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UNIVERSAL LIGHT CONE COMMUTATOR

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A B S T R A C T

An operator generalization of a previously considered universal light cone current commutator is proposed. It involves the vector $J^a_\mu(x)$ and scalar $S^a(x)$ elements of the U(12) algebra as well as the vector P_μ and scalar M^2 elements of the Poincaré enveloping algebra. The requirements imposed by the proposal on the light cone behaviour of the current commutator are deduced and discussed.

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About a year ago [1], we pointed out that the behaviour of scattering amplitudes in the scaling limit [2] is controlled by the behaviour of relevant matrix elements of current operators near the light cone (LC). At that time, we attempted to incorporate into this essentially kinematical observation a dynamical universality principle in order to relate the observed existence and magnitude of the electroproduction scaling functions to more familiar and understood concepts [3]. We shortly afterwards [4] established the consistency of our proposal with gauge invariance, Regge pole theory, etc., by showing that it could be accommodated by the most general LC singularity structure equivalent to scaling behaviour and consistent with these principles [5]. Our proposal was, however, specifically restricted to providing relations only between equal states at rest.

The purpose of the present paper is to continue this program by providing an operator formulation of our universality principle so that our relations are valid between arbitrary states. This will establish the consistency of the proposal with the general field-theoretic principles (Poincaré invariance, causality, etc.) and also with the operator structure of field products near the LC recently derived [6] in perturbation theory and in soluble models. Our main motivation for studying this model in such detail, in spite of the few experimental numbers presently available for comparison, is that if it (or some modification of it) is correct, then it provides an explanation for the observed electroproduction behaviour. This is in contrast to other models for electroproduction which have simply been able to accommodate the observed results. Also, our configuration space methods may lead to other simple dynamical principles which have complicated manifestations in momentum space. It will, in fact, be seen below that a striking consistency emerges between the non-local characters of quite different modifications of the right and left-sides of a LC commutation relation.

For orientation, we consider first the forward spin-averaged connected covariant retarded current-proton scattering amplitude

$$T^{ab}_{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\,\theta(x_0)\langle p|[J^a_\mu(x), J^b_\nu(0)]|p\rangle + g_{\mu i}\, g^i{}_\nu\, d^{abc}\Sigma^c(0)$$
$$= p_\mu p_\nu T_2^{ab}(\varkappa,\nu) + \dots \quad . \tag{1}$$

Here $\nu = q\cdot p$, $\varkappa = q^2$, $(p^2 = 1)$ and we have allowed for the presence of an operator Schwinger term of the form appearing in

$$\delta(x_0)[J^a_0(x), J^b_k(0)] = if^{abc}J^c_k(0)\delta^4(x) + id^{abc}\Sigma^c(0)\partial_k\delta^4(x) \, . \tag{2}$$

In this paper we shall explicitly consider only the SU(3) vector currents $J^a_\mu(x)$ although all our equations can be immediately generalized to include the axial vector currents. The absorptive part of (1) is

$$W^{ab}_{\mu\nu} = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle p|[J^a_\mu(x), J^b_\nu(0)]|p\rangle$$
$$= p_\mu p_\nu W_2^{ab}(\varkappa,\nu) + \dots \quad , \tag{3}$$

so that

$$W_2 = \frac{1}{\pi}\,\mathrm{Im}\,T_2 \quad . \tag{4}$$

We shall be concerned only with these double helicity flip invariant amplitudes. We write

$$T_2^{(\pm)ab} = \frac{1}{2}\left(T_2^{ab} \pm T_2^{ba}\right) ,$$

and similarly define $W_2^{(\pm)ab}$.

The currents are, of course, assumed to satisfy the local chiral algebra

$$\delta(x_0)\left[J_0^a(x), J_0^b(0)\right] = if^{abc} J_0^c(0)\,\delta^4(x)\,, \tag{5}$$

etc. When this relation is expressed in terms of a q_0 integral of (3) with $\underset{\sim}{q}$ fixed, and an infinite momentum limit is taken inside of this integral, there results the Fubini [7] - Dashen - Gell-Mann [8] sum rule

$$\int d\nu\, W_2^{(-)ab}(\varkappa,\nu) = -f^{abc} F^c\,, \tag{6}$$

where we have defined the forward form factor by

$$i p_\mu F^c = \langle p | J_\mu^c(0) | p \rangle \quad .$$

Equation (6) corresponds to the asymptotic behaviour

$$T_2^{(-)ab}(\varkappa,\nu) \xrightarrow[R]{} \frac{1}{\nu} f^{abc} F^c$$

in the Regge limit (R limit : $\nu \to \infty$, $\varkappa$ fixed) characteristic of a fixed pole in the complex j plane at $j = 1$ [9].

Because of the need for the above infinite momentum assumption, (6) is not equivalent to the equal-time commutation relation (5), but rather [10] to the LC commutation relation [11], [12]

$$\frac{1}{2}\int dx_+\, \delta(x_-)\left[J_-^a(x), J_-^b(0)\right] = if^{abc} J_-^c(0)\,\delta(x_-)\,\delta(\underset{\sim}{x})\,. \tag{7}$$

Equation (6) follows immediately from (7) by integrating (3) over ν in the frame $q_- = 0$, $p = (1,\underset{\sim}{0})$, in which

$$T_2^{ab}(\mu,\nu) = i\int d^4x\, e^{iq\cdot x}\,\theta(x_0)\langle p|[J_-^a(x), J_-^b(0)]|p\rangle - d^{abc}\Sigma^c(0), \tag{8}$$

$$W_2^{ab}(\mu,\nu) = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle p|[J_-^a(x), J_-^b(0)]|p\rangle . \tag{9}$$

The structure of (7) can be understood as follows. The left side is local in $\underline{x}$ and antisymmetric under the simultaneous transformations $a \to b$ and $\underline{x} \to -\underline{x}$. Thus, if it is proportional to $\delta(\underline{x})$, it must be antisymmetric in a, b and hence involve the antisymmetric coupling matrix f^{abc} [13]. Since J_μ^a is odd under the generalized charge conjugation operation $\mathcal{C}$, it follows that the right side can only involve another $\mathcal{C}$ odd operator (J_μ^c again in this case). The integral in (7) is not, however, the only covariant restriction of the commutator to the LC. We can also consider

$$\frac{1}{2}\int dx_+\,\delta(x_-)\,\epsilon(x_+)[J_-^a(x), J_-^b(0)] . \tag{10}$$

We assume for simplicity in this paper that all such x_+ integrals are rapidly convergent even though there is evidence to the contrary [see, e.g., Eq. (3.61) of Ref. 4]. We shall treat the more general case elsewhere. Contrary to (7), (10) is symmetric under $a \to b$, $\underline{x} \to -\underline{x}$ and so, if it is proportional to $\delta(\underline{x})$, it must be symmetric in a, b and therefore involve the symmetric coupling matrix d^{abc}. This means that the right side must involve a $\mathcal{C}$ even operator. It must, by covariance, also involve an odd number of Lorentz indices $-$. At this point an attempt to generalize (7) with (10) is frustrated because there exists no simple local $\mathcal{C}$ even even parity vector operator. We are therefore led to consider non-local operators and the simplest possibility seems to be to employ the momentum operator P_μ and write our operator universality proposal as

$$\frac{1}{2}\int dx_+\,\delta(x_-)\,\epsilon(x_+)[J_-^a(x), J_-^b(0)] = -i d^{abc}\frac{1}{2}\left\{M^{-1}P_-, S^c(0)\right\}\delta(x_-)\delta(\underline{x}) . \tag{11}$$

Here M is the mass operator $(M^2 \equiv P_\mu P^\mu)$ and $S^c(x)$ the local scalar density in $U(12)$ $(\frac{1}{2}\bar{\psi}\lambda^a\psi$ in the free quark model). The relation (11), consistent with Poincaré invariance, is the simplest $\mathcal{C}$ even generalization of (7) we can think of.

We shall see below that our need to introduce $M^{-1}P_-$ is not disasterous but is actually quite fortunate in view of the operator structure of (10) required by local field theory. Let us first, however, consider some properties of (11) in the abstract. The analogy between (7) and (11) is best seen by combining them into

$$\int dx_+ \delta(x_-)\theta(x_+)[J_-^a(x), J_-^b(0)] = \frac{i}{2}M^{-1}[f^{abc}MJ_-^c(0) - d^{abc}P_- S^c(0)]\delta(x_-)\delta(\underline{x}) - h.c. \quad . \tag{12}$$

This relation exhibits a highly-symmetric universal structure. As in Ref. 1), this universality is made precise by the $[SU(3) \times SU(3)]_\beta$ algebra

$$\delta(x_0)[S^a(x), S^b(0)] = if^{abc}J_0^c(0)\delta^4(x), \tag{13}$$

etc., which establishes the scale of S relative to J. Thus each side of (12) has equivalent weights of even and odd $\mathcal{C}$. Further motivation for (12) is the same as in Ref. 1). It is emphasized there how the second term in (12) represents an $SU(3)$ extension of the Pomeron, connected to the first term in (12) by a generalization of exchange degeneracy.

We are thus contemplating an analogy between the $\mathcal{C}$ even generalization (13) of (5) and the $\mathcal{C}$ even generalization (11) of the infinite momentum limit (7) of (5) [14]. Equations (12) and (13) lead to a universal behaviour in the deep inelastic limit just as (5) and the V-A form of the weak interaction Lagrangian $\mathcal{L}_W$ lead to a universal behaviour of low energy weak interactions. This analogy is further strengthened by comparing $\mathcal{L}_W$ with the supposed SU(3) breaking Lagrangian $\delta m S^8$. Finally we note that (12) incorporates a sort of combination of internal and space-time symmetries in the sense that it relates vector elements of SU(3) and scalar elements of its extension $[SU(3) \times SU(3)]_\beta$ with vector elements of the Poincaré algebra and scalar elements of its universal enveloping algebra.

Consideration of the single particle rest matrix elements of (12) easily leads to the predictions of Ref. 1). Following the methods of Refs. 1) and 4), (11) is seen to imply the existence of a finite Bjorken [2] asymptotic limit (A limit : $\nu \to \infty$, $\omega \equiv -q^2/\nu$ fixed) :

$$\nu W_2^{(+)ab}(x,\nu) \xrightarrow[A]{} F_2^{ab}(\omega) . \tag{14}$$

The asymptotic structure function is related to the Fourier transform of the light cone restriction of

$$\int d\underline{x} \langle p|[J_-^a(x), J_-^b(0)]|p\rangle = \theta(x_+x_-)\mathcal{M}^{ab}(x_+,x_-) \tag{15}$$

according to [15]

$$F_2^{ab}(\omega) = \frac{i}{2\pi}\int d\lambda\, e^{-i\lambda\omega}\,\varepsilon(\lambda)\mathcal{M}^{ab}(2\lambda,0) . \tag{16}$$

The presence of the $\theta(x_+x_-)$ factor in (15) follows from causality [4]. The existence of (16) follows from (11), which implies the existence of the integral [16]

$$\frac{1}{2}\int dx_+\,\varepsilon(x_+)\frac{1}{2}\mathcal{M}^{ab}(x_+,0) = -i d^{abc} D^c . \tag{17}$$

Here we have defined the forward scalar form factor

$$D^c \equiv \langle p | S^c(0) | p \rangle \; . \tag{18}$$

Equations (16) and (17) now give

$$F_2^{ab}(0) = \frac{1}{\pi} d^{abc} D^c \; , \tag{19}$$

and so (11) is seen to imply a constant asymptotic behaviour for $\omega \to 0$. Finally, (12) implies (when $\Sigma^c = 0$) that

$$T_2^{ab}(\varkappa, \nu) \xrightarrow[R']{} \frac{1}{\nu}\left[f^{abc} F^c + i d^{abc} D^c \right] \tag{20}$$

in the Regge limit with $|\varkappa|$ large (R' limit : $\nu \gg |\varkappa| \gg 1$).

Let us now return to (12) and compare it with the known operator LC singularity structure of quantum field theory. The appropriate tool for this endeavour is the LC operator product expansion formalism developed recently by Brandt and Preparata [6]. They showed the existence of such expansions in each order of any renormalizable field theory and in soluble models. Their methods show that

$$\left[J_-^a(x), J_-^b(0) \right] \xrightarrow[x^2 \to 0]{} \sum_{\pm} d_\pm^{abc} \Big[\mathcal{E}^{(\pm)c}(x)\, \partial_- \partial_- + 2i\, \mathcal{F}_-^{(\pm)c}(x)\, \partial_- + \mathcal{G}_{--}^{(\pm)c}(x) \Big] \Delta(x) \, , \tag{21}$$

where

$$\Delta(x) = \frac{1}{2\pi} \varepsilon(x_0)\, \delta(x^2) \, , \quad d_-^{abc} = f^{abc} \, , \quad d_+^{abc} = i d^{abc} \, ,$$

and

$$\mathcal{E}^{(\pm)c}(x) = \sum_{n=0}^{\infty} (i)^n x^{\alpha_1} \cdots x^{\alpha_n} E^{(\pm)c}_{\alpha_1 \cdots \alpha_n}(0) \, , \tag{22a}$$

$$\mathcal{F}_\mu^{(\pm)c}(x) = \sum_{n=0}^{\infty} (i)^n x^{\alpha_1} \cdots x^{\alpha_n} F^{(\pm)c}_{\mu \alpha_1 \cdots \alpha_n}(0) \, , \tag{22b}$$

$$\mathcal{G}^{(\pm)c}_{\mu\nu}(x) = \sum_{n=0}^{\infty} (i)^n x^{\alpha_1} \cdots x^{\alpha_n} G^{(\pm)c}_{\mu\nu\alpha_1\cdots\alpha_n}(0) , \tag{22c}$$

with the E's, F's, and G's Hermitian local field operators.

Equation (21) will be valid in renormalizable models like the gluon model in which the SU(3) symmetry breaking is due to mass terms. In any finite order of perturbation theory, (21) will actually also involve factors of $(\log x^2)^r$ for integer r. We shall ignore such factors here, partly because their effect to all orders is completely uncertain [17], and partly because their presence to a significant extent [18] is ruled out by the SLAC electroproduction and Columbia-BNL μ pair experimental results [6].

The symmetry properties

$$\mathcal{E}^{(\pm)c}(-x) = \pm \mathcal{E}^{(\pm)c}(x) \tag{23a}$$

$$\mathcal{F}^{(\pm)c}_{\mu}(-x) = \mp \mathcal{F}^{(\pm)c}_{\mu}(x) \tag{23b}$$

$$\mathcal{G}^{(\pm)c}_{\mu\nu}(-x) = \pm \mathcal{G}^{(\pm)c}_{\mu\nu}(x) \tag{23c}$$

require that

$$E^{(-)c} = E^{(+)c}_{\alpha} = E^{(-)c}_{\alpha\beta} = F^{(+)c}_{\mu} = F^{(-)c}_{\mu\alpha} = G^{(-)c}_{\mu\nu} = 0 , \tag{24}$$

and the equal-time commutation relations (2) and (5) imply that

$$E^{(+)c} = \Sigma^c , \qquad E^{(-)c}_{\alpha} = 0 , \qquad F^{(-)c}_{\mu} = J^c_{\mu} . \tag{25}$$

The remaining low-indexed terms in (22) contribute to the time derivative commutator according to

$$\int d^4x\, \delta(x_0)\left[\dot{J}_i^{\,a}(x),\, J_j^{\,b}(0)\right] = 2i d^{abc}\left[-E_{ij}^{(+)c}(0) + F_{ij}^{(+)c}(0) + \frac{1}{2} G_{ij}^{(+)c}(0)\right] \tag{26}$$

$$+\ \delta_{ij} \text{ terms} + (i \leftrightarrow j) \text{ antisymmetric terms.}$$

It is now a simple matter to use (21)-(25) to evaluate the light cone restrictions of the commutator. We find

$$\frac{1}{2}\int dx_+\, \delta(x_-)\left[\mathcal{J}_-^{\,a}(x),\, \mathcal{J}_-^{\,b}(0)\right] = \delta(x_-)\delta(\underline{x})\, i f^{abc}\left[\mathcal{J}_-^{\,c}(0) - \frac{i}{8}\int dx_+\, \epsilon(x_+)\, \mathcal{H}_{--}^{(-)c}(x)\right] \tag{27}$$

and

$$\frac{1}{2}\int dx_+\, \delta(x_-)\, \epsilon(x_+)\left[\mathcal{J}_-^{\,a}(x),\, \mathcal{J}_-^{\,b}(0)\right] = \delta(x_-)\delta(\underline{x})\, i d^{abc}\left[-2\delta(0)\Sigma^c(0) + \frac{1}{8}\int dx_+\, \mathcal{H}_{--}^{(+)c}(x)\right]. \tag{28}$$

We see that the general form (21), even though it is consistent with the equal-time commutation relations (2) and (5), does not necessarily obey the light cone commutation relation (7). It is hardly surprising then that (28) does not give (11). Because of the presence of $\delta(0) = \infty$, (28) does not, in fact, even exist. It fails to exist, however, in a well-defined way.

Before considering (27) and (28) further, let us directly compute from (21) the behaviour of the single particle rest matrix elements. We define

$$\langle p|\, \mathcal{E}^{(\pm)c}(x)\, |p\rangle = e^{(\pm)c}(x\cdot p) + \ldots \tag{29a}$$

$$\langle p|\, \mathcal{F}_-^{(\pm)c}(x)\, |p\rangle = f^{(\pm)c}(x\cdot p) + \ldots \tag{29b}$$

$$\langle p| \mathcal{A}^{(\pm)c}_{--}(x)|p\rangle = g^{(\pm)c}(x\cdot p) + \cdots , \tag{29c}$$

where e, f and g are scalar functions and the omitted terms do not contribute to the leading asymptotic behaviours. By the methods of Ref. 4), we find from (21) that (14) is valid with

$$F_2^{ab}(\omega) = \frac{i}{4\pi}\int d\lambda\, e^{-i\lambda\omega} h^{ab}(\lambda) , \tag{30}$$

where

$$h^{ab}(\lambda) = \sum_{\pm} d_{\pm}^{abc}\left[g^{(\pm)c}(\lambda) - 2if^{(\pm)c\prime}(\lambda) + e^{(\pm)c\prime\prime}(\lambda)\right] , \tag{31}$$

so that

$$F_2^{ab}(0) = -\frac{1}{4\pi} d^{abc}\int d\lambda\, g^{(+)c}(\lambda) . \tag{32}$$

Using (25), we find further that

$$T_2^{ab}(\varkappa,\nu) \xrightarrow[R']{} -\frac{i}{4\nu} d^{abc}\int d\lambda\, g^{(+)c}(\lambda) + \frac{1}{\nu} f^{abc}\left[F^c - \frac{1}{4}\int d\lambda\, \varepsilon(\lambda)\, g^{(-)c}(\lambda)\right] . \tag{33}$$

Equations (30) and (31) tell us that

$$\int d\omega\, F_2^{ab}(\omega) = \frac{i}{2} h^{ab}(0) = -\frac{1}{2} d^{abc}\left[g^{(+)c}(0) - 2if^{(+)c\prime}(0) + e^{(+)c\prime\prime}(0)\right] . \tag{34}$$

By virtue (26), this is just the Callan-Gross [19] result. We have derived it without taking an infinite momentum limit because in (29) there occur no $g_{\mu\nu}$ terms since $g_{--} = 0$.

The results (30)-(33) can be derived directly from (27) and (28) in the same way that (16)-(20) were derived from (7) and (11). [The three finite terms in (27)-(28) contribute the three $1/\nu$ pieces in (33) to (8) whereas the $\delta(0)\Sigma^c$ term contributes a constant piece which is exactly cancelled by the second term in (8).] We thus see from (33), as well as from (27), that the sum rule (6) need not be valid. Instead we have

$$\int d\nu\, W_2^{(-)ab}(x,\nu) = -f^{abc}\left[F^c - \tfrac{1}{4}\int d\lambda\, \varepsilon(\lambda)\, g^{(-)c}(\lambda)\right]. \tag{35}$$

This result is not surprising since it is known that the Gell-Mann algebra (5) plus Regge behaviour are not sufficient to derive (6). There can in general be additional "infinite mass" terms present [20] and this possibility is nicely exemplified by (35).

We want to accept (7), however, and so we set

$$\int dx_+\, \varepsilon(x_+)\, \mathcal{A}^{(-)c}_{--}(x_+, 0, \underline{0}) = 0. \tag{36}$$

We proceed to see what our proposal (11) tells us about (28). The <u>existence</u> of (10) immediately implies that [21]

$$\Sigma^c(0) = 0. \tag{37}$$

Thus our assumption excludes the presence of q number Schwinger terms. We are, needless to say, not unhappy about this. Equation (11) further requires the validity of the identity

$$\frac{1}{8}\int dx_+ \, \mathcal{U}^{(+)c}_{--}(x_+, 0, \underline{0}) = -\frac{1}{2}\left\{M^{-1}P_-, S^c(0)\right\} . \qquad (38)$$

Using (22c), this can be written as

$$\frac{1}{2}\int d\lambda \sum_n (-1)^n \lambda^{2n} G^c_{(2n)--}(0) = -\left\{M^{-1}P_-, S^c(0)\right\} , \qquad (39)$$

where

$$G^c_{(n)--}(0) \equiv G^{(+)c}_{\underbrace{--\cdots-}_{n+2}}(0) . \qquad (40)$$

Equation (39) is, of course, supposed to be a weak relation, valid between suitable hadronic states $|\alpha\rangle$ in the form

$$\frac{1}{2}\int d\lambda \sum_n (-1)^n \lambda^{2n} \langle\alpha| G^c_{(2n)--}(0) |\alpha\rangle = -2p_- \langle\alpha| S^c(0) |\alpha\rangle , \qquad (41)$$

where p_μ is the total momentum of $|\alpha\rangle$.

We must admit that (39) appears somewhat strange. It is saying that an infinite number of local fields add up to the product of the local field $S^c(0)$ and P_-/M. That is what must happen , however, if (11) is to be valid in the class of theories giving (21). It clearly illustrates the dynamical nature of (11), as opposed to the essentially kinematical nature of (7) [22]. In view of the fact that (11) represents an operator SU(3) generalization of the Pomeron - that celebrated dynamical object presumably representing the combined background effect of an infinity of channels - the form of (39) is not all that unexpected. It exhibits, in fact, a striking connection between our earlier need to introduce the non-local operator P_-/M

(in an attempt to construct a simple even-parity even $\mathcal{C}$ vector operator) and the occurrence of the infinite sum of local operators. If the sum were finite, then (39) would not be consistent with causality since a local field $\varphi(x)$ with $x^2 < 0$ would commute with the left side but not with the right side. So the presence of P_-/M on the right requires the existence of an infinite sum on the left. Conversely, the infinity of terms on the left strongly suggests that the sum is non-local.

Apart from these indications that (39) is not (obviously) inconsistent, we can do little to convince anyone that it is correct. For matrix elements within the $\frac{1}{2}^+$ baryon octet, however, slightly more can be said. We label these octet states by $e = N, \Xi, \Sigma$ or Λ and write

$$\langle p, e | G^c_{(2n)--}(0) | p, e \rangle = (p_-)^{2n+2} K^c_{ne} \, . \tag{42}$$

Defining now

$$K^c_e(\lambda) = \sum_n (-1)^n \lambda^{2n} K^c_{ne} \, , \tag{43}$$

Equation (39) implies that

$$\frac{1}{2} \int d\lambda \, K^c_e(\lambda) = -\frac{2}{m_e} D^c_e \, , \tag{44}$$

where

$$D^c_e \equiv \langle e | S^c(0) | e \rangle \, . \tag{45}$$

The function (43) should be strongly peaked at $\lambda = 0$ so that the $SU(3)$ behaviour of $K_e^c(0) = K_{0e}^c$ should dominate the $SU(3)$ behaviour of the left side of (44). The $SU(3)$ behaviour of the right side, on the other hand, is dominated by that of D_e^c. We thus see that our result (44) is consistent with and suggested by the conclusion of Brandt and Preparata [23] that in the canonical gluon model, (26) and (45) have the same d/f ratios.

It would be interesting to check the validity of (12) in various models. It is unfortunate that perturbation theory seems unsuited for this purpose because of the absence of a simple Pomeron in that theory and because of our inability to deal with the logarithmic factors which occur and which ruin scaling.

Although in this paper we have emphasized the possible validity of the universal relation (12), a more conservative approach is also possible and, since it is based on only rather familiar and tested ideas, is perhaps more likely to be correct. Thus, a strict consequence of (28) and the usual Regge picture for absorptive parts is that $\int dx_+ \mathcal{A}_{--}^{(+)c}(x_+,0,\underline{0})$ is non-vanishing only for $c = 0$ and $c = 8$. In this case, we still maintain the LC interpretation of the Pomeron embodied in (21), (28), (33), etc., and (22c) continues to illustrate the complicated dynamical nature of this diffractive mechanism. In view of the underlying $SU(3)$ structure of (21) and its (admittedly mainly kinematical) persistence in (27) and (36), it may not be completely unreasonable for this $SU(3)$ structure to persist also in (10) so that highly symmetric asymptotic behaviour like (20) are obtained. The specific operator implementation of this idea given by (11) is, of course, even more speculative and its detailed structure should probably not be taken too seriously. A comprehensive field theoretic understanding of the Pomeron would undoubtedly either confirm (11) or provide a suitable alternative.

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12) For a four-vector A_{μ} we write $A_{\pm} = A_0 \pm A_3$ and $\underline{A} = (A_1, A_2)$. Also $\delta(\underline{x}) = \delta(x_1)\,\delta(x_2)$.

13) We are assuming an asymptotic SU(3) symmetry or, more precisely, that leading LC singularities are mass independent. See the discussion following Eq. (22c).

14) Note that we cannot generalize (7) simply by taking the infinite momentum limit of (13) because (13), being a bad-bad relation, presumably has no infinite momentum limit.

15) Some care must be exercized in deducing (16) from (11). The clearest way to proceed is to derive (16) from the general form (21) made consistent with (11). This is done below.

16) We are assuming that the $\delta(x_-)$ in (11) is defined as the limit of a sequence of symmetric functions.

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18) That is, to an extent which would significantly change the singularity structure in (21).

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21) We are assuming that the second term in the bracket is finite.

22) Note, however, that (7) is not purely kinematical since it requires the validity of the dynamical condition (34).

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We note that the numerical value for (34) predicted in this paper for deep-inelastic electron-neutron scattering is in good agreement with the preliminary SLAC results reported by R. Taylor at the XVth International Conference on High Energy Physics in Kiev.