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PROBING THE LIGHT CONE

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1. - INTRODUCTION

In these lectures we shall be concerned with a class of phenomena in particle physics which can be related to the behaviour of quantum field operators near the light cone (LC). This class includes lepton-hadron scattering experiments in which a very massive current interacts with a hadronic system in a high energy inelastic collision. We shall develop an appropriate operator formalism for describing light cone behaviour in quantum field theory and apply it to study a number of interesting examples in the above class : electroproduction, μ pair production, vector meson dominance.

That the LC in configuration space corresponds to the scaling limit for the electroproduction structure functions was pointed out some time ago [1),2)]. Knowing this, one can, of course, simply take the Fourier transform of the Bjorken [3)] scaling laws to determine the LC behaviour of the proton-proton matrix element of the product of electromagnetic currents [4),5)]. To understand the reason for existence of scaling, however, this information is useless and some dynamical principles are needed. An attempt at this in terms of a universality principle is described in Refs. 2) and 4).

A recent study of the behaviour of products of local fields near the LC in renormalized perturbation theory has established the existence of operator product expansions which describe this behaviour [6)]. These expansions provide an understanding of the scaling behaviour in terms of the canonical field dimensions and singularity structure of renormalizable field theories. They predict the strength of the light cone singularities and thereby provide a means of measuring the dimensions of interacting fields, they determine properties of amplitudes in several variables, and they provide relations between form factors describing different experiments corresponding to different matrix elements of the current products. It is concluded from several such applications that the present experimental results are in good agreement with the naïve field dimensions and canonical singularity structure of renormalizable field theory.

A number of experiments which probe the LC are described in Section 2. Short distance behaviour in quantum field theory is reviewed in Section 3 and the LC behaviour is exhibited and discussed in Section 4. The dimensionality concept is discussed in Section 5. In Section 6, we describe our treatment of mass dispersion relations. Sections 7 - 9 describe application to deep inelastic electroproduction, massive μ pair production, and vector meson dominance.

2. - EXPERIMENTS WHICH PROBE THE LIGHT CONE [7)]

Let us first consider the weak or electromagnetic scattering of a lepton off of a hadronic system α to produce an arbitrary final hadronic state. Calling q the momentum transferred to the lepton and p the total momentum of the system α, the total cross-section of interest has the form

$$\frac{d^2\sigma^r}{dq^2 d\nu} \propto \int d^4k\, \theta(k_0)\, \delta(k^2-q^2)\,\delta(k\cdot p-\nu) \int d^4x\, e^{-ik\cdot x}\, {}_{in}\langle\alpha|J_\mu(x) J_\nu(0)|\alpha\rangle_{in}\, \varepsilon_r^\mu \varepsilon_r^\nu . \tag{2.1}$$

Here $\nu \equiv q\cdot p$ is the initial energy variable, r is the polarization of the leptons, and J_μ is the hadronic current to which the leptons couple. The matrix element in (2.1) is connected and spin averaged. Changing the orders of integration in (2.1) gives

$$\frac{d^2\sigma^r}{dq^2 d\nu} \propto \int d^4x\, \Delta_+(x,p,\nu,q^2) \langle\alpha|J_\mu(x) J_\nu(0)|\alpha\rangle\, \varepsilon_r^\mu \varepsilon_r^\nu , \tag{2.2}$$

where we have defined

$$\Delta_+(x,p,\nu,q^2) = \int d^4k\, e^{-ik\cdot x}\, \theta(k_0)\, \delta(k^2-q^2)\,\delta(k\cdot p-\nu) . \tag{2.3}$$

The form (2.2) is very useful for our configuration space purposes. The integral (2.3) can be simply evaluated in the frame $p=(1,\vec{0}) \equiv \xi$ (we take $p^2=1$) to give

$$\Delta_+(x,\xi,\nu,q^2) = \frac{\pi}{ir}\left[e^{i(\nu^2+q^2)^{1/2} r} - h.c.\right] e^{-i\nu t} , \tag{2.4}$$

where we have written $r = |\underset{\sim}{x}|$, $t = x_0$. We refer to the Bjorken[3] scaling limit as the A limit

$$\nu \to \infty, \; q^2 \to \infty, \quad \omega \equiv \frac{q^2}{2\nu} \text{ fixed}, \tag{2.5}$$

and obtain

$$\Delta_+(x, p, \nu, q^2) \xrightarrow[A]{} \frac{\pi}{ir}\left[e^{i\nu(r-t)+i\omega r} - e^{-i\nu(r+t)-i\omega r}\right]. \tag{2.6}$$

Only the regions $r = \pm t$ in the (first/second) term are important and so we can deduce the covariant result

$$\Delta_+(x, p, \nu, q^2) \xrightarrow[A]{} \frac{\pi}{i\tau} \sin\left(\frac{\nu x^2}{2\tau} + \omega\tau\right) \tag{2.7}$$

in terms of the "transverse" variable

$$\tau^2 \equiv (p\cdot x)^2 - x^2 . \tag{2.8}$$

It follows from (2.7) that, in the A limit, $\Delta_+(x,p,\nu,q^2)$ is highly oscillating outside of the region

$$\frac{x^2}{2\tau} \lesssim \frac{1}{\nu}, \quad \tau \lesssim \frac{1}{\omega} \quad \text{or} \quad x^2 \lesssim \frac{2}{q^2}$$

and, therefore, has effective support on the LC $x^2 \sim 0$. Referring back to (2.2), we see that the A limit of the cross-section can be obtained simply from the behaviour of $<\alpha|J_\mu(x)J_\nu(o)|\beta>$ on the LC. This is precisely the behaviour we have studied in the previous sections.

For reactions in which a hadronic system produces a lepton pair and an arbitrary final hadronic state, the analysis becomes more complicated because of kinematic restrictions on the k space integration region in the analogue of (2.1). In the frame $p = (\sqrt{s}, \underset{\sim}{0})$, the physical k space region will have the form $\{ |\underset{\sim}{k}| < \mathcal{H}(q^2,s) \}$ for some functions $\mathcal{H}(q^2,s)$. Proceeding as above it can be shown that the relevant configuration space region is given by

$$|x^2 - \varkappa^{-2}| \lesssim \frac{1}{q^2} \; . \tag{2.9}$$

So, for large q^2 and $\varkappa$, the LC is again the dominant region. The extra condition that $\varkappa(q^2,s)$ be large can be satisfied in many cases of interest.

Another important use of the LC is to determine the behaviour of amplitudes for large values of a single mass variable. We illustrate this by consideration of a scalar vertex function

$$A(q^2,k^2) = \int d^4x \, e^{-iq\cdot x} \langle 0|T[A(x),B(0)]|p\rangle \; . \tag{2.10}$$

Here $A(x)$ and $B(x)$ are scalar currents and $|p\rangle$ is a state of one scalar particle of momentum $p = q-k$ and mass $p^2 = m^2$. We can write (2.10) as

$$A(q^2,k^2) = \frac{1}{4\pi}(\nu^2 - m^2 p^2)^{-\frac{1}{2}} \int d^4x \, \Delta_+(x,p,\nu,q^2) \langle 0|T[A(x),B(0)]|p\rangle , \tag{2.11}$$

where $\nu = q\cdot p = \frac{1}{2}(q^2 + k^2 - m^2)$. Thus, the behaviour of $A(q^2,k^2)$ for $q^2 \to \infty$ and fixed $q^2/2\nu$, i.e., for fixed q^2/k^2, is determined by the LC behaviour of $A(x)B(0)$. A special case of this limit is the old Bjorken [8)] limit $q^2/k^2 \to 0$, in which case only the equal time behaviour of $A(x)B(0)$ is relevant. Another special case is the limit $q^2 \to \infty$ with k^2 fixed so that $q^2/2\nu \to 1$. This limit is important because it determines the number of subtractions needed in fixed k^2 dispersion relations.

The analysis of this Section can clearly be extended to many kinematically more complicated processes. Our operator expansions can thus be used to derive and relate these processes. Correspondingly, the observed nature of these processes can be used to learn about the existence and properties of the field theory describing the hadronic structure.

3. - SHORT DISTANCE BEHAVIOUR IN QUANTUM FIELD THEORY

In order to introduce short distance operator product expansions in as simple a way as possible, we consider first a free scalar field :

$$(\Box + m^2)\varphi(x) = 0 \quad , \qquad [\varphi(x), \varphi(0)] = \Delta(x; m^2) \; . \tag{3.1}$$

The equal time behaviour $\dot{\Delta}(x^2,m)|_{x_0=0} = \delta(\underset{\sim}{x})$ gives the canonical equal time commutation relation

$$[\dot{\varphi}(x), \varphi(0)]_{x_0=0} = \delta(\underset{\sim}{x}) I \; , \tag{3.2}$$

where we have explicitly indicated the unit operator by I to emphasize that the commutator is a c number. A property of this field theory which we shall make use of is the existence of the Wick product

$$j(x) \equiv :\varphi(x)\varphi(x): \; = \lim_{\xi \to 0} \left[\varphi(x+\xi)\varphi(x) - \Delta_+(\xi) \right] \tag{3.3}$$

as a finite local field operator. Here, of course,

$$\Delta_+(x) \equiv \langle 0 | \varphi(x)\varphi(0) | 0 \rangle = \frac{1}{(2\pi)^3} \int d^4k \, e^{-ik\cdot x} \theta(k_0) \delta(k^2 - m^2) \tag{3.4}$$

is the free field Wightman function.

Note that, in view of the fact that

$$\Delta_+(x) \xrightarrow[x \to 0]{} \frac{-1}{4\pi^2} \frac{1}{x^2 - i\varepsilon x_0} + O(m^2 \log x^2) \; , \tag{3.5}$$

the formal expression $\varphi(x)\varphi(x)$ is meaningless, whereas the Wick product $:\varphi(x)\varphi(x):$ is a well-defined operator. In (3.3), divergences in the ordinary product $\varphi(x+\xi)\varphi(x)$ for $\xi \to 0$ are cancelled by the divergences in $\Delta_+(\xi)$ for $\xi \to 0$.

Equation (3.3) can be rewritten as

$$\varphi(x)\varphi(0) \xrightarrow[x\to 0]{} \Delta_+(x)\, I + j(0) \tag{3.6}$$

$$\sim \frac{-1}{4\pi^2}\frac{1}{x^2} I + j(0) , \tag{3.7}$$

where we have used (3.5). Here and elsewhere x^2 means $x^2 - i\varepsilon x_0$. Note that (3.2) follows directly from (3.7). The forms (3.6) and (3.7) are interesting because they express $\varphi(x)\varphi(0)$ for $x \sim 0$ as a sum of finite local fields (I and j) with c number coefficients which (perhaps) diverge for $x \to 0$. The nature of the expansion (3.7) can be elegantly described in terms of the "dimensionality" concept. One assigns to each local field in the theory a dimension in mass units. Thus dim I = 0 and, from (3.2) for example, dim φ = 1. Also dim j = 2 and dim $\partial_\alpha \varphi$ = 2. Then the nature of the c number singularities in (3.7) is determined from the fact [illustrated in (3.5)] that leading singularities in the theory are mass independent.

The behaviour of the product of any two local fields in the theory can be determined in a similar way. One simply expands in terms of all other local fields with dimensions small enough to give singularities. As an example, we have

$$j(x)j(0) \xrightarrow[x\to 0]{} c_0\left(\tfrac{1}{x^2}\right)^2 I + c_1\left(\tfrac{1}{x^2}\right) j(0) + c_2\left(\tfrac{1}{x^2}\right) x^\alpha :\varphi\partial_\alpha\varphi: + c_3 :jj: . \tag{3.8}$$

From this follows the equal time commutation relation

$$[j(x), j(0)]_{x_0=0} = c\, j(0)\, \delta(\underset{\sim}{x}) + c\text{-number} . \tag{3.9}$$

Similar results hold in any free field theory.

If, instead of free fields, we consider fields interacting according to a renormalizable interaction, then in any order of perturbation theory, expansion of the above forms remain valid apart from the presence of factors of power of $\log x^2$ [9),10),11)]. For example, in any finite order of φ^4 theory, the product (3.7) of renormalized fields is replaced by

$$\varphi(x)\varphi(0) \xrightarrow[x\to 0]{} F_0(x^2)\frac{1}{x^2}I + F_1(x^2)j(0) , \tag{3.10}$$

for suitable functions $F_i(x^2)$ with logarithmic singularities for $x \to 0$. Thus

$$\varphi(x)\varphi(0) \xrightarrow[x\to 0]{} b(\log x^2)^a \frac{1}{x^2} I + b'(\log x^2)^{a'} j(0) \tag{3.11}$$

for some integers a and a'. Note that $j(x)$ is no longer given by (3.3), but rather by

$$j(x) = \lim_{\xi\to 0} [F_1(\xi^2)]^{-1}\left[\varphi(x+\xi)\varphi(0) - F_0(\xi)\frac{1}{\xi^2} I\right] \tag{3.12}$$

in the given order. Note also that (3.2) is no longer valid - the equal time commutator is even divergent if $a > 0$ (as it is).

The general behaviour of products $A(x)B(o)$ of (renormalized) local field operators at short distances $x^\mu \to 0$ in renormalized perturbation theory and in soluble field theoretic models is similar. One obtains operator expansions of the form [9),10),11)]

$$A(x)B(0) \xrightarrow[x\to 0]{} \sum_{i=0}^{N} F_i(x)\, O_i(0) , \tag{3.13}$$

where $O_1,\dots,O_N$ is a finite set of local field operators and the $F_i(x)$ are functions with singularities $(1/x)^{d_i-d_A-d_B}$ (apart from logs), where the d's are the dimensions of the fields [12)]. We use the colon notation $:A(o)B(o):$ to denote a generalized Wick product of renormalized fields obtained from the ordinary product $A(x)B(o)$ by first subtracting off the singular expansion (3.13) (or a trivial modification of it) and then taking the limit $x^{\mu} \to 0$. The resulting quantity can be shown to be a finite local field operator having the same quantum numbers as the free field ordinary Wick product $:A(o)B(o):$ [9),10),11)]. All of the divergences encountered in unrenormalized perturbation theory arise from its use of divergent expressions like $A(o)B(o)$ rather than $:A(o)B(o):$.

Thus, for example, in φ^4 theory, the short distance behaviour of $j(x)j(o)$ is again of the form (3.8) except that the c_i's are replaced by $c_i(\log x^2)^{a_i}$ for suitable integers a_i. All this follows from the fact that the leading short distance singularities are mass independent and hence given by dimensional analysis. A more precise treatment of this dimensionality concept will be given in Section 5.

4. - LIGHT CONE BEHAVIOUR IN QUANTUM FIELD THEORY [7)]

The difficulty encountered in going from short distance behaviour to LC behaviour can be seen from Eq. (3.8). For notational simplicity, we shall first ignore all logarithmic factors. We shall discuss their possible effects later on. Note, in (3.8) that, for $x \to 0$, $(1/x^2)$ is a power more singular than $(1/x^2)x^{\alpha}$. Near the LC, however, each function has the same singularity and, in fact, an infinite number of terms with this singularity occurs in the LC expansion. The result is

$$j(x)j(o) \xrightarrow[x^2 \to 0]{} c_1\left(\frac{1}{x^2}\right)^2 I + \frac{1}{x^2}\sum_{n=0}^{\infty} x^{\alpha_1}\cdots x^{\alpha_n} O^{(n)}_{\alpha_1\cdots\alpha_n}(0) , \qquad (4.1)$$

where $\dim O^{(n)} = n+2$. Thus, each term in the sum has dimension two and carries a LC singularity $1/x^2$. For consistency with (3.8), we must have

$$O^{(0)} = c_2 j \quad \text{and} \quad O^{(1)}_{\alpha_1} = c_3 : \varphi \partial_{\alpha_1} \varphi : \quad .$$

The other terms in (4.1) do not contribute to the short distance limit (3.8), but they are necessary to describe the LC limit (4.1).

We can now calculate the LC behaviour of, for example, the expectation value of $j(x)j(o)$ in the one-particle state of momentum p. We can write

$$\langle p | O^{(n)}_{\alpha_1 \ldots \alpha_n}(0) | p \rangle = a_n p_{\alpha_1} \cdots p_{\alpha_n} + b_n g_{\alpha_1 \alpha_2} p_{\alpha_3} \cdots p_{\alpha_n} + \ldots , \tag{4.2}$$

where the omitted terms each involve at least one $g_{\alpha\beta}$. Only the first term in (4.2) therefore contributes to the leading LC singularity of (4.1). Thus, defining

$$f(\lambda) = \sum_{m=0}^{\infty} a_m \lambda^m , \tag{4.3}$$

we obtain

$$\langle p | j(x) j(0) | p \rangle_c \xrightarrow[x^2 \to 0]{} \frac{1}{x^2} f(x \cdot p) \tag{4.4}$$

as the leading LC singularity of the connected matrix element.

Expansions of the form (4.1) exist and describe the LC behaviour of the product of any local field operators in each order of renormalized perturbation theory and, more generally, in any theory in which expansions of the form (3.13) exist for all local field products at short distances. They might therefore be abstracted from these models and assumed to be true in the real world.

We shall not have time here to derive these expansions. Derivations are given in Ref. 6) [13]. This reference also contains derivations of similar expansions for the product $j_\mu(x) j_\nu(o)$ of vector currents and for other interesting products in φ^4 theory, the gluon model, all other renormalizable models, and in soluble field theoretic models. Properties of the expansions are discussed in detail, including the nature of the basis fields $\theta_{\alpha_1 \cdots \alpha_n}$ and consistency with causality and translation invariance.

As a second example, we consider a vector current $j_\mu(x)$ of dimension three in φ^4 theory, for example $:\varphi(x)\partial_\mu\varphi(x):$. Ignoring logs, we obtain the LC expansion

$$
\begin{aligned}
j_\mu(x) j_\nu(0) \xrightarrow[x^2\to 0]{} E_{c\mu\nu}(x) I \; + \; \sum_n \Big\{ & x^{-6} x_\mu x_\nu x_{\alpha_1} \cdots x_{\alpha_n} \mathcal{P}_1^{\alpha_1 \cdots \alpha_n}(0) \\
& + x^{-4}\Big[g_{\mu\nu} x_{\alpha_1} \cdots x_{\alpha_n} \mathcal{P}_2^{\alpha_1 \cdots \alpha_n}(0) + x_\nu x_{\alpha_1} \cdots x_{\alpha_n} \mathcal{P}_{3\mu}^{\alpha_1 \cdots \alpha_n}(0) + x_\mu x_{\alpha_1} \cdots x_{\alpha_n} \mathcal{P}_{4\nu}^{\alpha_1 \cdots \alpha_n}(0) \Big] \\
& + x^{-2} x_{\alpha_1} \cdots x_{\alpha_n} \mathcal{P}_{5\mu\nu}^{\alpha_1 \cdots \alpha_n}(0) \Big\} ,
\end{aligned}
\tag{4.5}
$$

where we have written $x^{-6} = (x^2)^{-3}$, etc.

If the current $j_\mu(x)$ is conserved, then the expansion (4.5) can be simplified further. The current conservation condition $\partial^\mu j_\mu(x) = 0$ places essentially two constraints on (4.5) and reduces the number of operator sequences from five to three. The final result can be conveniently written in the manifestly conserved form

$$
\begin{aligned}
j_\mu(x) j_\nu(0) \xrightarrow[x^2\to 0]{} & (\partial_\mu \partial_\nu - g_{\mu\nu}\Box) x^{-2} \sum_n x^{\alpha_1} \cdots x^{\alpha_n} \mathcal{R}_{0\alpha_1 \cdots \alpha_n}(0) \\
& + i\varepsilon_{\mu\nu\alpha\beta} \partial^\alpha x^{-2} \sum_n x^{\alpha_1} \cdots x^{\alpha_n} \mathcal{R}^{\beta}_{1\alpha_1 \cdots \alpha_n}(0) \\
& + \big[g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\alpha\nu}\partial_\beta\partial_\mu - g_{\alpha\mu}\partial_\beta\partial_\nu + g_{\alpha\mu} g_{\beta\nu}\Box \big] (\log - x^2) \sum_n x^{\alpha_1} \cdots x^{\alpha_n} \mathcal{R}^{\alpha\beta}_{2\alpha_1 \cdots \alpha_n}(0).
\end{aligned}
\tag{4.6}
$$

Here the $(\log x^2)$ term does not violate our neglect of logs since the log goes away after it is differentiated. The relation between the $\mathcal{R}$'s and the $\mathcal{P}$'s can be found by explicitly performing the differentiations exhibited in (4.6) but will not be given here.

5. - DIMENSIONALITY [7)]

In this Section we shall indicate how one can be more precise about the notion of dimensionality which we have been using. One says that a local field $\chi(x)$ has dimension d if there exists a one-parameter group $U(s)$ of unitary transformations such that

$$U(s)\chi(x)U^{-1}(s) = s^d \chi(sx) \quad . \tag{5.1}$$

Examples are the free massless scalar field with $d = 1$ and free spinor field with $d = \frac{3}{2}$. We shall refer to this notion of dimension as "dynamical" dimension. For the usual fields in free field theories, dynamical dimension coïncides with naïve dimension. We shall say a field has canonical dimension if it has a dynamical dimension equal to that of the corresponding free field. In a theory in which all local fields have dimensions and short distance expansions such as (3.13) are valid, application of (5.1) to (3.13) implies that the $F_i(x)$ behave as stated like $(1/x)^{d_i-d_A-d_B}$, with no logarithmic factor. This is what happens in free massless field theories.

In an exactly scale invariant theory, the structure of any two-point function is fixed up to some constants. For example, application of (5.1) to the Wightman function $<0|\chi(x)\chi(o)|0>$ gives

$$<0|\chi(x)\chi(0)|0> = \text{const} \ (x^2)^{-d} \quad . \tag{5.2}$$

Also, if $Q \equiv \int d^3x \ j_o(x)$ is the generator of an exact internal symmetry, then $\dim j_o(x) = 3$, as can be seen by applying scale transformations to a relation like $[\chi(x), Q] = q\,\chi(x)$.

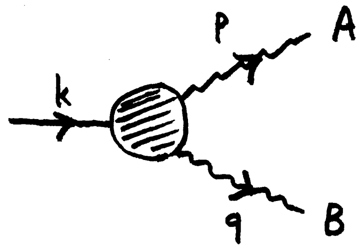
In any finite order of a renormalizable perturbation theory, because of the occurrence of logarithmic factors, the renormalized fields do not have well-defined dynamical dimensions. Nevertheless, the short distance behaviour of any Whightman function is, apart from logarithmic factors, the same as it would be if the fields did have canonical dynamical dimensions. Put differently, the short distance behaviour is determined, apart from logs, by the naïve dimensions of the fields. In particular, the nature of short distance expansions, and, by our analysis, of LC expansions are so determined. We shall describe this situation by saying that the fields have effective canonical dimensions.

In any theory with effective canonical dimensionality and with short distance expansions, LC expansions very similar to those given in Section 4 will exist. Included in such theories are free field models, renormalizable perturbation theories in any finite order, and most of the known exactly soluble models. In theories with effective non-canonical dynamical dimensionality and with short distance expansions, our derivations show that LC expansions will also exist. In these theories, the singular functions will, of course, be somewhat different from those encountered in Section 4. The Thirring model is the only one we know of that exhibits non-canonical dynamical dimensionality [14].

A final point we should mention concerns the nature of the sum (if it exists) of the perturbative expansions of the renormalizable field theories. It is possible, and has been suggested, that the logarithmic factors occurring in each order sum up to a power and so change the dynamical dimensions of the fields. This is what happens in the Thirring model. We suspect that this Thirring model phenomenon arises because of the zero mass particle present and will not occur in realistic models with no massless particles. There is no evidence that the logs of renormalizable perturbation theories add up to a power [15]. In the following section we shall point out the existence of empirical indications of the validity of effective canonical dimensionality.

6. - MASS DISPERSION RELATIONS [16)]

Consider a vertex function



where the scalar particle (k) is on-shell $(k^2 = m^2)$ and p and q $(k = p+q)$ are the momenta carried by two scalar currents $A(x)$ and $B(x)$:

$$A(p^2, q^2) = \int d^4x\, e^{-ip\cdot x} \langle 0|T[A(x)B(0)]|k\rangle \; . \tag{6.1}$$

We saw, in Section 2, that the behaviour of $A(p^2,q^2)$ in the limit $p^2 \to \infty$ with $\omega \equiv p^2/2\nu \equiv [1 - (q^2-m^2/p^2)]^{-1}$ fixed (∞ included) is determined by the behaviour of $A(x)B(0)$ near (namely, within $1/p^2$) the light cone $x^2 = 0$. Thus, to compute this limit, we can use the general light cone expansion

$$A(x)B(0) \xrightarrow[x^2\to 0]{} E(x^2 - i\epsilon x_0) \sum_n x^{\alpha_1} \cdots x^{\alpha_n} O^{(n)}_{\alpha_1\cdots\alpha_n}(0) \; , \tag{6.2}$$

where the $O^{(n)}_{\alpha_1\cdots\alpha_n}(0)$ are local operators. Defining

$$\sum_n x^{\alpha_1} \cdots x^{\alpha_n} \langle 0| O^{(n)}_{\alpha_1\cdots\alpha_n}(0)|k\rangle = f(k\cdot x) + \text{order } x^2 \; , \tag{6.3}$$

we obtain

$$A(q^2, p^2) \longrightarrow \int d^4x\, e^{-ip\cdot x} E(x^2 - i\epsilon) f(k\cdot x) \; . \tag{6.4}$$

Thus the general singularity $E(z) = z^{-r}$ gives the result

$$A(p^2; q^2) \longrightarrow -\frac{e^{i\pi r/2}(2\pi)}{\Gamma(r)} (p^2)^{r-2} \omega^{2-r} F_r(\omega) , \tag{6.5}$$

where

$$F_r(\omega) \equiv \int_0^\infty d\lambda\, e^{i\omega\lambda} \lambda^{1-r} f(\lambda) \quad . \tag{6.6}$$

Two interesting special cases are the limit $\omega \to 1$ (so that $q^2/p^2 \to 0$ and the limit is $p^2 \to \infty$ with q^2 fixed), in which

$$A(p^2; q^2) \xrightarrow[q^2 \text{ fixed}]{p^2 \to \infty} (p^2)^{r-2} F_r(1) , \tag{6.7}$$

and the limit $\omega \to \infty$ (so that $q^2/p^2 \to 1$ and the limit is the Bjorken [8] limit $p_0 \to \infty$ with $\vec{p}$ fixed), in which

$$A(p^2, q^2) \xrightarrow[\vec{p} \text{ fixed}]{p_0 \to \infty} (p_0)^{2r-4} f(0) \quad . \tag{6.8}$$

As expected, the limit (6.8) is controlled by the first non-vanishing (although perhaps infinite) equal-time commutator as determined by (6.2).

In simple perturbation theories, one finds $r = 1$ (within logs) and $f(\lambda) \sim e^{i\lambda}$ so that $F_1(\omega)$ has a pole at $\omega = 1$ and (6.7) becomes meaningless, the correct behaviour being $A \to$ const. We explicitly assume that our $F_r(\omega)$ do not develop such poles. This assumption accounts for the observed rapid decrease of empirical form factors and the smooth behaviour of the structure functions measured at SLAC and amounts to assuming a composite structure for the hadrons. It is, presumably, the same mechanism which reggeizes the fixed poles of perturbation theory that eliminates the poles in $F_r(\omega)$.

We shall make a second assumption in order to determine the values of r relevant in specific cases. We assume that all relevant field and current operators have the same (canonical) dimensions that they have in the gluon model (ignoring logs) (triplet quarks coupled to a massive neutral vector meson via the baryon number current). The gluon model thus treated has been very successful in accounting for many aspects of processes like the ones we are considering [17], and, as we shall see in Sections 8 and 9, this specific assumption gives the essentially unique singularity structure for electromagnetic currents consistent with the SLAC and Columbia-BNL experiments.

Our final assumption will be that asymptopia sets in quite quickly, namely for $p^2 \sim 2\ \mathrm{GeV}^2$. This assumption is strikingly supported by the results of SLAC and Columbia-BNL. Its implications for our purposes are that (6.7) becomes valid for $p^2 > 2\ \mathrm{GeV}^2$ and that $f(\lambda)$ has support concentrated very near $\lambda = 0$. This last statement accounts for the rapid approach of the electroproduction scaling function to its (constant) asymptotic limit. It means, in particular, that $F_1(1)$ is of the order of $f(o)$.

We proceed to apply these ideas to discuss mass dispersion relations. The amplitude $A(p^2,q^2)$ is assumed to be analytic in the cut p^2 plane, with a cut starting at $p^2 = \alpha > 0$. We can, therefore, write the "finite mass dispersion relation"

$$A(p^2;q^2) = \frac{1}{\pi}\int_\alpha^\Lambda dp'^2 \frac{a(p'^2;q^2)}{p'^2-p^2+i\epsilon} + \frac{1}{2\pi i}\int_{C_\Lambda} dp'^2 \frac{A(p'^2;q^2)}{p'^2-p^2+i\epsilon} , \qquad (6.9)$$

where $a(p^2,q^2) \equiv \mathrm{abs}\, A(p^2,q^2)$ and c_Λ is the circular contour $|p^2| = \Lambda$. For $\Lambda > 2\ \mathrm{GeV}^2$, we thus obtain

$$A(p^2;q^2) \simeq \frac{1}{\pi}\int_\alpha^\Lambda dp'^2 \frac{a(p'^2;q^2)}{p'^2-p^2+i\epsilon} + \frac{F_r(1)}{2\pi i}\int_{C_\Lambda} dp'^2 \frac{(p'^2)^{r-2}}{p'^2-p^2+i\epsilon} . \qquad (6.10)$$

Integrating A over c_Λ , we get the further useful relation

$$0 = \frac{1}{\pi}\int_\alpha^\Lambda dp'^2 a(p'^2, q^2) + \frac{F_r(1)}{2\pi i}\int_{c_\Lambda} dp'^2 (p'^2)^{r-2} \quad . \tag{6.11}$$

We are thus paralleling the "finite energy sum rule" treatment of four-point functions. The important fact that Λ can be as small as 2 GeV^2 is analogous to the usefulness of the concept of "duality".

Let us suppose that there is a low-lying particle of mass μ with the quantum numbers of $A(x)$ so that

$$a(p^2; q^2) = \pi\delta(p^2-\mu^2)\, a_p(q^2) + a_N(p^2; q^2) \; . \tag{6.12}$$

Then, (6.10) and (6.11) become (canonical dimensionality implies that r is an integer)

$$A(0,q^2) = \frac{a_p(q^2)}{\mu^2+i\epsilon} + \frac{1}{\pi}\int_\alpha^\Lambda dp'^2 \frac{a_N(p'^2; q^2)}{p'^2+i\epsilon} + \delta_{r2} F_r(1) \tag{6.13}$$

and

$$0 = a_p(q^2) + \frac{1}{\pi}\int_\alpha^\Lambda dp'^2 a_N(p'^2; q^2) + \delta_{r1} F_r(1) \; . \tag{6.14}$$

We can use the above equations to approximately calculate both the "infinite" mass contributions and the continuum contributions. These contributions give corrections to the result of simply saturating the mass dispersion relation with a low lying meson. In this way we can understand, for example, pion pole dominance of matrix elements of the divergence of the axial vector current and we can estimate corrections to vector meson dominance which are in good agreement with experiment. An example of this will be discussed in Section 9.

7. - DEEP INELASTIC ELECTRON PROTON SCATTERING [18]

Our purpose here is to use the expansion (4.6) to study the process $e + p \to e +$ anything. The relevance of the LC behaviour of the matrix element $\langle p|[J_\mu(x), J_\nu(o)]|p\rangle$ to the A limit this reaction has been known for some time [1]. Our use of the operator expansion will, however, enable us to deduce a number of new results from the observed scaling behaviour. We follow the notation of Ref. [4], where more details and references can be found. We shall first work with the result (4.6) of ignoring logarithmic factors and afterwards discuss the effect of these logs and of their possible role in changing the singularity structure.

The total cross-section (2.1) of interest can be written

$$\frac{d^2\sigma}{dq^2 d\nu} = \frac{\pi\alpha^2}{E^2|q|^2\sin^2\frac{\theta}{2}}\left[W_2(q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(q^2,\nu)\sin^2\frac{\theta}{2}\right], \tag{7.1}$$

where E is the initial electron energy and θ the scattering angle and we have set the proton mass equal to unity: $p^2 = 1$. The structure functions are defined by

$$\frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle p|[J_\mu(x), J_\nu(o)]|p\rangle = (p_\mu - \rho q_\mu)(p_\nu - \rho q_\nu)W_2(q^2,\nu) - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1(q^2,\nu), \tag{7.2}$$

where $\rho \equiv -\nu/q^2 = (2\omega)^{-1}$, and the A limits are

$$\lim_A \nu W_2(q^2,\nu) = F_2(\rho), \tag{7.3}$$

$$\lim_A W_1(q^2,\nu) = F_1(\rho). \tag{7.4}$$

The transverse and longitudinal structure functions are

$$F_T = F_1, \qquad F_L = \rho F_2 - F_1. \tag{7.5}$$

Experimentally [19], (7.3) is well satisfied in a non-trivial way $[F_2(\rho) \sim \mathrm{const.}$ for $\rho \geq 2]$ and F_L/F_T is small, as suggested by the gluon model [20].

It is convenient to introduce new structure functions by writing

$$\frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle p|[J_\mu(x), J_\nu(0)]|p\rangle = [q^2 p_\mu p_\nu - \nu(p_\mu q_\nu + q_\mu p_\nu) + \nu^2 g_{\mu\nu}]V_2(q^2,\nu)$$
$$-(q^2 g_{\mu\nu} - q_\mu q_\nu)V_1(q^2,\nu) . \tag{7.6}$$

Equations (7.2)-(7.6) imply

$$\lim_A (-\nu^2)V_2(q^2,\nu) = \rho F_2(\rho) \tag{7.7}$$

$$\lim_A \nu V_1(q^2,\nu) = \rho F_L(\rho) . \tag{7.8}$$

In configuration space (7.6) reads

$$\frac{1}{2\pi}\langle p|[J_\mu(x), J_\nu(0)]|p\rangle = -[\Box p_\mu p_\nu - (p\cdot\partial)(p_\mu\partial_\nu + p_\nu\partial_\mu) + (p\cdot\partial)^2 g_{\mu\nu}]\hat{V}_2(x^2, x\cdot p)$$
$$-(\partial_\mu\partial_\nu - \Box g_{\mu\nu})\hat{V}_1(x^2, x\cdot p) , \tag{7.9}$$

in terms of the Fourier transforms

$$V_i(q^2,\nu) = \int d^4x\, e^{iq\cdot x}\hat{V}_i(x^2, x\cdot p) . \tag{7.10}$$

The A limit of the V_i's is given by the LC behaviour of the $\hat{V}_i$'s and these can be determined from (4.6). We define as in (4.1)-(4.4) the matrix elements

$$\langle p|\sum_n x^{\alpha_1}\cdots x^{\alpha_n}\mathcal{R}_{0\alpha_1\ldots\alpha_n}(0)|p\rangle = f_0(x\cdot p) + O(x^2) , \tag{7.11}$$

$$\langle p|\sum_n x^{\alpha_1}\cdots x^{\alpha_n}\mathcal{R}^{\alpha\beta}_{2\alpha_1\ldots\alpha_n}(0)|p\rangle = g^{\alpha\beta}f(x\cdot p) + p^\alpha p^\beta f_2(x\cdot p) + O(x^2). \tag{7.12}$$

Comparison with (7.9), using

$$\mathrm{Im}(x^2 - i\varepsilon x_0)^{-1} = \pi\,\varepsilon(x_0)\,\delta(x^2)\,, \quad \mathrm{Im}\log(-x^2 + i\varepsilon x_0) = \pi\,\varepsilon(x_0)\,\Theta(x^2)\,, \tag{7.13}$$

gives the results

$$\hat{V}_2(x^2, x\cdot p) \xrightarrow[x^2\to 0]{} -\varepsilon(x_0)\,\Theta(x^2)\,f_2(x\cdot p) \tag{7.14}$$

$$\hat{V}_1(x^2, x\cdot p) \xrightarrow[x^2\to 0]{} -\varepsilon(x_0)\,\delta(x^2)\,f_0(x\cdot p)\,. \tag{7.15}$$

These are precisely the LC behaviours shown in Ref. 4) to be equivalent to the scaling laws (7.7) and (7.8) or (7.3) and (7.4). Indeed, direct substitution of (7.14) and (7.15) into (7.10) gives the results (7.7) and (7.8) with

$$\rho F_2(\rho) = -2\pi\int d\lambda\, e^{-i\lambda/2\rho}\,\lambda f_2(\lambda)\,, \tag{7.16}$$

$$\rho F_L(\rho) = -\frac{\pi i}{2}\int d\lambda\, e^{-i\lambda/2\rho} f_0(\lambda)\,. \tag{7.17}$$

We have thus derived the validity of the scaling laws (7.7) and (7.8) in the large class of theories in which (4.6) holds. Strictly speaking, because these theories really give extra logarithmic factors, we can only deduce that (7.7) and (7.8) are valid apart from powers of $\log q^2$. Indeed, it is a known fact that in low orders these theories give scaling apart from logs. This is satisfactory since the presence of such logarithmic factors could easily escape experimental detection at present. One might think that this result is trivial because we have built in scaling via the mass independence of (4.6). The point is, however, that, because of the possibility of non-canonical dimensions, mass independence is not equivalent to scaling. We can thus reach the strong conclusion that the observed scaling behaviour is consistent with canonical dimensionality but not with many types of non-canonical dimensionality. If a single operator with a non-vanishing proton-proton matrix element in (4.6) had a dimension significantly less than its canonical one, then the scaling limits (7.7) and/or (7.8) would be divergent by the corresponding power of q^2. If we further believe in the relevance of a perturbative model, then we can conclude that the logarithmic factors in the model do not add up to significantly change the singularity structure. It is therefore an interesting and non-trivial fact that <u>non-trivial scaling is equivalent to the presence of the LC singularity structure required by canonical dimensionality</u>.

We have thus seen how the existence of operator product expansions like (4.6) enables one to correlate experimental results with the nature of possible field theoretic models for the hadrons. In addition to providing evidence for essentially canonical dimensionality, the observed scaling strongly suggests the relevance of renormalizable field theories. Non-renormalizable models, made finite, say, by the introduction of infinitely many subtraction constants, possess much worse LC singularities. A further major advantage of our formalism is that, unlike the matrix element statements like (7.14) and (7.15), it enables one to compare and relate different processes since these processes simply involve different matrix elements of the same operators.

Before leaving electroproduction, we wish to comment on what happens if $F_L = 0$. It is clear from the above analysis that, neglecting the unlikely possibility that the proton-proton matrix element of each $\mathcal{R}_{0\alpha_1\ldots\alpha_n}$ in (4.6) vanishes, $F_L = 0$ means that the leading allowed singularity $1/x^2$ in the first piece of (4 . 6) is not present. Assuming canonical dimensionality, this means that the $(1/x^2)$ must be replaced by $(\log x^2)$. This comes from both the non-leading contributions of the given operators satisfying $\dim \mathcal{R}_{\alpha_1\ldots\alpha_n} = n+2$ and from the leading contributions of additional operators satisfying $\dim \mathcal{R}_{\alpha_1\ldots\alpha_n} = n+4$. Calling the matrix element of the sum of these operators still $f_0(x\cdot p)$ as in (7.11), (7.15) becomes replaced by

$$\hat{V}_1(x^2, x\cdot p) \xrightarrow[x^2\to 0]{} -\varepsilon(x_0)\,\theta(x^2)\left[f_0(x\cdot p) + 2f'(x\cdot p)\right]. \tag{7.18}$$

Another use of the LC in connection with electroproduction is to relate the A limit with the Regge limit (R limit ; $\nu \to \infty$, q^2 fixed) and thereby understand the asymptotic behaviours of the $F_i(\rho)$ for large ρ [1),4),21)]. Conventional Regge theory and Pomeron dominance predict that

$$\lim_R V_2(q^2,\nu) = \mathcal{N}_2(q^2)\,\nu^{-1} \tag{7.19}$$

$$\lim_R V_1(q^2,\nu) = \mathcal{N}_1(q^2)\,\nu . \tag{7.20}$$

It is easily seen [4)] that in (7.10) the R limit of the V_i for $\nu \gg |q^2| \gg 1$ is again controlled by the LC behaviours of the $\hat{V}_i$. Equations (7.19) and (7.20) together with (7.14) and (7.15) require then that [4)]

$$f_2(\lambda) \sim |\lambda|^{-1} \quad , \qquad f_0(\lambda) \sim |\lambda| \tag{7.21}$$

for $\lambda \to \infty$. Use of these results in (7.16) and (7.17) then gives

$$F_2(\rho) \xrightarrow[\rho\to\infty]{} \text{const} \tag{7.22}$$

$$F_L(\rho) \xrightarrow[\rho\to\infty]{} \text{const}\ \rho \ , \tag{7.23}$$

in good agreement with experiment [19)].

8. - MASSIVE μ PAIR PRODUCTION [22]

The several numbers obtained from SLAC are insufficient to really test our LC ideas. The recent Columbia-BNL experiment measuring massive muon pair production from high-energy proton-proton collisions [23] is therefore extremely useful theoretically since it involves an initial state different from SLAC's and provides additional experimental constraints. Several theoretical investigations of this process have already been given [24]. In this note, we shall apply our theoretical results [6] on the behaviour of current products near the light cone to study the Columbia experiment. Our predictions turn out to be in excellent agreement with experiment.

We consider the reaction proton + proton $\rightarrow \mu^+ + \mu^- +$ anything and call p and p' the momenta of the initial nucleons ($p^2 = p'^2 = m^2$) and q the momentum of the muon pair. We define the invariants $s = (p+p')^2$, $\nu = p \cdot q$, and $\nu' = p' \cdot q$. The cross-section is (neglecting the muon mass)

$$\frac{d\sigma^r}{dq^2} = \frac{\alpha^2}{6\pi^3} \frac{1}{[s(s-4m^2)]^{1/2}} \int \frac{d^3q}{q_0} \frac{1}{q^2} W^{\mu\nu} \varepsilon_\mu^r \varepsilon_\nu^r \quad , \tag{8.1}$$

where

$$W_{\mu\nu} = E_1 E_2 \int d^4x \, e^{-iq\cdot x} {}_{in}\langle pp' | J_\mu(x) J_\nu(0) | pp' \rangle_{in} \; . \tag{8.2}$$

In (8.1), $\varepsilon_\mu^r(q)$ describes the polarization $r = T_1, T_2$, or L of the μ pair and the integration region in the centre of mass (CM) frame is described by the inequality

$$\sqrt{q^2} < q_0 < (s + q^2 - 4m^2)/2\sqrt{s} \equiv \mathcal{H}_0(q^2, s) \; . \tag{8.3}$$

In (8.2), J_μ is the electromagnetic current, a spin average is (here and everywhere) understood, and only the connected part of the matrix element occurs. The total cross-section is

$$\frac{d\sigma}{dq^2} = -\frac{\alpha^2}{6\pi^3}\frac{1}{[s(s-4m^2)]^{1/2}}\int\frac{d^3q}{q_0}\frac{1}{q^2}W^\mu{}_\mu \ . \tag{8.4}$$

In the physical region for our reaction we must have $q^2 = q_0^2 - \vec{q}^{\,2} > 0$. This is in contrast to the SLAC kinematics where $q^2 < 0$. Using current conservation and the reflection property $W_{\nu\mu} = W^*_{\mu\nu}$, we can write $W_{\mu\nu} = (q_\mu q_\nu - g_{\mu\nu}\, q^2)W_1(s, q^2, \nu, \nu') + \ldots$.

The SLAC experiment can be nicely described by the assumption that the appropriate dimensionless functions $F_i(q^2, \nu)$ become functions of only the ratio $\rho \equiv \nu/q^2$ in the limit $-q^2 \to \infty$, $\nu \to \infty$, ρ fixed. This corresponds to the expectation that a massive photon should only probe the short-distance (mass-independent) structure of the target. We should like to apply this same idea to the μ pair process but note that, because of purely hadronic non-scale invariant effects, this need not imply that the dimensionless structure functions $F_i = q^2 W_i$ become functions of only the ratios $\rho \equiv \nu/q^2$, $\rho' \equiv \nu'/q^2$, and $\sigma \equiv s/q^2$ in the limit

$$q^2, s, \nu, \nu' \longrightarrow \infty \text{ with } \rho, \rho', \text{ and } \sigma \text{ fixed}. \tag{8.5}$$

We shall rather implement the electromagnetic scale-invariance principle by assuming that the short-distance behaviour of the product $J_\mu(x)J_\nu(0)$ is mass-independent. This is part of the content of our operator expansion.

Combining (8.1) and (8.2), we obtain the expression

$$\frac{d\sigma^r}{dq^2} \propto \frac{1}{q^2} \int d^4x \, \Delta_R^+(-x;q^2) \langle pp' | J^\mu(x) J^\nu(0) | pp' \rangle \, \varepsilon_\mu^r \varepsilon_\nu^r , \tag{8.6}$$

where

$$\Delta_R^+(-x;q^2) \equiv \int_R d^4k \, e^{-ik\cdot x} \, \theta(k_0) \, \delta(k^2 - q^2) , \tag{8.7}$$

with the integration R specified by $|\vec{k}| < [\varkappa_0^2 - q^2]^{\frac{1}{2}} \equiv \varkappa$ in the CM frame. Now, when $s \to \infty$ and $q^2 \to \infty$, we see from (8.3) that $R \to$ (all space) and so $\Delta_R^+(x\,;q^2) \to \Delta^+(x\,;q^2)$, the ordinary free field Wightman function. Thus, in the specified limit, only the integration region $x^2 \simeq 0$ is important in (8.6). More quantitatively, it can be shown that (8.7) has support essentially in the region

$$\left| x^2 - \frac{1}{\varkappa^2} \right| < \frac{1}{q^2} . \tag{8.8}$$

Elsewhere Δ_R gives exponential decrease or rapid oscillation which damps the integrand <u>in the physical region</u>. This region is near the light cone provided q^2 and $\varkappa$ are both big.

We can thus use our expansion (4.6) in (8.6) to determine the behaviour of $d\sigma/dq^2$ in the A limit. For simplicity we shall use the form valid when $F_L = 0$ although we would obtain the same results without this assumption. We call $O_{1,2}(x)$ the coefficient of first (third) differential operator in (4.6).

We next define

$$\langle pp' | O_1(x) | pp' \rangle = E_1(x^2) \, f_0 \tag{8.9}$$

and

$$\langle pp'|O_2^{\alpha\beta}(x)|pp'\rangle = E_2(x^2)[f_1 g^{\alpha\beta} + f_2 p^\alpha p^\beta + f_3 p'_\alpha p'_\beta$$
$$+ f_4(p^\alpha p'^\beta + p'^\alpha p^\beta)], \tag{8.10}$$

where $f_i = f_i(s, x\cdot p, x\cdot p') + O(x^2)$, the x^2 dependence being irrelevant for our purposes since it leads to weaker singularities. Fermi statistics require that

$$f_i(s, x\cdot p, x\cdot p') = f_i(s, x\cdot p', x\cdot p), \quad i = 0, 1, 4,$$
$$f_2(s, x\cdot p, x\cdot p') = f_3(s, x\cdot p', x\cdot p), \tag{8.11}$$

and crossing gives

$$f_i(s, x\cdot p, x\cdot p') = f_i(s, -x\cdot p, -x\cdot p'), \quad i = 0-5. \tag{8.12}$$

We thus obtain

$$W^{\mu\nu}\epsilon_\mu^r\epsilon_\nu^r \simeq \int d^4x\, e^{-iq\cdot x} E(x^2)\{-q^2(2f_0+f_1) + (-q_\alpha q_\beta + \epsilon_\alpha^r \epsilon_\beta^r q^2)$$
$$\cdot [f_1 g^{\alpha\beta} + \ldots]\}, \tag{8.13}$$

where we have written $E_1(x^2) = E_2(x^2) = \log x^2 \equiv E(x^2)$.

Consider the contribution of

$$\langle pp'|R_{0\alpha_1\ldots\alpha_n}(0)|pp'\rangle = F^{(n)}(s)(p_{\alpha_1}\cdots p_{\alpha_n} + p'_{\alpha_1}\cdots p'_{\alpha_n}) + \ldots \tag{8.14}$$

to $f_0(s, x\cdot p, x\cdot p')$. We assume that the large s behaviour of such amplitudes (corresponding to the emission of a zero four-momentum particle with Lorentz indices $\alpha_1 \ldots \alpha_n$) is governed by Regge theory [25]. Then $F^{(n)}(s) \to c_n s^\alpha$ whereas the omitted terms (involving the mixed polynomials $p_{\alpha_1} \ldots p_{\alpha_m}\ p'_{\alpha_{m+1}} \ldots p'_{\alpha_n}$) behave like $s^{\alpha-(n-m)}$, where α is the $t = 0$ intercept of the leading contributing Regge trajectory (presumably the Pomeron with $\alpha = 1$). Thus we can write

$$f_0(s, x\cdot p, x\cdot p') \longrightarrow s^\alpha \left[f_0(x\cdot p) + f_0(x\cdot p') \right], \tag{8.15}$$

where $f_0(x\cdot p) \equiv \sum_n c_n (x\cdot p)^n$. Similarly considering (8.10) leads to the behaviour (8.15) for f_1 and $f_2 = f_3$ and to (8.15) but with $s^{\alpha-1}$ for f_4.

Returning to (8.13), we are led to consider the behaviour of the pole contributions to integrals of the form $I \equiv \int d^4x e^{-iq\cdot x} E(x^2 - i\varepsilon x_0) f(x\cdot p)$ in the limit (8.5). We obtain $I \simeq (1/m)(1/\eta)(\partial/\partial\eta)(1/\eta)\tilde{f}(q_0 - \eta/m)$, where $\tilde{f}(\omega)$ is the FT of $f(\lambda)$ and $\eta \equiv |\vec{q}|$. In the limit (8.5), we thus obtain $I \simeq (1/q^4)(1/\rho^2) f'(1/\rho) \equiv (1/q^4) g(\rho)$. In this way, (8.13) is seen to have the asymptotic form

$$\begin{aligned} W^{\mu\nu} \varepsilon^r_\mu \varepsilon^r_\nu \longrightarrow \frac{s^\alpha}{q^4} \Big\{ & q^2 g_0(\rho) + \left[\nu^2 - (p\cdot\varepsilon^r)^2 q^2 \right] g_1(\rho) \\ & + 2s^{-1} \left[\nu\nu' - (p\cdot\varepsilon^r)(p'\cdot\varepsilon^r) q^2 \right] g_2(\rho) + (p \leftrightarrow p') \Big\}. \end{aligned} \tag{8.16}$$

The form (8.16) which we have obtained has a simple physical interpretation in terms of the Regge picture which accounts well for the SLAC data. The SLAC results for $\rho \geq 2$ can be described by the assumption that they correspond to (Pomeron) Regge pole dominated behaviour with the q^2 dependence of the photon-Pomeron-photon vertex given by scale invariance. If we adapt this picture for the present situation, and further use Regge theory to conclude that the (pp') - Pomeron - (pp') vertex has the large s behaviour $s^{\alpha} = s^1$ [thus obtaining a Regge squared description (see Fig. 1) corresponding to the two large sub-energies ν or ν' and s], we obtain precisely the form (8.16) for large ρ with the further information that $g_0(\rho) \to A_0 \rho$, $g_1(\rho) \to A_1/\rho$, and $g_2(\rho) \to A_2/\rho$ for some constants A_i. Our final assumption will be that these asymptotic behaviours set in at the SLAC points $\rho \sim 2$. Then we can neglect g_2 in (8.16) and obtain for (8.1)

$$\frac{d\sigma^r}{dq^2} \longrightarrow \text{const}\, \frac{1}{q^6} \int \frac{d^3q}{q_0} \Big\{ (A_0 + q^2 A_1)(P\cdot q) + q^2 A_1 \Big[\frac{(p\cdot\epsilon^r)^2}{\rho} + \frac{(p'\cdot\epsilon^r)^2}{\rho'} \Big] \Big\} , \tag{8.17}$$

and for (8.4)

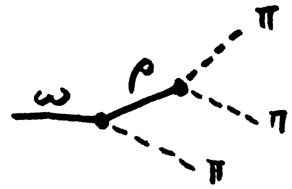
$$\frac{d\sigma}{dq^2} \longrightarrow \text{const}\, \frac{1}{q^6} (3A_0 + 2A_1 q^2) \int \frac{d^3q}{q_0} (P\cdot q) , \tag{8.18}$$

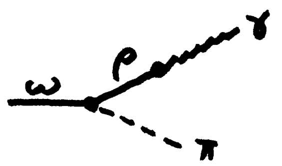
where $P = p + p'$.

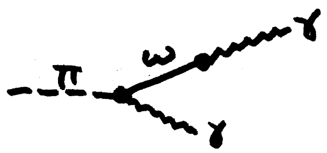
We have compared our prediction (8.17) with the experimental results in Fig. 2. The experimentalists do not measure the total unconstrained cross-section (8.17), but have an angle cut $\cos\theta \geq 0.998$ and a momentum cut $12 < p_{lab} < 29$ GeV/c for $E_{lab} = 29.5$ GeV. It can be shown that these cuts do not significantly affect the relevance of the light cone. The theoretical curve we have plotted corresponds to performing the integral (8.17) over the cut region and taking $3A_0/2A_1 \sim 20$. Our result is seen to be in good agreement with experiment [26]. We can obtain an even better fit by suitably adjusting the functions $g_i(\rho)$ below the value ($\rho \sim 2$) suggested by SLAC for the onset of the asymptotic behaviour.

9. - CORRECTING VECTOR MESON DOMINANCE [16)]

In this final Section we shall apply the considerations of Section 6 to the related processes $\omega \to 3\pi$, $\omega \to \pi\gamma$, and $\pi \to 2\gamma$. The old Gell-Mann - Sharp - Wagner [27)] model uses vector meson dominance to relate the amplitudes for these processes as follows :

$A(\omega \to 3\pi) \simeq$  $= K_1\, g_{\omega\pi\rho}\, g_\rho$ (9.1a)

$A(\omega \to \pi\gamma) \simeq$  $= K_2\, g_{\omega\pi\rho}\, \gamma_\rho^{-1}$ (9.1b)

$A(\pi \to 2\gamma) \simeq$  $= K_3 A(\omega \to \pi\gamma)\, \gamma_\omega^{-1}$ (9.1c)

Here the K_i's are known kinematical constants and the rest of the notation is standard.

To compare these predictions with experiment, we shall use the recent Orsay [28)] colliding beam determinations of γ_ρ and γ_ω. These experiments give the most accurate measurements of these vector meson (V) mass shell photon-vector meson junctions. They find $\gamma_\rho^2/4\pi = 0.52 \pm 0.03$ and $\gamma_\omega^2/4\pi = 3.7 \pm 0.7$. [We mention that, whereas the Orsay value for γ_ρ agrees with other on-shell determinations, their value for γ_ω is not consistent with all other determinations [29)].] According to the usual vector meson dominance hypothesis, these on-shell $(q^2 = m_V^2)$ values are to be used in (9.1) even though the γ-V junctions in (9.1) are at the photon mass-shell $q^2 = 0$.

Thus comparing (9.1) with experiment, one finds [28]

$$\left.\frac{\Gamma(\omega\to\pi\gamma)}{\Gamma(\pi\to 2\gamma)}\right|_{VMD} \sim \frac{1}{2}\left.\frac{\Gamma(\omega\to\pi\gamma)}{\Gamma(\omega\to 2\gamma)}\right|_{EXP} \tag{9.2}$$

and

$$\left.\frac{\Gamma(\omega\to\pi\gamma)}{\Gamma(\omega\to 3\pi)}\right|_{VMD} \sim 2\left.\frac{\Gamma(\omega\to\pi\gamma)}{\Gamma(\omega\to 3\pi)}\right|_{EXP} . \tag{9.3}$$

We see that the VMD model gives a quite unsatisfactory account of these processes. Let us therefore see how our LC approach can improve things.

We define the $\pi^0 \to 2\gamma$ amplitude in terms of the $\pi^0 \to \gamma^3(k_1) + \gamma^8(k_2)$ one $A(k_1k_2)$:

$$\frac{2}{\sqrt{3}} A(k_1,k_2) = \varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^{\mu}(k_1)\,\varepsilon^{\nu}(k_2)\,k_1^{\alpha} k_2^{\beta} F(k_1^2,k_2^2) . \tag{9.4}$$

The relevant operator product expansion is the second piece of (4.6) :

$$J_\mu^3(x)\,J_\nu^8(0) \longrightarrow \varepsilon_{\mu\nu\alpha\beta}\, x^{-4} x^{\alpha} \sum_{n=0}^{\infty} x^{\alpha_1}\cdots x^{\alpha_n} O^{\beta(n)}_{\alpha_1\cdots\alpha_n} . \tag{9.5}$$

In the gluon model $O^{(0)}_\beta(0) \propto A^3_\beta(0)$. Thus, Eqs. (6.7), (6.13) and (6.14) give

$$F(k_1^2,k_2^2) \xrightarrow[\substack{k_1^2\to\infty \\ k_2^2 \text{ fixed}}]{} \frac{1}{k_1^2} F_1(1) \sim \frac{1}{k_1^2}\frac{2 f_\pi e^2}{3\sqrt{2}} , \tag{9.6}$$

$$F(0,0) \simeq \frac{e}{\gamma_\omega} A(\omega \to \pi\gamma) + \frac{A_c}{m_c^2} , \qquad (9.7)$$

$$\frac{\sqrt{2}}{3} f_\pi \simeq \frac{e m_\omega^2}{\gamma_\omega} A(\omega \to \pi\gamma) + A_c , \qquad (9.8)$$

where

$$\frac{e m_\omega^2}{2\gamma_\omega (m_\omega^2)} = \frac{e m_\omega^2}{2\gamma_\omega}$$

is the on-shell ω - γ junction and A_c and m_c represent an average of the continuum effects so that we expect $m^2 \leq m_c^2 \leq \Lambda = 2\ \mathrm{GeV}^2$. Equations (9.7) and (9.8) give

$$F(0,0) \simeq \frac{e}{\gamma_\omega}\left(1 - \frac{m_\omega^2}{m_c^2}\right) A(\omega \to \pi\gamma) + \frac{\sqrt{2} f_\pi e^2}{3 m_c^2} . \qquad (9.9)$$

Note that the first term in (9.9) is the usual Gell-Mann - Sharp - Wagner term (9.1c), but with a correction factor $(1 - m_\omega^2/m_c^2)$, and the second term comes from the light cone behaviour. We shall compare this prediction with experiment below.

We call $A(k_1^2)$ the off-shell $\omega \to \pi\gamma$ invariant amplitude so that the Feynman amplitude is $A(0)\, \varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^{\mu}(k)\, \varepsilon^{\nu}(k_1) k_1^{\alpha} k_2^{\beta}$. We obtain as above

$$A(0) \simeq g_{\omega\rho\pi} \frac{e}{2\gamma_\rho}\left(1 - \frac{m_\rho^2}{m_c^2}\right) . \qquad (9.10)$$

This result coïncides with the GMSW model [27)] provided

$$\gamma_\rho(0) = \left(1 - \frac{m_\rho^2}{m_c^2}\right)^{-1} \gamma_\rho \quad . \tag{9.11}$$

We now compare our predictions (9.9) and (9.10) with experiment. We leave the usual VMD prediction (9.1a) unchanged since most of the physical events occur such that the ρ is nearly on-shell. By choosing $m_c^2 \simeq 2$, we find that both (9.9) and (9.10) are in good agreement with the experimental results. This value of m_c^2 is quite reasonable since the vector mesons should saturate the dispersion relations in their neighbourhoods and thus restrict m_c^2 to be near its upper limit.

We see that the effect of the continuum is to give a k^2 dependence to the γ vector meson junction which is quite appreciable. The striking fact is that with $m_c^2 \simeq 2\ \mathrm{GeV}^2$, one can account for the discrepancies of the VMD model discussed above.

We believe that the applications described in the last three Sections demonstrate the usefulness of operator product expansions near the light cone and support the correctness of the assumption that the hadrons can be described by a renormalizable field theory with effective canonical dimensionality. It is clearly desirable to explore additional applications in order to further test these ideas and to gain more familiarity with the concepts involved. It is possible that the LC will provide an increase in the usefulness of configuration space field theoretic methods in understanding some aspects of particle physics.

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25) We are using the easily derived fact that $\langle \text{in} | O | \text{in} \rangle$ has the same asymptotic behaviour as the scattering amplitude $\langle \text{in} | O | \text{out} \rangle$ if O satisfies the asymptotic condition.

26) Our results are, however, in disagreement with all of those of Ref. 24). Matveev et al. assume the F_i are functions of the ratios or use a vector meson dominance (VMD) model which predicts that $(d\sigma/dq^2) \sim (s/q^6)$. As can be seen from (8.16), our F_i differ from functions of the ratios by powers of s [and have the striking from $g(\rho) + g(\rho')$]. By explicitly evaluating a particular Feynman diagram, Berman et al. obtain a special case of (8.16) corresponding to $g_0 \equiv 1$, $g_1 = g_2 = 0$. Sakurai, of course, uses a VMD model and concludes that there is predominantly longitudinal polarization. Our prediction for σ_L/σ_T can be obtained from (8.17) by using the A_0 and A_1 which fit $d\sigma/dq^2$. It is seen to be a function of q^2 which does not make σ_L/σ_T particularly large or small. This prediction should be testable in future experiments. Finally, the model of Drell and Yan predicts that $(d\sigma/dq^2) \propto (1/q^4)\, F(s/q^2)$, whereas we predict that $(d\sigma/dq^2) \propto (1/q^2) F_1(s/q^2) + F_2(s/q^2)$.

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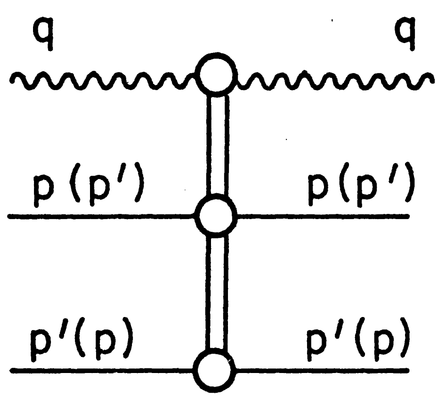
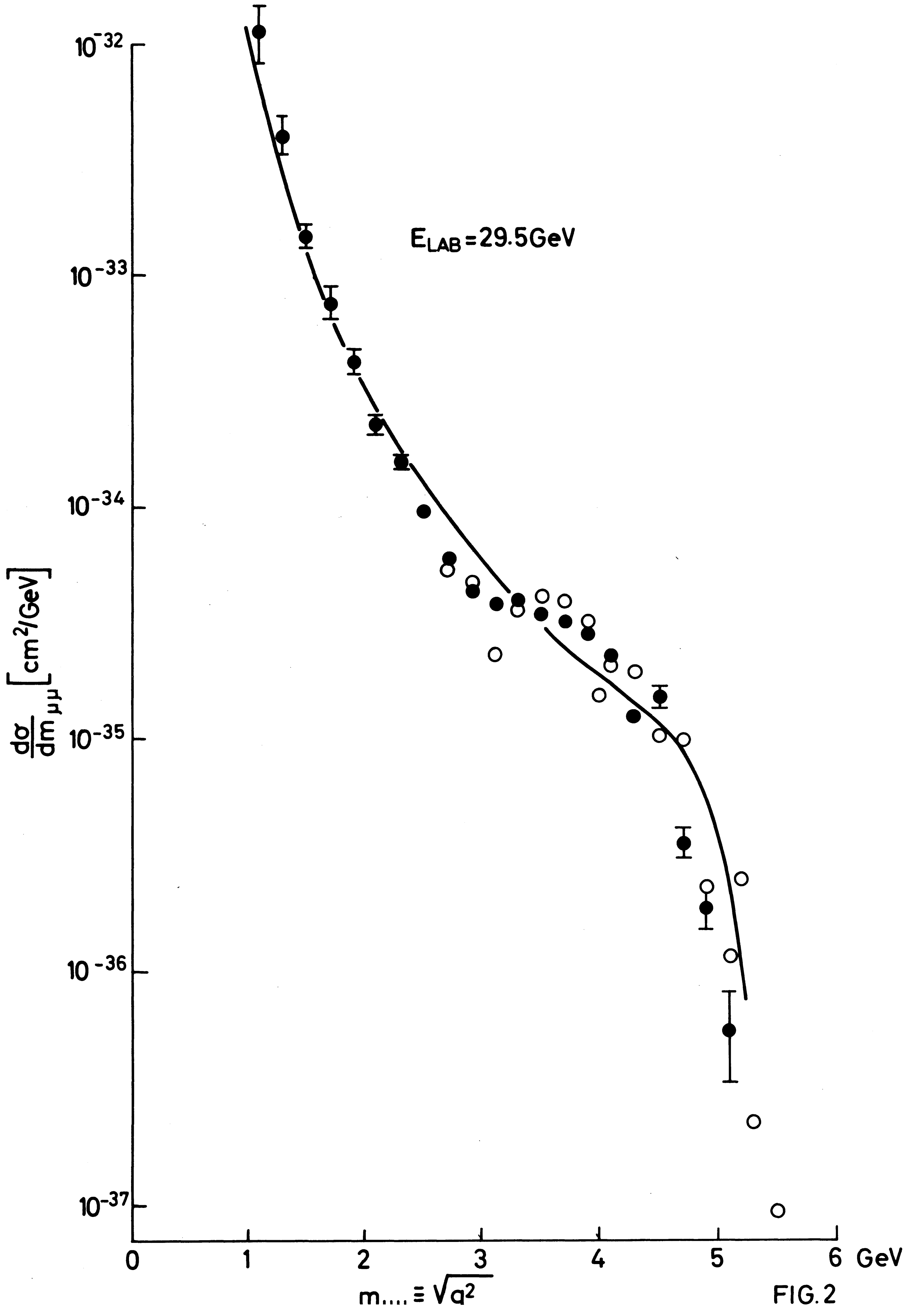


Fig. 1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10^{-32}
10^{-33}
10^{-34}
10^{-35}
10^{-36}
10^{-37}
$\frac{d\sigma}{dm_{\mu\mu}}$ [cm^2/GeV]
0
1
2
3
4
5
6 GeV
$m_{....} \equiv \sqrt{q^2}$

FIG. 2