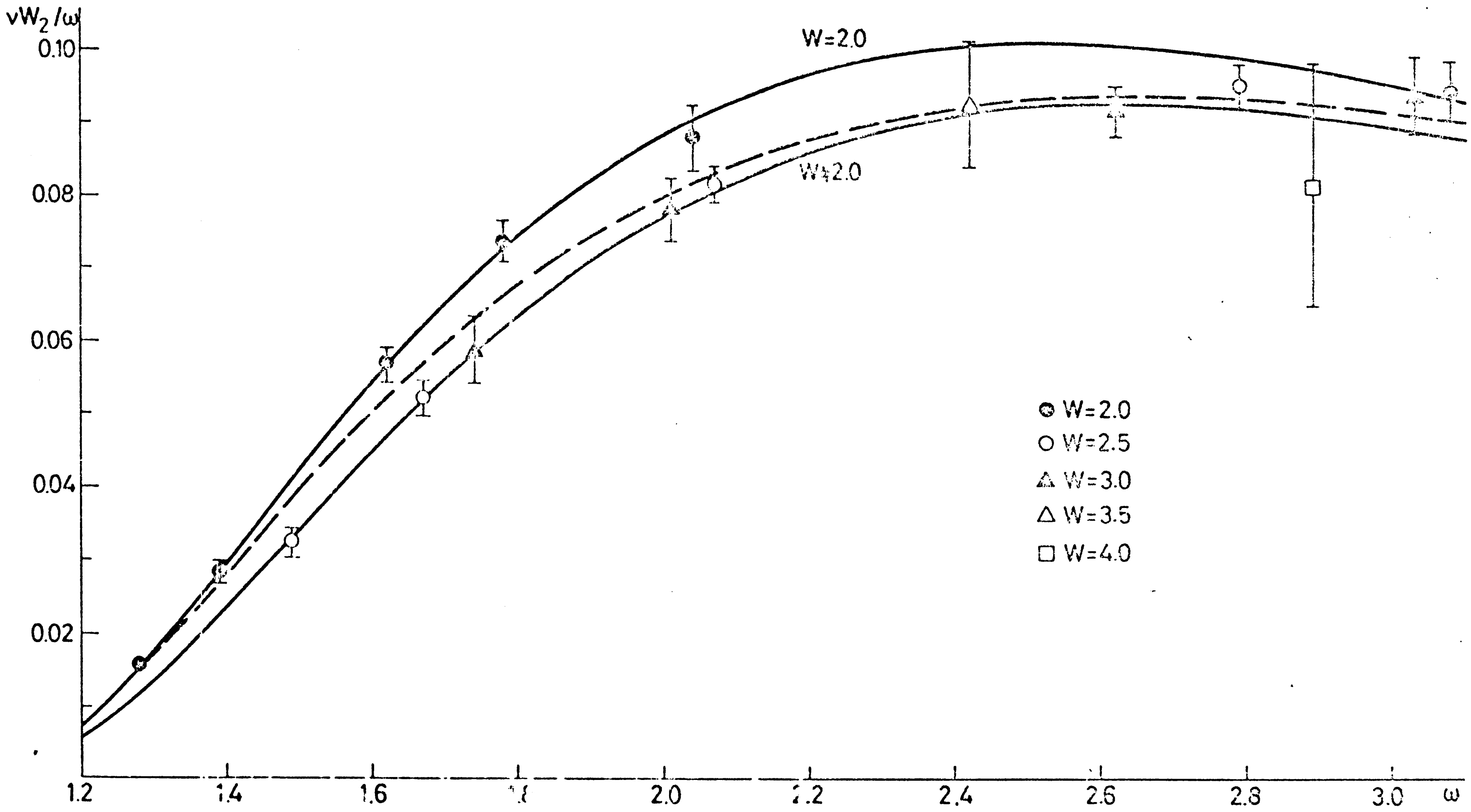
$\nu W_2/\omega$
0.10
0.08
0.06
0.04
0.02
1.2
1.4
1.6
2.0
2.4
2.6
2.8
3.0
ω
W=2.0
W≠2.0
W=2.0
W=2.5
W=3.0
W=3.5
W=4.0

FIG.1

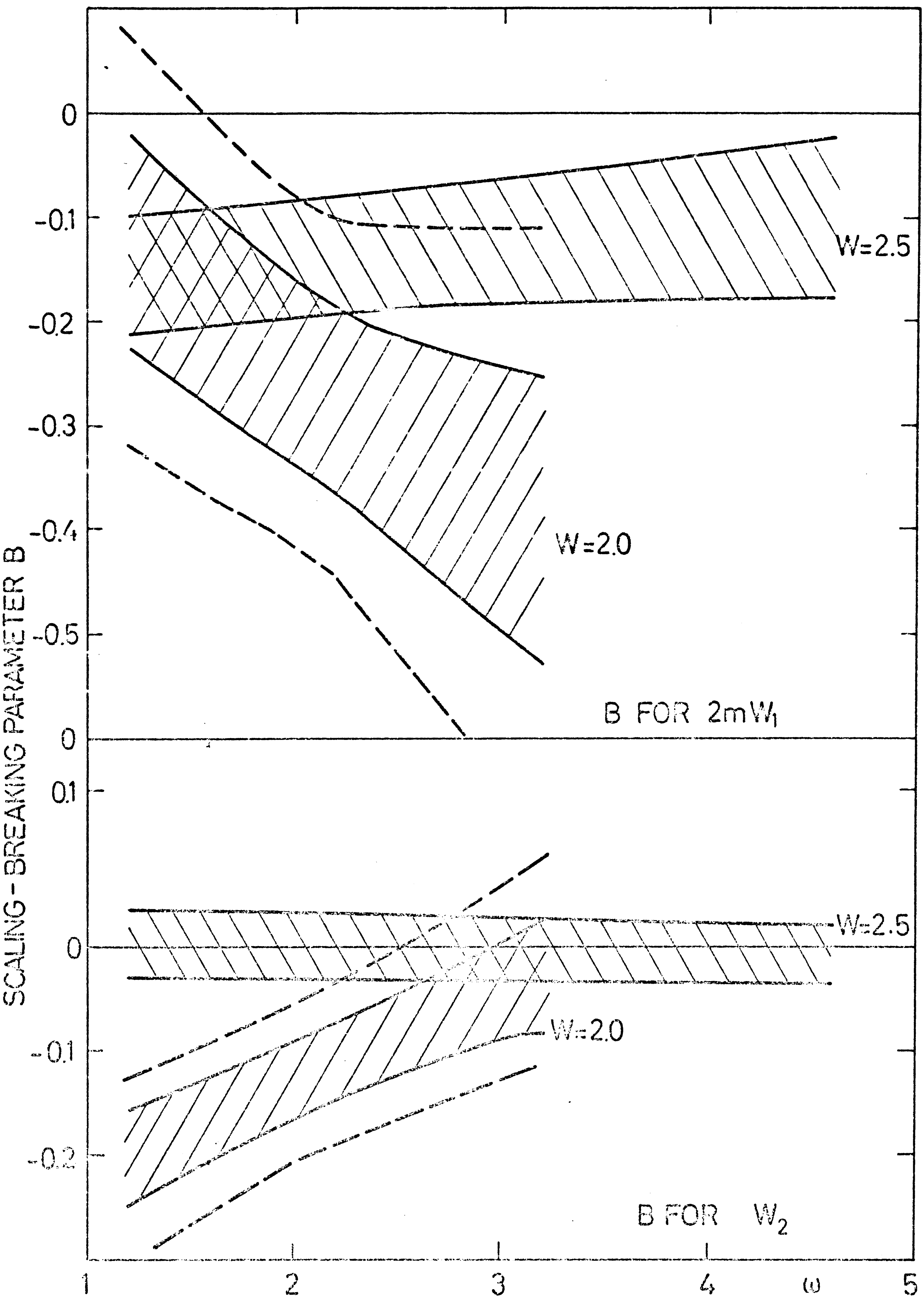
SCALING - BREAKING PARAMETER B
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
0.1
0
-0.1
-0.2
W=2.5
W=2.0
B FOR 2mW1
W=2.5
W=2.0
B FOR W2
1
2
3
4
ω
5

FIG.2

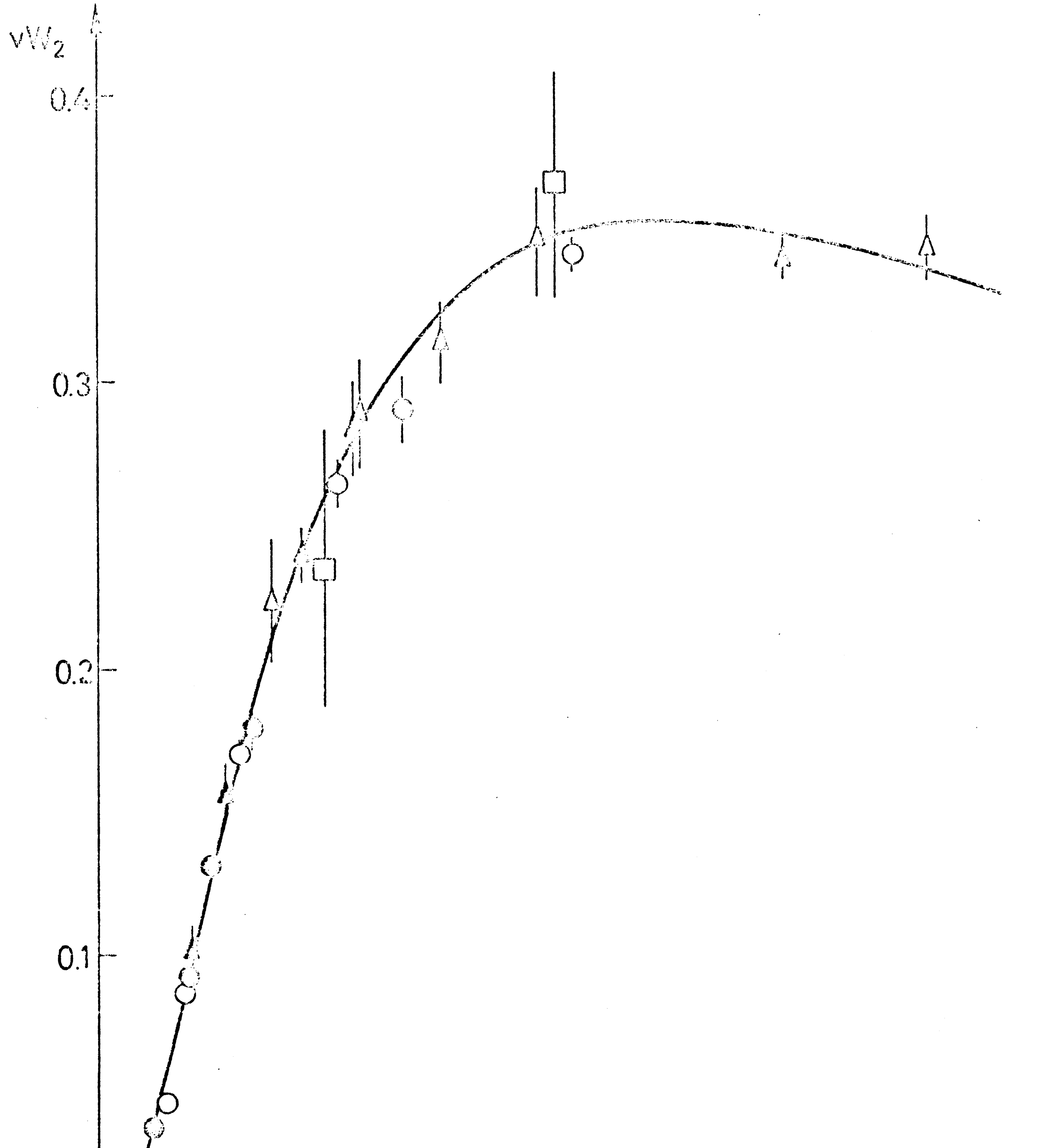

FIG.3