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CALCULATION OF EFFICIENCY OF SI-DETECTOR IN THE RANGE 0.04 - 1.5 MeV

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1. INTRODUCTION

In the feasibility studies on r.f. superconducting cavities currently performed at CERN [1], it has proved desirable to reach a better understanding of the phenomena, which cause the limitations on the accelerating electrical field strenght. Consequently the CERN EF Radio Frequency Group has placed on the outer surface of the cavity:

(a) an array of 39 carbon resistors for temperature mapping (described in ref. [2];

(b) an array of ten solid state X-ray detectors for measuring the intensity of X-rays emitted by electrons impacting on the cavity walls.

The detectors have so far been used in integrating mode, giving a current proportional to the power deposited in the detector by γ rays, thus only providing information on the relative energy deposition by X-rays at a certain time.

As electron-impacting is one very plausible cause for the breakdown of the cavity, the phenomenon calls for closer investigation, and as one way of doing this is by studying the spectrum of the X-rays emitted, the wish for more detailed information from the detectors emerges. This report is a first approach to this goal, presenting a calculation of the absolute efficiency of the X-ray detectors used. The reason for performing a theoretical calculation rather than doing a neat little experiment is the very low γ ray efficiency of the detectors, making it impossible to do a calibration with ordinary radioactive sources, as any signals produced are swallowed up by electronic noise. The detectors are nevertheless useful

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for X-ray measurements in connection with the cavity, as the amount of radiation produced here is very high. We estimate the radiation intensity produced at maximum acceleration field level $\sim$ 4 MV/m and at the position of detectors ranging between 300 rad/h at the peaks of electron impact and 16 rad/h at minimum impact region. Furthermore, at He temperature, the leakage current of the detectors used drops to an unmeasurable value.

2. THE PRINCIPLE OF OPERATION

The solid state X-ray detector consists of a slice of semiconductor material between two electrodes. A semiconductor has an electron structure as indicated in fig. 1. The permissible quantum states for electrons are characterized by a division of the allowed energy values into bands, of which in the ground state the lowest lying are completely filled while higher bands are completely empty. The filled and empty bands are separated by an energy gap E_g. For Si E_g = 1.10 eV, for Ge E_g = 0.67 eV at 300°K. A slight temperature dependence is observed, the corresponding numbers at 0°K being 1.1 eV and 0.75 eV.

As no conduction of the material can take place by help of the electrons in a filled band (a well known fact from standard solid state physics, see for instance ref. [3]), the departure of the actual state of the semiconducting sample from the ground state is crucial for the electrical properties. The most important cause of deviation from the ground state is due to the finite temperature of the sample, causing electrons from the highest filled band (often referred to as the valence band) to be excited into the lowest empty band (the conduction band) leaving an empty place in the valence band which effectively reacts as a positively charged particle, called a hole. The hole in the valence band and the electron in the conduction band can conduct the electrical current, thus giving rise to a non-infinite resistivity. The number of electronhole pairs in equilibrium at temperature T is given by a characteristic Boltzmann factor.

$$N_{e\ell} = N_{hole} \propto \exp(-E_g/2kT) \quad . \qquad (1)$$

Another way of creating electron-hole pairs is by interaction with an incident particle (e.g. an electron) which looses energy in the semi-conducting sample by exciting electrons into the conduction band, as indicated in fig. 1. These events are called ionizations. The electrons (and holes) thus created interact very rapidly with the crystal-lattice giving away energy to phonons themselves falling to the bottom of the conduction band (respectively rising to the top of the valence band). A typical time for such a phonon scattering is 10^{-13} - 10^{-12} s. Taking into account the energy carried away by phonons, the average energy required to create an electron hole pair is ε_0 = 3.66 eV in Si and ε_0 = 2.96 eV in Ge. These numbers should be compared to a typical value of 30 eV for the ion pair creation in a gas ionization chamber.

An X-ray detection takes place when an incident X-ray interacts with the atomic electrons, producing photoelectrons, Compton electrons or electron-positron pairs. These primarily produced electrons - of high energy on an atomic scale - interacts with other electrons in the sample creating electron hole pairs, which hopefully are collected by the electrodes thus giving rise to a short charge pulse. According to the above, the charge created is given by

$$\text{charge} = \text{energy deposited}/\varepsilon_0 \qquad (2)$$

the response thus being directly proportional to the energy deposit.

The collection of charges created should be effective to provide a good resolution of the detector. This can be achieved by applying a sufficiently high voltage across the electrodes, so that newly created pairs are promptly swept away, the time of travel being small compared to the recombination time. Also the active region depends on the applied voltage. The active region is that spatial volume, in which the electric field is strong enough to ensure an effective collection (that is the volume within which an X-ray interaction should take place to be a good event). The effective thickness of the detector varies as

$$W = k(\rho V_{bias})^{1/2} \qquad (3)$$

as long as saturation (full depletion) is absent. Here k is a constant of order unity, ρ the resistivity of the material and V_{bias} the applied voltage.

As the operating temperature is never exactly 0°K, some thermally created pairs are always present, so that a small current, the leakage current, is flowing, and the signal marking the X-ray detection appears as a small enhancement of this constant d.c. leakage current. To keep the leakage current low, it is often required to work at temperatures below room temperature, and high purity samples (or effectively compensated as in lithium drifted Si and Ge detectors) should be used.

Impurities also affect the performance of the detector in the sense, that they behave as potential wells, in which electrons or holes are trapped and held for times greater than the collection time; that is, when the electron or hole is released from the impurity trap, the signal marking the event is already over. Too high impurity concentration makes this a serious effect, disturbing the proportionality between charge collected and energy deposited.

Further information on solid state detectors can be obtained from ref. [4] and references therein.

3. THE DETECTORS USED

The actual detectors used in the r.f. superconducting cavity experiment are described in ref. [5]. The material is high purity Si, the shape a circular plate of radius 3.1 mm and thickness 0.1 mm (that is the maximal depletion layer thickness obtainable).

The detectors were designed for an operating voltage of 50 V at 300°K, but during the actual measurements with the cavity, no bias voltage was supplied. This is due to the fact, that at He temperature and below; the resistivity of Si becomes so large, that the bias voltage needed for complete depletion according to eq. (3) is lower than the natural electrostatic potential of the detector (which can also be considered as a p-i-n junction diode).

The detectors are situated approximately 6 mm from the cavity wall (consisting of 2 mm Nb). They are in direct contact with the liquid helium of the bath, the performance in this configuration giving no rise to complaints.

The energies of X-rays expected range from 0.04 MeV to 1.5 MeV. the lower limit is set by the absorption of soft X-rays within the 2 mm Nb cavity walls, while the upper limit is due to the kinematics of the acceleration of electrons in the cavity.

4. THE CALCULATIONS

Fig. 2 shows a typical X-ray detection event. From a pointlike isotropic source a distance R from the detector, an X-ray (of energy E_γ is emitted with an angle α with respect to the axis. It penetrates a length of p into the detector, before it interacts. The probability of interaction in an interval dp around p is given by

$$P\ (\text{interaction}) = (\sigma_{\text{Compton}} + \sigma_{\text{photo}} + \sigma_{\text{pair}}) \cdot N \cdot dp\ , \qquad (4)$$

where the σ's are the respective cross sections (in cm^2/atom), N the density of atoms in the detector material. By the interaction, either a photoelectron, a Compton electron (or an electron-positron pair, but this process is unlikely in the energy range considered) is emitted in a direction specified by (θ,ϕ) with a certain probability, $P_i(\theta,\phi)$ (i = photo or Compton). Here θ is the polar angle with respect to the direction of the incoming photon and ϕ is the azimuthal angle. The path length of the electrons is determined by the geometry of the event

$$\text{length} = \ell(\alpha, p, \theta, \phi) \qquad (5)$$

and consequently the energy deposited in the detector in this event is determined by the electron stopping power in Si. To a first approximation

$$E_{\text{deposited}} = \ell(\alpha, p, \theta, \phi)\ \frac{dE}{dx}\ \ (E_{\text{electron}})^{(*)}, \qquad (6)$$

(*) This formula for the deposited energy is only valid as long as the right hand side is lower than E_{electron}. Otherwise eq. (6) should read: $E_{\text{deposited}} = E_{\text{electron}}$.

where $E_{electron}$ is the kinetic energy of the electron. In case of photoelectric production

$$E_{electron} = E_{\gamma} - I \quad , \tag{7}$$

where I is the ionization potential for the electron kicked out of the atom. In a Compton scattering event, the electron energy depends on the scattering angle

$$E_{electron} = E_{\gamma} \frac{2\gamma \cos^2\theta}{(1+\gamma)^2 - \gamma^2\cos^2\theta} \quad ; \quad \gamma = \frac{E_{\gamma}}{0.511 \text{ MeV}} \quad . \tag{8}$$

For higher accuracy it can become necessary to replace eq. (6) by an iteration, in say n steps, calculating the energy loss of the electron by traversing a length of $\ell(\alpha,p,\theta,\phi)/n$, recalculate the stopping power for the new, lower energy and so on.

The calculations performed contain an averaging over α and integrations over p, θ and φ as well as the above mentioned iteration of electron traversal. The number of steps in each integration has been chosen so as to give results accurate to an estimated 1%. Three values of R were used: R = 6, 8 and 10 mm.

5. FORMULAS USED

5.1 Stopping power of electrons

$$-\frac{dE}{dx} = \frac{2\pi \, NZE_0 \, r_0^2}{\beta^2} \left\{ \ell n \left[\frac{(\gamma-1)\beta 2E^2}{2\bar{I}^2} \right] + \frac{1}{\gamma^2} \left[\frac{(\gamma^2-2\gamma+9)}{\gamma} \right] - (2\gamma-1)\ell n2 \right\} \tag{9}$$

NZ electron density in detector ($7.0 \cdot 10^{24}$ cm^{-3})

E_0 rest energy of electron (0.511 MeV)

r_0 classical electron radius ($2.82 \cdot 10^{-13}$ cm)

I mean ionization potential of electrons in detector material (12 Ry)

E total energy of electron (including rest energy)

γ,β usual kinematic factors of electron

$$\beta = \frac{v}{c} \; ; \; \gamma = (1-\beta^2)^{-1/2} = E/E_0 \; . \tag{10}$$

5.2 Angular distribution of photoelectrons

$$P_{ph}(\theta,\phi) = A \frac{\sin^3\theta}{4(1-\beta\cos\theta)^3} \left[\frac{2}{\gamma(1-\beta\cos\theta)} + (\gamma-1)(\gamma-2)\right], \quad (11)$$

where

$$A^{-1} = \frac{2}{3}\gamma^3 + \frac{(\gamma-1)(\gamma-2)}{4} \left[\frac{1}{\beta^3} \ln \frac{1-\beta}{1+\beta} + 2\frac{\gamma^2}{\beta^2}\right]. \quad (12)$$

5.3 Angular distribution of Compton electrons

$$P_c(\theta,\phi) = \frac{4(1+\gamma)^2\cot\theta}{[(1+\gamma)^2 + \cot^2\theta]^2 \sin^3\theta} \cdot \frac{1}{\sigma_{Compton}} \cdot \left(\frac{d\sigma}{d\Omega}\right)_{KN}, \quad (13)$$

where $(d\sigma/d\Omega)_{KN}$ is the Klein Nishina differential cross section for the photon to scatter into $d\Omega$. Taking the angle of the scattered photon to be u

$$u = 2 \arctan \left(\frac{\cot\theta}{1+\gamma}\right); \quad (14)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{KN} = \frac{r_0^2}{2} \frac{(1+\cos^2u)}{[1+\gamma(1-\cos u)]^2} \left\{1 + \frac{\gamma^2(1-\cos u)^2}{(1+\cos^2u)[1+\gamma(1-\cos u)]}\right\}; \quad (15)$$

and

$$\sigma_{Compton} = \int_0^{\pi} \left(\frac{d\sigma}{d\Omega}\right)_{KN} \cdot 2\pi \sin u \, du ; \quad (16)$$

In formulas (13)-(15) γ is E_γ/E_o. Both P_c and P_{ph} are normalized according to

$$\oint P_c(\theta,\phi)d\Omega = \oint P_{ph}(\theta,\phi)d\Omega = 1 \quad (17)$$

where $d\Omega = \sin\theta \, d\theta \, d\phi$. All formulas are taken from ref. [7].

The values of total photoelectric and Compton scattering cross sections are taken from ref. [6].

6. RESULTS

Fig. 3 shows the calculated energy deposit as a function of incoming X-ray energy. A steep rise is seen at very low energies, due to the high photoelectric cross section and the high stopping power for low energetic electrons. To get the current produced in the detector by a flux of $S(E_\gamma)$ (photons of energy E_γ/s) being emitted into the spherical angle of the detector, one should perform the calculation

$$i = S(E_\gamma) \cdot \frac{E_{deposited}(E_\gamma)}{\varepsilon_0} \cdot (1.602 \cdot 10^{-19} \text{ Coulomb}) , \qquad (18)$$

where $E_{deposited}(E_\gamma)$ is the ordinate of fig. 3 and ε_0 is defined as in sect. 2.

Fig. 4 shows the efficiency of the detector, simply defined by the fraction of incoming energy, which is absorbed in the detector

$$\text{efficiency} = \frac{E_{deposited}}{E_\gamma} . \qquad (19)$$

Fig. 5 shows the dependence of energy deposit on source-detector distance. Fig. 6 shows the results of an additional calculation of the electron efficiency of the detector (again defined by formula (19), carried out by an accurate iteration (100 steps) in the sense described in connection with formula (6).

The calculations ignore the ionization potential in formula (7). This can be estimated according to

$$I \sim Z^2 R_y = 2.7 \text{ keV} \qquad (20)$$

for K-shell electrons. This omission in the calculations is justified by the fact, that the soft 2.7 keV X-ray following the ionization (or rather a cascade of X-rays of total energy 2.7 keV) is very likely to be absorbed in the detector giving away its energy to create electron hole pairs.

Also no secondary events of Compton photons are considered. As according to formule (4) the probability for one event with typical values of $\sigma_{tot} = 10^{-23}$ cm^2/atom and dp = t = thickness of detector = 0.01 cm is

$$P(\text{event}) = 10^{-23} \,.\, 5 \,.\, 10^{22} \,.\, 0.01 = 5 \,.\, 10^{-3} \tag{21}$$

it is seen, that the probability for multiple events is very low, of the order of 10^{-5}.

Furthermore, no pair production events have been taken into account. According to ref. [6) the cross section for pair production at 1.5 MeV is $8.8 \,.\, 10^{-27}$ cm^2/atom, less than 0.5% of the Compton scattering cross section.

Acknowledgements

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REFERENCES

[1] Ph. Bernard, H. Heinrichs, H. Lengeler, E. Picasso and H. Piel, First Results with a Superconducting 500 MHz Cavity, CERN/LEP-Note 209 (1980);

Ph. Bernard, G. Cavallari, E. Chiaveri, E. Haebel, H. Heinrichs, H. Lengeler, E. Picasso, V. Picciarelli and H. Piel, First Results on a Superconducting r.f. Test Cavity for LEP, presented at the XIth International Conference on High Energy Accelerators, Geneva, Switzerland, and CERN/EF 80-3 (1980).

[2] H. Piel, R. Romijn, Temperature Mapping on a Superconducting r.f. Cavity in Subcooled He, Internal Note CERN/EF/RF 80-3 (1980).

[3] N. Ashcroft and M. Mermin, Solid State Physics, Holt, Rinehart and Winston (1976).

[4] P. Jarron (in preparation).

[5] G.T. Ewan, Nucl. Instr. & Meth. 162 (1979) 75.

[6] E. Storm, E. Gilbert and H. Israel, Los Alamos Laboratory, University of California, LA-2237, 18 November 1958.

[7] R. Roy and R. Reed, Interactions of Photons and Leptons with Matter, Academic Press (1968).

FIGURE CAPTIONS

Fig. 1 The energy band structure of a semiconductor with some ionizing events indicated.

Fig. 2 Geometry of calculations:

R = 8 mm,
d = 3.1 mm,
t = 0.1 mm.

Fig. 3 The average deposited energy, E_{dep} by an X-ray falling into the solid angle of detector, plotted against the X-ray energy (R = 8 mm).

Fig. 4 X-ray efficiency of detector, defined as E_{dep}/E_{γ} (R = 8 mm).

Fig. 5 Dependence of the energy deposit on the source-detector distance. The abscissa is the X-ray energy while the ordinate is the ratio of energy deposit at the specified value of R to the deposit for R = 8 mm.

Fig. 6 Electron efficiency of the detector i.e. the fraction of its kinetic energy that an electron hitting the detector looses as a function of the kinetic energy of the electron (R = 10 mm).

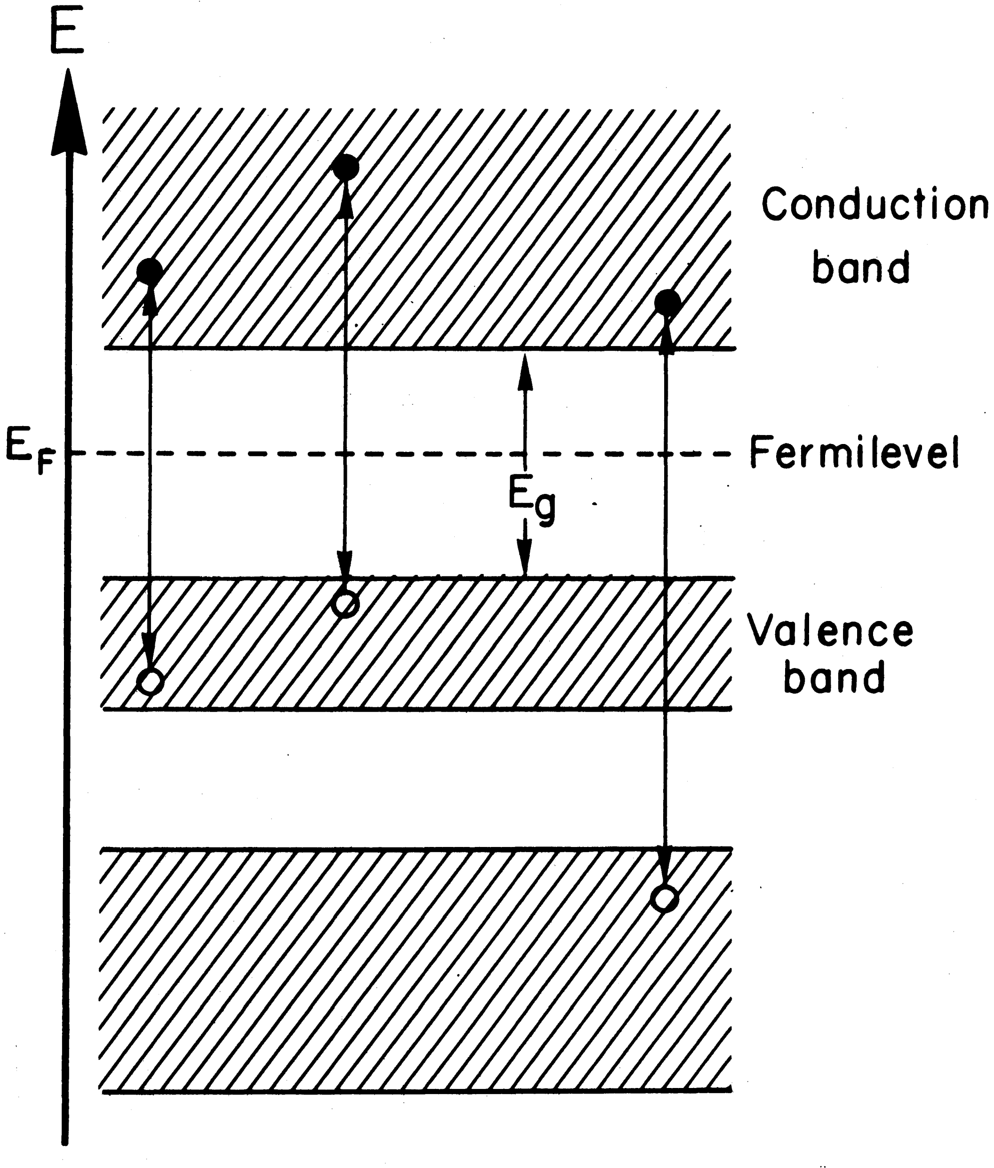

Fig. 1

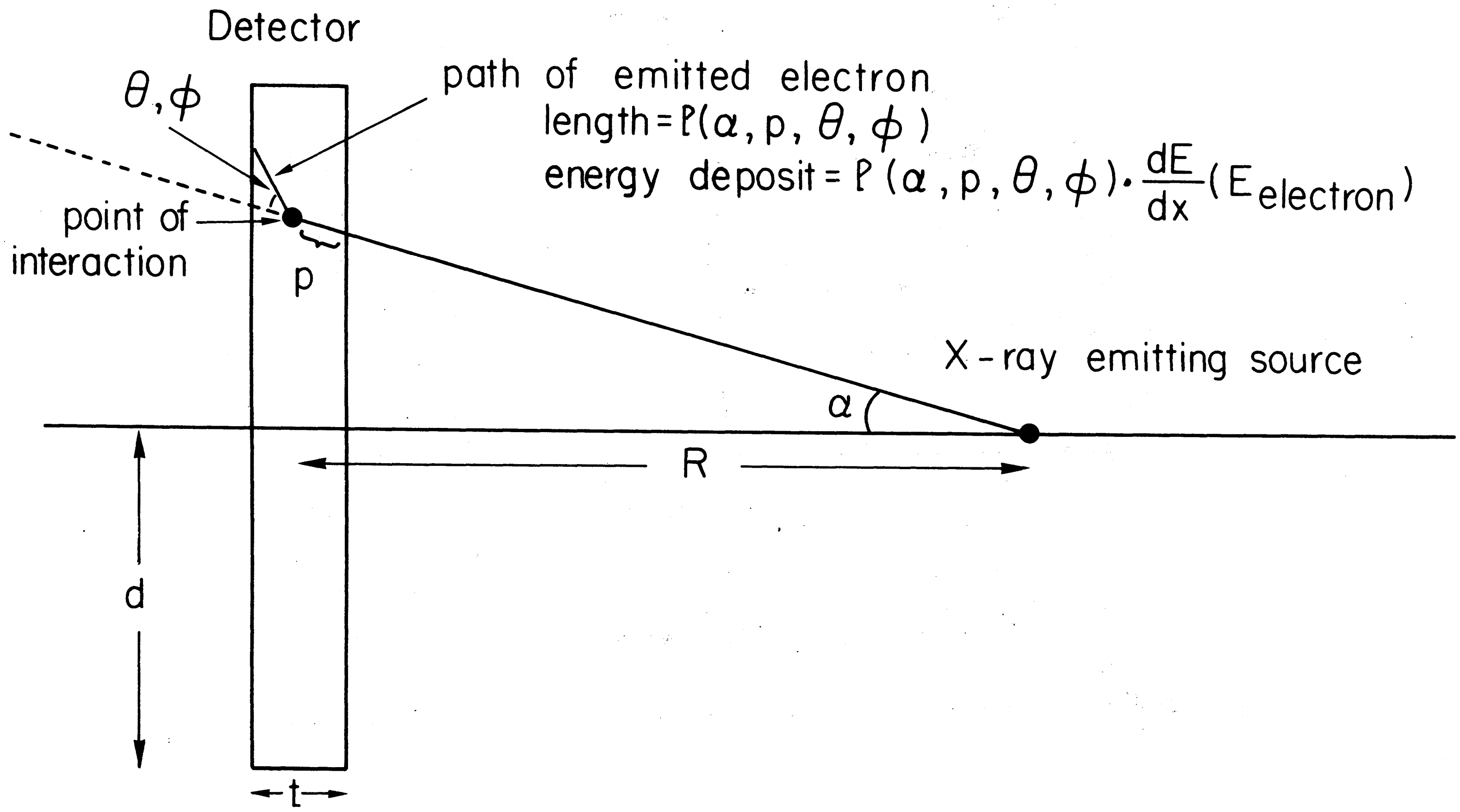
Detector
θ, φ
path of emitted electron
length = ℓ(α, p, θ, φ)
energy deposit = ℓ(α, p, θ, φ) · dE/dx (E electron)
point of interaction
p
X-ray emitting source
α
R
d
t

Fig. 2

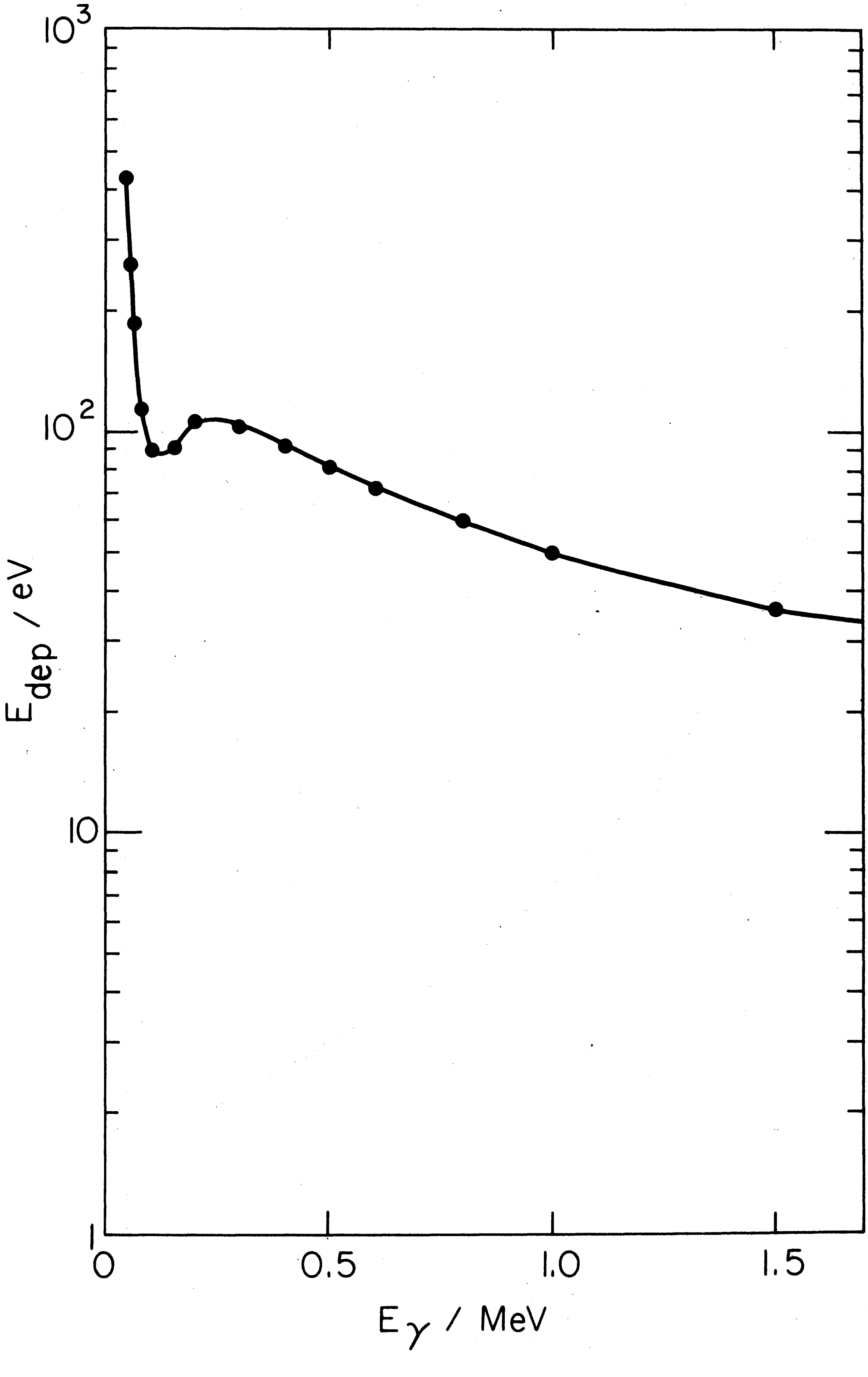

Fig. 3

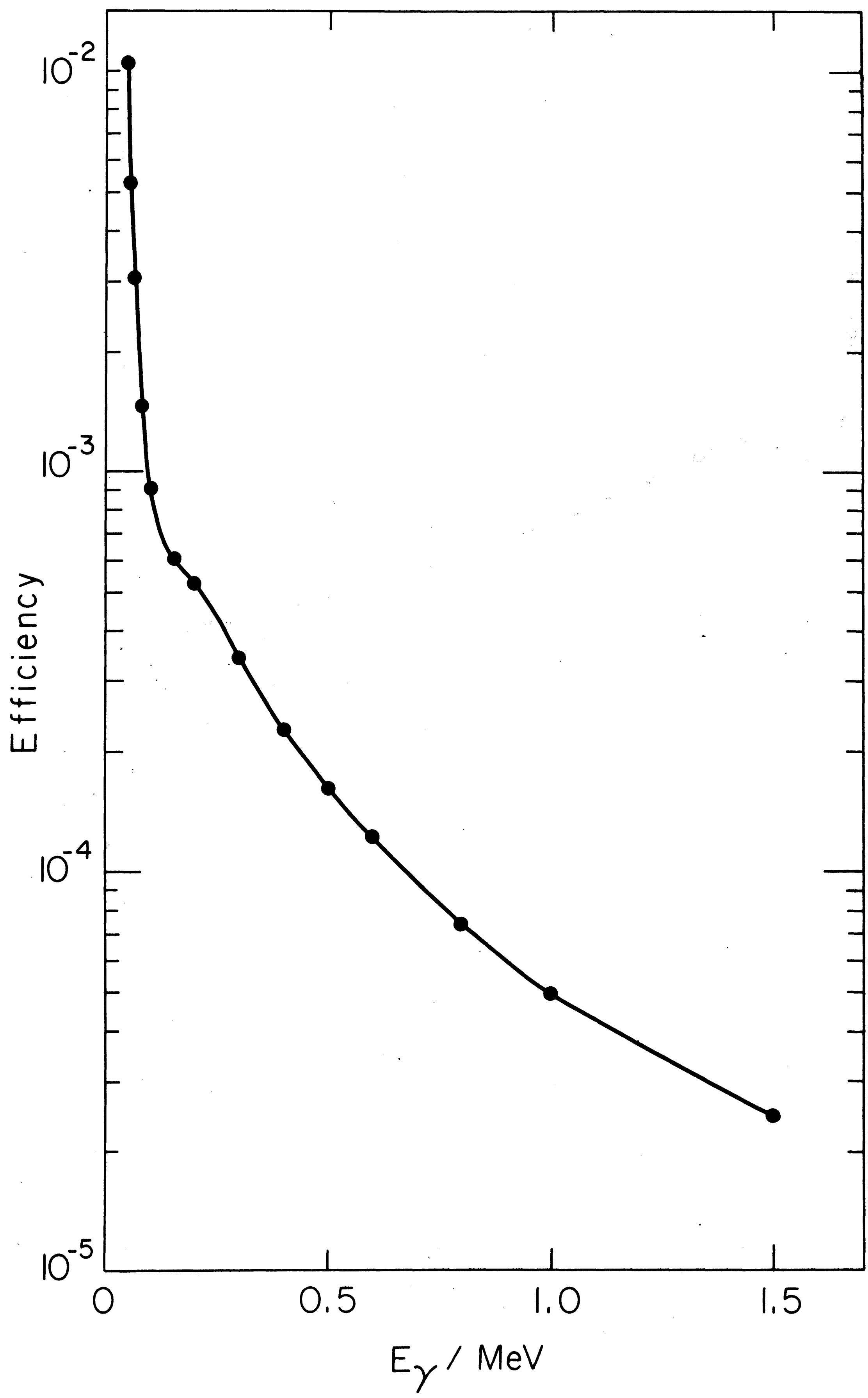

Fig. 4

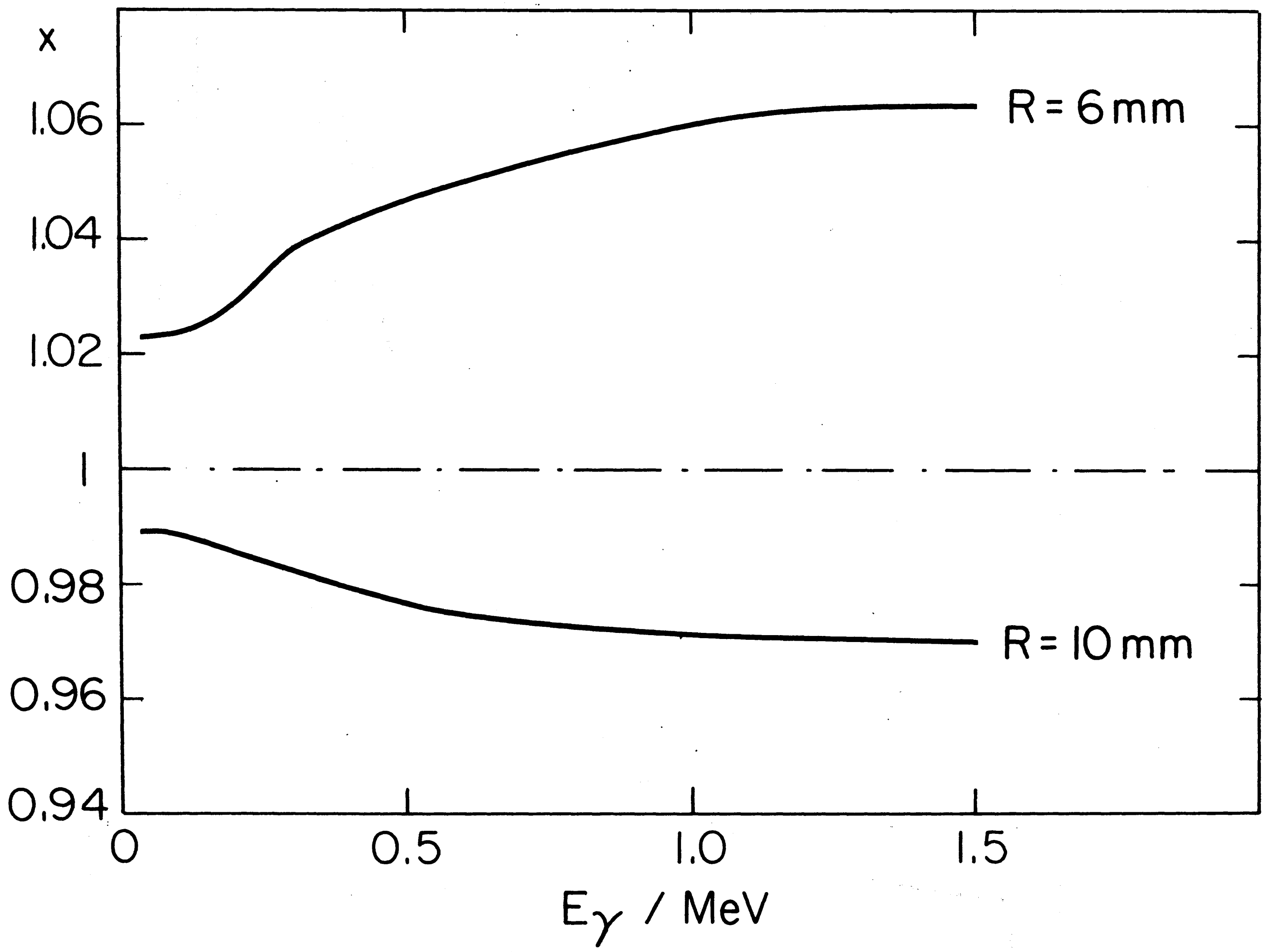

Fig.5

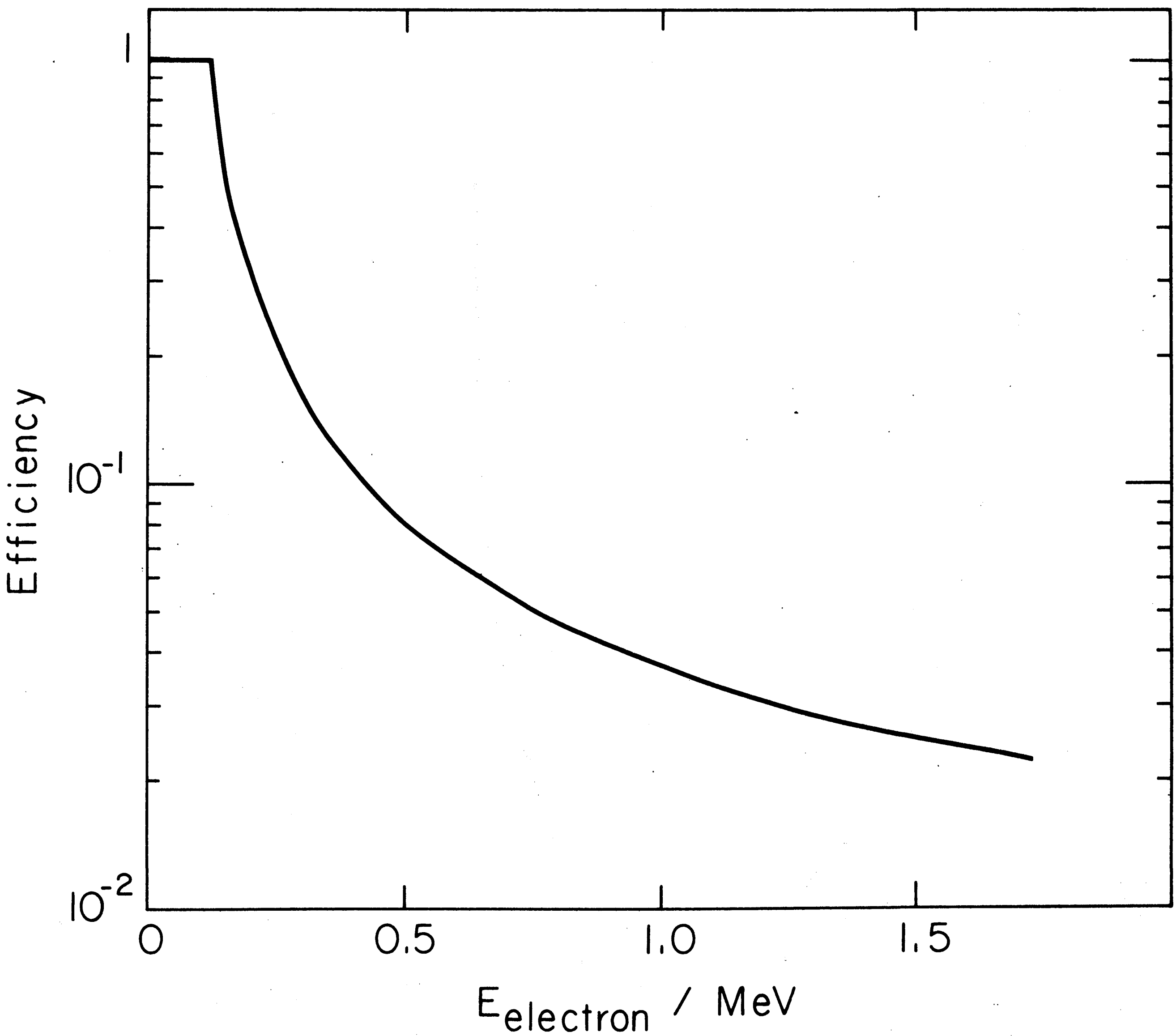

Fig. 6