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Indications of the CMSSM Mass Scale from Precision Electroweak Data[*]

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Abstract

We discuss the sensitivities of present-day electroweak precision data to the possible scale of supersymmetry within the constrained minimal supersymmetric extension of the Standard Model (CMSSM). Our analysis is based on M_W, $\sin^2\theta_{\rm eff}$, $(g-2)_\mu$, ${\rm BR}(b \to s\gamma)$, and the lightest MSSM Higgs boson mass, M_h. We display the impact of the recent reduction in m_t from 178.0 ± 4.3 GeV to 172.7 ± 2.9 GeV on the interpretation of the precision observables. We show the currently preferred values of the CMSSM mass scale $m_{1/2}$ based on a global χ^2 fit, assuming that the lightest supersymmetric particle (LSP) is a neutralino, and fixing m_0 so as to obtain the cold dark matter density allowed by WMAP and other cosmological data for specific values of A_0, $\tan\beta$ and $\mu > 0$. The recent reduction in m_t reinforces previous indications for relatively light soft supersymmetry-breaking masses, offering good prospects for the LHC and the ILC, and in some cases also for the Tevatron. Finally, we discuss the sensitivity of the global χ^2 function to possible future evolution in the experimental central value of m_t and its error.

1. Introduction

We have recently analyzed the indications provided by current experimental data concerning the possible scale of supersymmetry [1–3] within the framework of the minimal supersymmetric extension of the Standard Model (MSSM) [4, 5]. We focus on the constrained MSSM (CMSSM), in which it is assumed that the soft supersymmetry-breaking scalar masses m_0, gaugino masses $m_{1/2}$ and tri-linear parameters A_0 are each constrained to be universal at the input GUT scale, with the gravitino heavy and the lightest supersymmetric particle (LSP) being the lightest neutralino $\tilde{\chi}^0_1$.

It is well known that predicting the masses of supersymmetric particles using precision low-energy data is more difficult than it was for the top quark or even the Higgs boson. This is because the Standard Model (SM) is renormalizable, so decoupling theorems imply that many low-energy observables are insensitive to heavy sparticles [6]. On the other hand, supersymmetry may provide an important contribution to loop-induced processes. In fact, it was found [1,3] that present data on the electroweak precision observables M_W and $\sin^2\theta_{\rm eff}$, as well as the loop-induced quantities $(g-2)_\mu$ and ${\rm BR}(b \to s\gamma)$ (see [7] for a review), may already be providing interesting indirect information on the scale of supersymmetry breaking, at least within the context of the CMSSM with a neutralino LSP. In that framework, the range of m_0 is very restricted by the cold dark matter density $\Omega_\chi h^2$ determined by WMAP and other observations, for any set of assumed values of $\tan\beta, m_{1/2}$ and the trilinear soft supersymmetry-breaking parameter A_0 [8, 9]: in our analysis we have fixed m_0 to satisfy the cold dark matter density constraint, $0.094 < \Omega_{\rm CDM}h^2 < 0.129$ [10] [1].

Within the CMSSM and using the (then) preferred range $m_t = 178.0 \pm 4.3$ GeV [12], we found previously [1, 2] a preference for low values of $m_{1/2}$, particularly for $\tan\beta = 10$, that exhibited only a moderate sensitivity to A_0 [2]. Here we focus on the change induced by the decrease of the experimental value of m_t. The new analysis [3] updates our previous analysis [1], taking into account the experimental result of $m_t = 172.7 \pm 2.9$ GeV [14], and provides a *vade mecum* for understanding the implications of any further evolution in the preferred range and experimental error of m_t [3].

As we show here explicitly, the new experimental value of m_t has a non-trivial effect on the ranges of $m_{1/2}$ preferred by the experimental measurements of M_W and $\sin^2\theta_{\rm eff}$. Moreover, it reduces substantially the mass expected for the lightest MSSM Higgs boson, M_h, for any given values of $m_{1/2}, m_0, \tan\beta$ and A_0, thereby strengthening the constraints on $m_{1/2}$. We therefore improve our analysis by incorporating the full likelihood information provided by the final results of the LEP search for a Standard Model-like Higgs boson [16, 17] (see [18] for other recent analyses in the framework of the CMSSM, which differ from our analysis by the treatment of certain observables such as M_W, $\sin^2\theta_{\rm eff}$ or M_h, or in their treatment of the 95% C.L. exclusion bound for M_h.)

2. Current experimental data

In this Section we review briefly the experimental data set that has been used for the fits. We focus on parameter points that yield the correct value of the cold dark matter density, $0.094 < \Omega_{\rm CDM}h^2 < 0.129$ [10], which is, however, not included in the fit itself. The data set furthermore comprises the following observables: the mass of the W boson, M_W, the effective leptonic weak mixing angle, $\sin^2\theta_{\rm eff}$, the anomalous magnetic moment of the muon, $(g-2)_\mu$, the radiative B-decay branching ratio ${\rm BR}(b \to s\gamma)$, and the lightest MSSM Higgs boson mass, M_h. A detailed description of the first four observables can be found in [1, 7]. We limit ourselves here to recalling the current precision of the experimental results and the theoretical predictions. The experimental values of these observables have not changed significantly compared to [1, 7], and neither have the theoretical calculations. However, the lower experimental value for m_t affects the interpretation of M_W and $\sin^2\theta_{\rm eff}$, in particular, changing the room available for contributions from possible physics beyond the Standard Model, such as supersymmetry. Moreover, as already commented, the new, lower experimental value of m_t necessitates the incorporation of more complete experimental information about M_h into the fit.

The uncertainties in the precision observables are given as follows:

- *The W boson mass*:
  The intrinsic theoretical uncertainty in the prediction for M_W within the MSSM with real

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[1] The central value of $\Omega_{\rm CDM}h^2$ indicated by the recent three-year WMAP data is very similar, whilst the uncertainty is now somewhat reduced [11].

[2] Our notation for the A_0 parameter follows that which is standard in supergravity models (see e.g. [4]), namely the coupling in the scalar potential is given by $A_0\, g^{(3)}$ for the tri-linear superpotential term $g^{(3)}$. This differs from the sign convention used in many publicly available codes, see e.g. [13].

[3] We also briefly comment on the effect of using the most up-to-date value of $m_t = 172.5 \pm 2.3$ GeV [15].

parameters has been estimated as [19]

$$\Delta M_W^{\rm intr,current} \lesssim 9 \text{ MeV} , \tag{1}$$

depending on the mass scale of the supersymmetric particles. A recent reevaluation of M_W [20], taking into account all existing corrections yields results very similar (within ~ 5 MeV) to our calculation. The parametric uncertainties are dominated by the experimental error of the top-quark mass and the hadronic contribution to the shift in the fine structure constant. Their current errors induce the following parametric uncertainties [7,21]

$$\delta m_t^{\rm current} = 2.9 \; (2.3) \text{ GeV} \;\; \Rightarrow \;\; \Delta M_W^{{\rm para},m_t,{\rm current}} \approx 17.5 \; (14) \text{ MeV}, \tag{2}$$

$$\delta(\Delta\alpha_{\rm had}^{\rm current}) = 36 \times 10^{-5} \;\; \Rightarrow \;\; \Delta M_W^{{\rm para},\Delta\alpha_{\rm had},{\rm current}} \approx 6.5 \text{ MeV} . \tag{3}$$

The experimental value of M_W used in this analysis is [22,23] [4]

$$M_W^{\rm exp,current} = 80.410 \pm 0.032 \text{ GeV}. \tag{4}$$

The experimental and theoretical errors for M_W are added in quadrature in our analysis.

- *The effective leptonic weak mixing angle*:
  In the MSSM, the remaining intrinsic theoretical uncertainty in the prediction for $\sin^2\theta_{\rm eff}$ has been estimated as [19]

$$\Delta \sin^2\theta_{\rm eff}^{\rm intr,current} \lesssim 7 \times 10^{-5}, \tag{5}$$

depending on the supersymmetry mass scale. The current experimental errors of m_t and $\Delta\alpha_{\rm had}$ induce the following parametric uncertainties

$$\delta m_t^{\rm current} = 2.9 \; (2.3) \text{ GeV} \;\; \Rightarrow \;\; \Delta \sin^2\theta_{\rm eff}^{{\rm para},m_t,{\rm current}} \approx 10 \; (8) \times 10^{-5}, \tag{6}$$

$$\delta(\Delta\alpha_{\rm had}^{\rm current}) = 36 \times 10^{-5} \;\; \Rightarrow \;\; \Delta \sin^2\theta_{\rm eff}^{{\rm para},\Delta\alpha_{\rm had},{\rm current}} \approx 13 \times 10^{-5}. \tag{7}$$

The experimental value is [22,23]

$$\sin^2\theta_{\rm eff}^{\rm exp,current} = 0.23153 \pm 0.00016 . \tag{8}$$

The experimental and theoretical errors for $\sin^2\theta_{\rm eff}$ are added in quadrature in our analysis.

- *The anomalous magnetic moment of the muon*:
  We use here the latest estimate based on e^+e^- data [24] (see [25,26] for reviews):

$$a_\mu^{\rm theo} = (11\,659\,182.8 \pm 6.3_{\rm had} \pm 3.5_{\rm LBL} \pm 0.3_{\rm QED+EW}) \times 10^{-10}, \tag{9}$$

where the source of each error is labelled.
The result for the SM prediction is to be compared with the final result of the Brookhaven $(g-2)_\mu$ experiment E821 [27,28], namely:

$$a_\mu^{\rm exp} = (11\,659\,208.0 \pm 5.8) \times 10^{-10}, \tag{10}$$

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[4] The newest experimental value of $M_W^{\rm exp} = 80.404 \pm 0.030$ GeV [23] yields practically identical results.

leading to an estimated discrepancy

$$a_\mu^{\rm exp} - a_\mu^{\rm theo} = (25.2 \pm 9.2) \times 10^{-10}, \tag{11}$$

equivalent to a 2.7 σ effect. While it would be premature to regard this deviation as a firm evidence for new physics, it does indicate a preference for a non-zero supersymmetric contribution.

We note that new e^+e^- data sets have recently been published in [29–32], but not yet used in an updated estimate of $(g-2)_\mu$. Their inclusion is not expected to alter substantially the estimate given in (9). In particular, we note that the SND data [31] have recently been revised significantly [32], following a re-evaluation of the background processes $e^+e^- \to \pi^+\pi^-\gamma$ and $\mu^+\mu^-\gamma$. They are now in much better agreement with the CMD2 data [30], and show an increased disagreement with the τ decay data [5].

- *The decay $b \to s\gamma$*:

  Since this decay occurs at the loop level in the SM, the MSSM contribution might *a priori* be of similar magnitude. A recent theoretical estimate of the SM contribution to the branching ratio is [33]

  $$\mathrm{BR}(b \to s\gamma) = (3.70 \pm 0.46) \times 10^{-4}, \tag{12}$$

  where the calculations have been carried out completely to NLO in the $\overline{\rm MS}$ renormalization scheme [34–36], and the error is dominated by higher-order QCD uncertainties. We record, however, that the error estimate for $\mathrm{BR}(b \to s\gamma)$ is still under theoretical debate, see also [37, 38].

  For the experimental value, we assume [3] the estimate [39]

  $$\mathrm{BR}(b \to s\gamma) = (3.39^{+0.30}_{-0.027}) \times 10^{-4}, \tag{13}$$

  whereas the present experimental value estimated just recently by the Heavy Flavour Averaging Group (HFAG) is $\mathrm{BR}(b \to s\gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4}$ [40]. The uncertainties are combined statistical and systematic errors, the systematic error due to the spectral shape function, and the uncertainty due to the $d\gamma$ fraction, respectively. The new central value is somewhat closer to that in the SM (12), imposing a somewhat stronger constraint on the supersymmetric mass scale, but we do not expect the conclusion to differ greatly from this analysis.

- *The lightest MSSM Higgs boson mass*:

  The mass of the lightest $\mathcal{CP}$-even MSSM Higgs boson can be predicted in terms of the other CMSSM parameters. At the tree level, the two $\mathcal{CP}$-even Higgs boson masses are obtained as functions of M_Z, the $\mathcal{CP}$-odd Higgs boson mass M_A, and $\tan\beta$. For the theoretical prediction of M_h we employ the Feynman-diagrammatic method using the code `FeynHiggs` [41, 42], which includes all numerically relevant known higher-order corrections. The current intrinsic error of M_h due to unknown higher-order corrections has been estimated to be [7, 43, 44]

  $$\Delta M_h^{\rm intr,current} = 3~\mathrm{GeV}~. \tag{14}$$

  Details about the inclusion of M_h and the evaluation of the corresponding χ^2 values obtained from the direct searches for a Standard Model (SM) Higgs boson at LEP [16] can be found in [3].

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[5] We thank Lee Roberts for information on this point.

Assuming that the five observables listed above are uncorrelated, a χ^2 fit has been performed with

$$\chi^2 \equiv \sum_{n=1}^{4} \left(\frac{R_n^{\rm exp} - R_n^{\rm theo}}{\sigma_n} \right)^2 + \chi^2_{M_h}. \tag{15}$$

Here $R_n^{\rm exp}$ denotes the experimental central value of the nth observable (M_W, $\sin^2\theta_{\rm eff}$, $(g-2)_\mu$ and ${\rm BR}(b \to s\gamma)$), $R_n^{\rm theo}$ is the corresponding CMSSM prediction and σ_n denotes the combined error, and $\chi^2_{M_h}$ denotes the χ^2 contribution coming from the lightest MSSM Higgs boson mass [3].

3. CMSSM analysis for $m_t = 172.7$ GeV

As already mentioned, in our old analysis of the CMSSM [1] we used the range $m_t = 178.0 \pm 4.3$ GeV that was then preferred by direct measurements [12]. The preferred range has subsequently evolved to 172.7 ± 2.9 GeV [14] (and very recently to 172.5 ± 2.3 GeV [15]). The effect of this lower m_t value is twofold.

First, it drives the SM prediction of M_W and $\sin^2\theta_{\rm eff}$ further away from the current experimental value [6]. This effect is shown in Figs. 1 – 4 for $\tan\beta = 10, 50$. In the right plots of Figs. 1 and 2 we have also updated the experimental value of M_W. The change in the SM prediction elevates the experimental discrepancy to about 1.5 σ, despite the change in the preferred experimental range of M_W, which does not compensate completely for the change in m_t. The net effect is therefore to increase the favoured magnitude of the supersymmetric contribution, i.e., to lower the preferred supersymmetric mass scale. In the case of $\sin^2\theta_{\rm eff}$, the reduction in m_t has increased the SM prediction whereas the experimental value has not changed significantly. Once again, the discrepancy with the SM has increased to about 1.5 σ, and the preference for a small value of $m_{1/2}$ has therefore also increased.

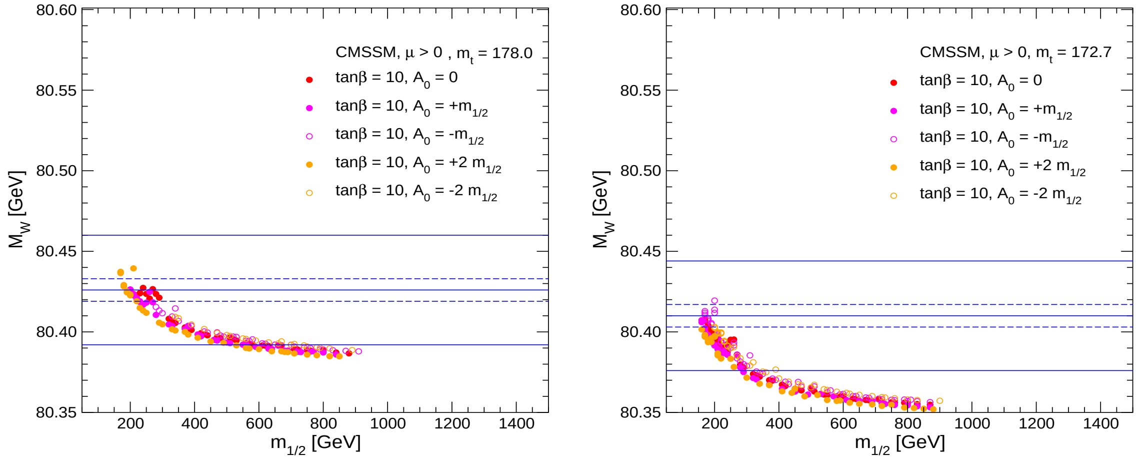

Fig. 1: *The CMSSM predictions for M_W as functions of $m_{1/2}$ for $\tan\beta = 10$ for various A_0. The top quark mass has been set to $m_t = 178.0$ GeV (left) and $m_t = 172.7$ GeV (right). The experimental measurements indicated in the plots are the previous one, $M_W = 80.426 \pm 0.034$ GeV (left) and the newer one, $M_W = 80.410 \pm 0.032$ GeV (right).*

Secondly, the predicted value of the lightest Higgs boson mass in the MSSM is lowered by the new m_t value, see, e.g., [45]. The effects on the electroweak precision observables of

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[6] Whereas $(g-2)_\mu$ and ${\rm BR}(b \to s\gamma)$ are little affected.

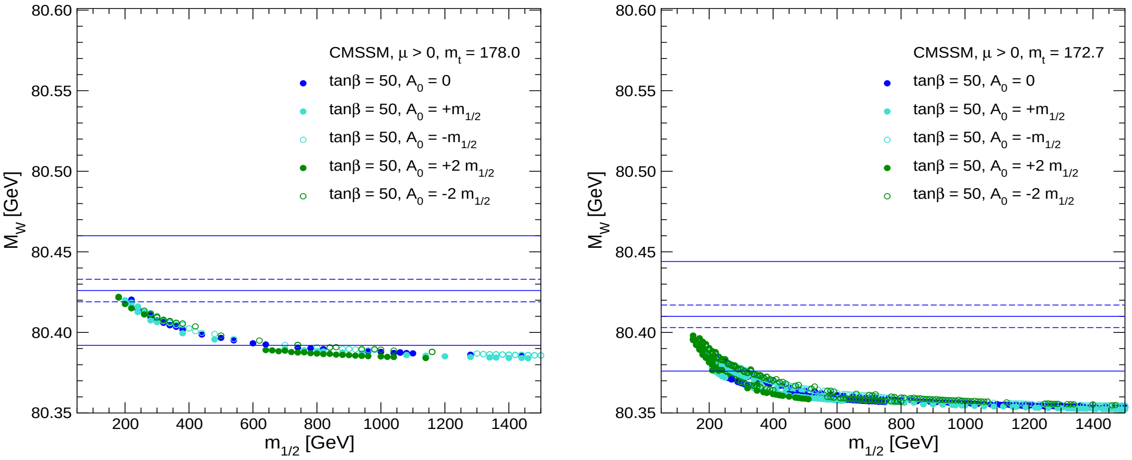

Fig. 2: *Same as in Fig. 1, but for* $\tan\beta = 50$.

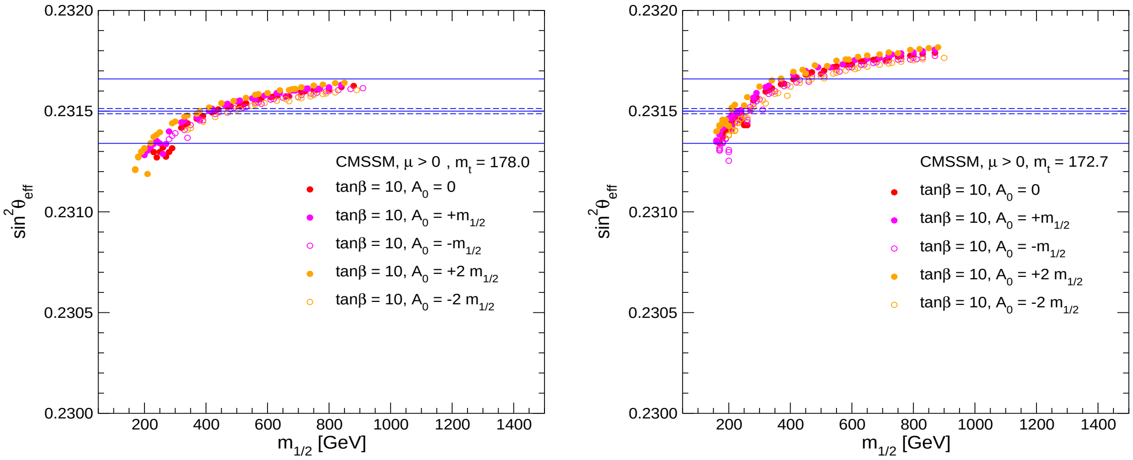

Fig. 3: *The CMSSM predictions for* $\sin^2\theta_{\rm eff}$ *as functions of* $m_{1/2}$ *for* $\tan\beta = 10$ *for various* A_0. *The top quark mass has been set to* $m_t = 178.0$ GeV *(left) and* $m_t = 172.7$ GeV *(right).*

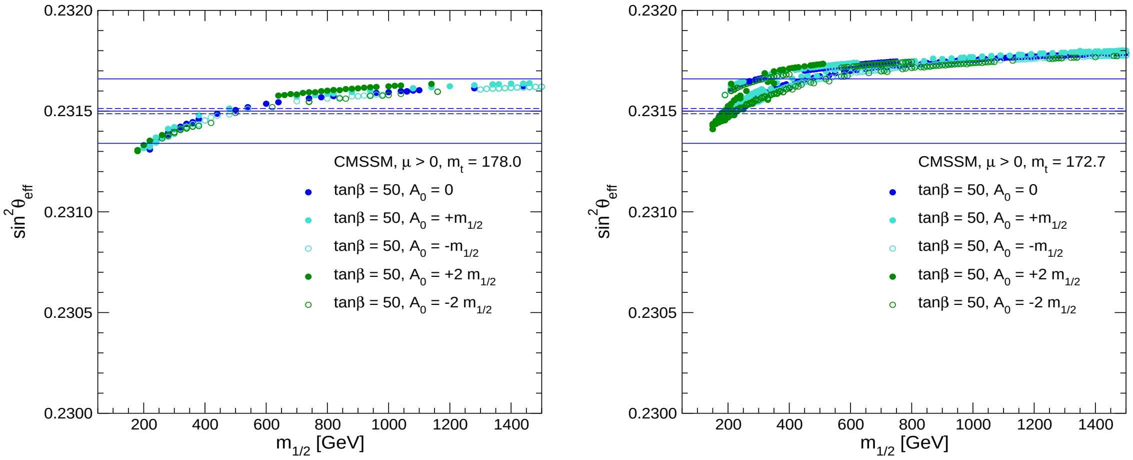

Fig. 4: *Same as in Fig. 3, but for* $\tan\beta = 50$.

the downward shift in M_h are minimal, but the LEP Higgs bounds [16, 17] now impose a more important constraint on the MSSM parameter space, notably on $m_{1/2}$. This is visualized in Figs. 5 and Fig. 6, where we show the results for $\tan\beta = 10, 50$ for $m_t = 178.0$ GeV (left plots) and $m_t = 172.7$ GeV (right plots). A hypothetical LHC measurement is also shown, namely $M_h = 116.4 \pm 0.2$ GeV, as well as the present 95% C.L. exclusion limit of 114.4 GeV. For $\tan\beta = 10$ as shown in Fig. 5, with the lower m_t value for small $m_{1/2}$, a positive value

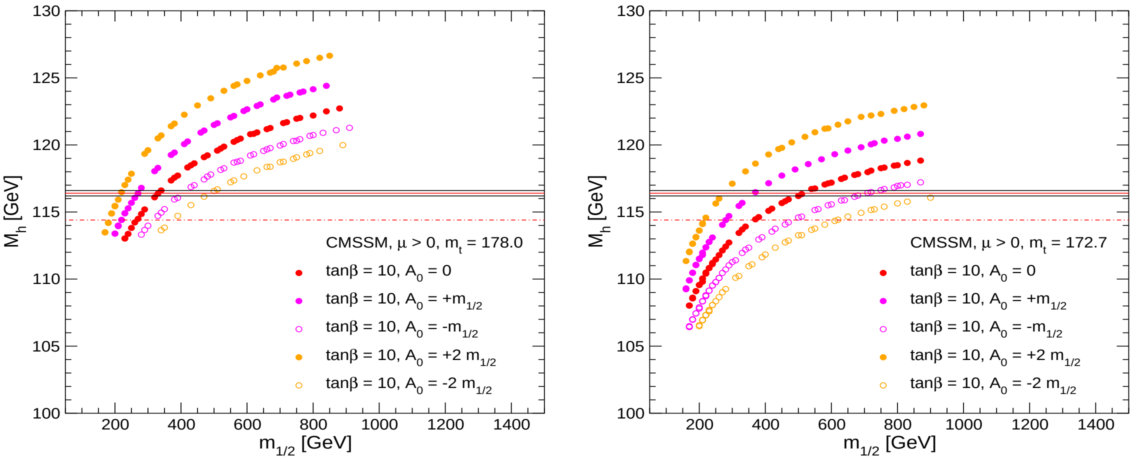

Fig. 5: *The CMSSM predictions for* M_h *as functions of* $m_{1/2}$ *for* $\tan\beta = 10$ *for various* A_0*. The top quark mass has been set to* $m_t = 178.0$ GeV *(left) and* $m_t = 172.7$ GeV *(right). A hypothetical LHC measurement is also shown, namely* $M_h = 116.4 \pm 0.2$ GeV*, as well as the present 95% C.L. exclusion limit of* 114.4 GeV.

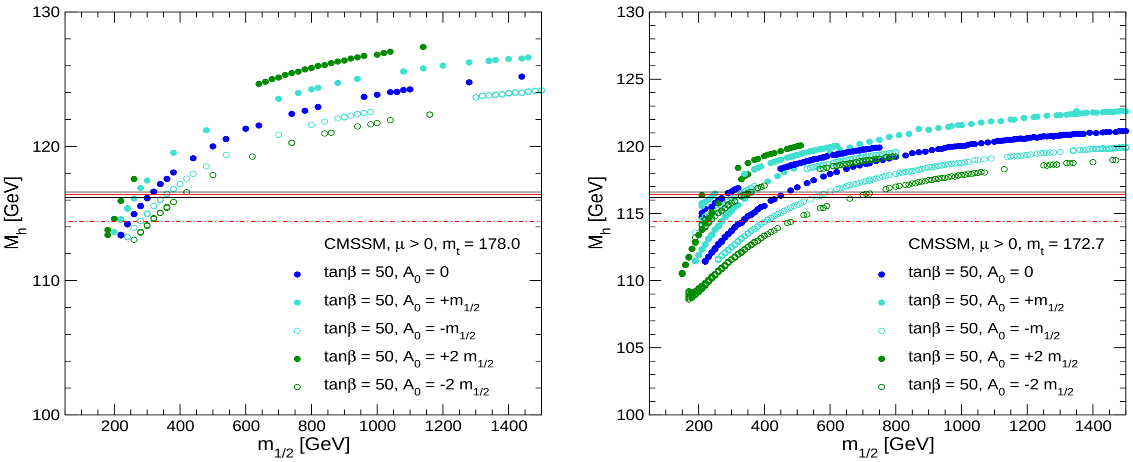

Fig. 6: *Same as in Fig. 5, but for* $\tan\beta = 50$.

of A_0 is needed in order to satisfy the LEP Higgs exclusion bounds. For $\tan\beta = 50$, see Fig. 6, on the other hand, this effect is much less severe. In our previous analysis, we rejected all parameter points for which `FeynHiggs` yielded $M_h < 113$ GeV. The best fit values in [1] corresponded to relatively small values of M_h, a feature that is even more pronounced for the new m_t value. In view of all these effects, we have updated [3] our old analysis of the phenomenological constraints on the supersymmetric mass scale $m_{1/2}$ in the CMSSM using the new, lower value [7] of m_t and including a χ^2 contribution from M_h.

We now present the updated results [3] for the χ^2 fit, which includes the χ^2 contribution for M_h for $m_t = 172.7 \pm 2.9$ GeV.[8] As seen in the first panel of Fig. 7, the qualitative feature observed in [1] of a pronounced minimum in χ^2 at $m_{1/2}$ for $\tan\beta = 10$ is also present for the new value of m_t. However, the χ^2 curve now depends more strongly on the value of A_0,

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[7] See also [2], where a lower bound of $M_h > 111.4$ GeV has been used.

[8] The results for $m_t = 172.5 \pm 2.3$ GeV are very similar, see also Sect. 4.

corresponding to its strong impact on M_h. Values of $A_0/m_{1/2} < -1$ are disfavoured at the $\sim 90\%$ C.L., essentially because of their lower M_h values (see Fig. 5), but $A_0/m_{1/2} = 2$ and 1 give equally good fits and descriptions of the data. The old best fit point in [1] had $A_0/m_{1/2} = -1$, but there all $A_0/m_{1/2}$ gave a similarly good description of the experimental data. The minimum χ^2 value is about 2.5. This is somewhat higher than the result in [1], but still represents a good overall fit to the experimental data. The rise in the minimum value of χ^2, compared to [1], is essentially a consequence of the lower experimental central value of m_t, and the consequent greater impact of the LEP constraint on M_h [16, 17]. In the cases of the observables M_W and $\sin^2\theta_{\rm eff}$, a smaller value of m_t induces a preference for a smaller value of $m_{1/2}$, but the opposite is true for the Higgs mass bound. The rise in the minimum value of χ^2 reflects the correspondingly increased tension between the electroweak precision observables and the M_h constraint.

A breakdown of the contributions to χ^2 from the different observables can be found for some example points in Table 1. We concentrate here on parameter sets with relatively bad fit qualities that either have large $m_{1/2}$ values or lie in the focus-point region (see below). One can see that, for large $m_{1/2}$ values, $(g-2)_\mu$ always gives the dominant contribution. However, with the new lower experimental value of m_t also M_W and $\sin^2\theta_{\rm eff}$ give a substantial contribution, adding up to more than 50% of the $(g-2)_\mu$ contribution. On the other hand, M_h and $\mathrm{BR}(b \to s\gamma)$ make negligible contributions to χ^2 at these points. As seen from the example shown in the last line of the Table, focus points may yield similar results for the electroweak precision observables as in the SM, resulting in a relatively high χ^2 value. This region is mostly disfavoured at the $\sim 90\%$ C.L. level, as also seen in Fig. 8.

$\tan\beta$	$m_{1/2}$	m_0	A_0	$\chi^2_{\rm tot}$	M_W	$\sin^2\theta_{\rm eff}$	$(g-2)_\mu$	$\mathrm{BR}(b \to s\gamma)$	M_h
10	880	270	1760	9.71	2.29	1.28	6.14	0.01	0
50	1910	1500	-1910	9.61	2.21	1.11	6.29	0.01	0
50	800	2970	-800	8.73	1.92	0.72	6.05	0.04	0

Table 1: *Breakdown of χ^2 contributions from the different precision observables to $\chi^2_{\rm tot}$ for some example points. All masses are in GeV. The last row is representative of the focus-point region.*

The remaining panels of Fig. 7 update our old analyses [1] of the χ^2 functions for various sparticle masses within the CMSSM, namely the lightest neutralino $\tilde\chi^0_1$, the second-lightest neutralino $\tilde\chi^0_2$ and the (almost degenerate) lighter chargino $\tilde\chi^\pm_1$, the lightest slepton which is the lighter stau $\tilde\tau_1$, the lighter stop squark $\tilde t_1$, and the gluino $\tilde g$. Reflecting the behaviour of the global χ^2 function in the first panel of Fig. 7, the changes in the optimal values of the sparticle masses are not large. The 90% C.L. upper bounds on the particle masses are nearly unchanged compared to the results for $m_t = 178.0 \pm 4.3$ GeV given in [1].

The corresponding results for the case $\tan\beta = 50$ are shown in Fig. 8. We see in panel (a) that the minimum value of χ^2 for the fit with $m_t = 172.7 \pm 2.9$ GeV is larger by about a unit than in our previous analysis with $m_t = 178.0 \pm 4.3$ GeV. Because of the rise in χ^2 for the $\tan\beta = 10$ case, however, the minimum values of χ^2 are now very similar for the two values of $\tan\beta$ shown here. The dip in the χ^2 function for $\tan\beta = 50$ is somewhat steeper than in the previous analysis, since the high values of $m_{1/2}$ are slightly more disfavoured due to their M_W

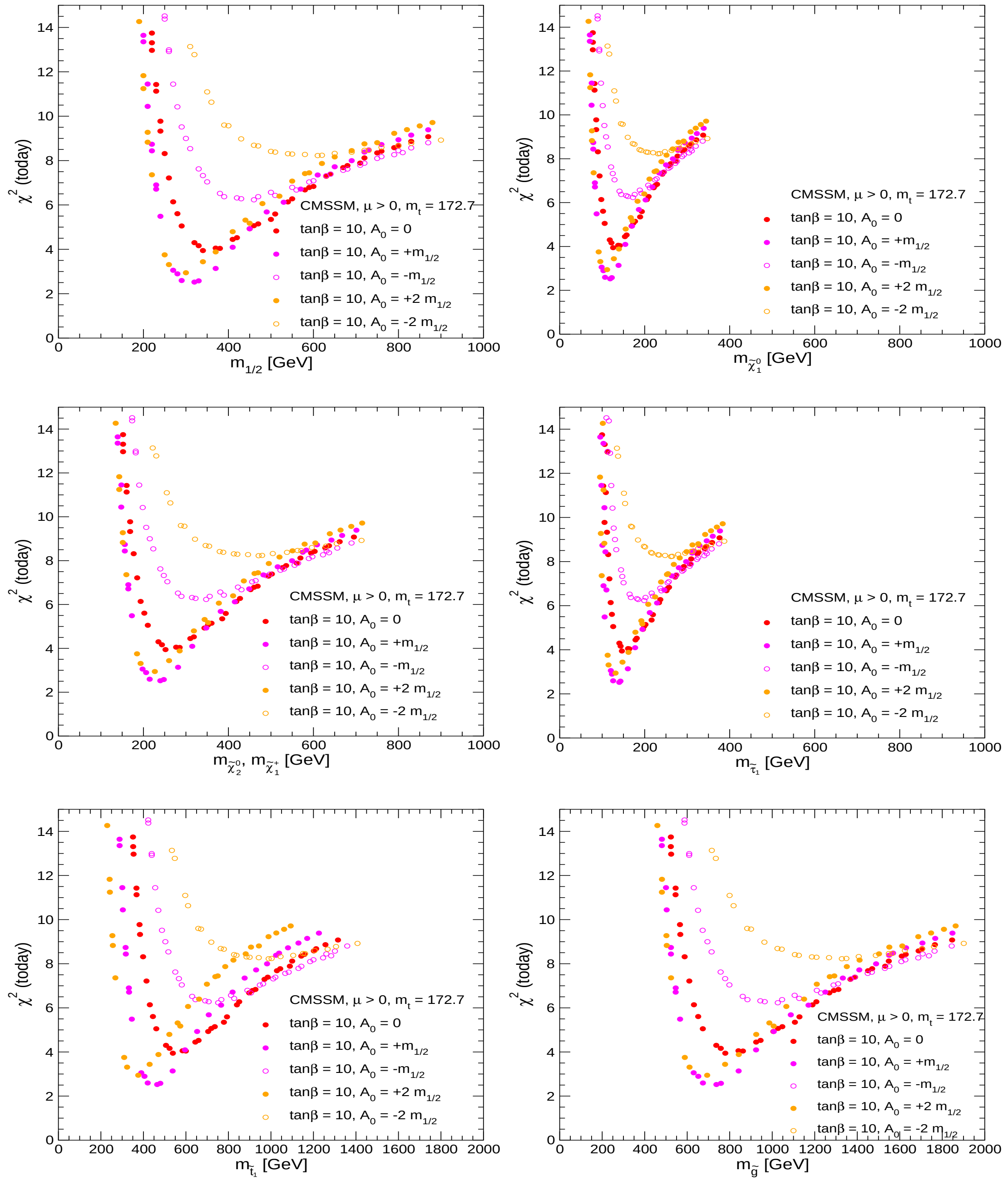

Fig. 7: *The combined likelihood function* χ^2 *for the electroweak observables* M_W, $\sin^2\theta_{\rm eff}$, $(g-2)_\mu$, $\mathrm{BR}(b \to s\gamma)$, *and* M_h *evaluated in the CMSSM for* $\tan\beta = 10$, $m_t = 172.7 \pm 2.9$ GeV *and various discrete values of* A_0*, with* m_0 *then chosen to yield the central value of the relic neutralino density indicated by WMAP and other observations. We display the* χ^2 *function for (a)* $m_{1/2}$*, (b)* $m_{\tilde\chi^0_1}$*, (c)* $m_{\tilde\chi^0_2}, m_{\tilde\chi^\pm_1}$*, (d)* $m_{\tilde\tau_1}$*, (e)* $m_{\tilde t_1}$ *and (f)* $m_{\tilde g}$ *[3].*

and $\sin^2\theta_{\rm eff}$ values. The best fit values of $m_{1/2}$ are very similar to their previous values. The preferred values of the sparticle masses are shown in the remaining panels of Fig. 8.

We note one novel feature, namely the appearance of a group of points with moderately high χ^2 that have relatively small $m_{1/2} \sim 200 - 800$ GeV. These points have relatively large values of m_0, as reflected in the relatively large values of $m_{\tilde\tau_1}$ and $m_{\tilde t_1}$ seen in panels (d) and (e) of Fig. 8. These points are located in the focus-point region of the $(m_{1/2}, m_0)$ plane [46], where the LSP has a larger Higgsino content, whose enhanced annihilation rate brings the relic

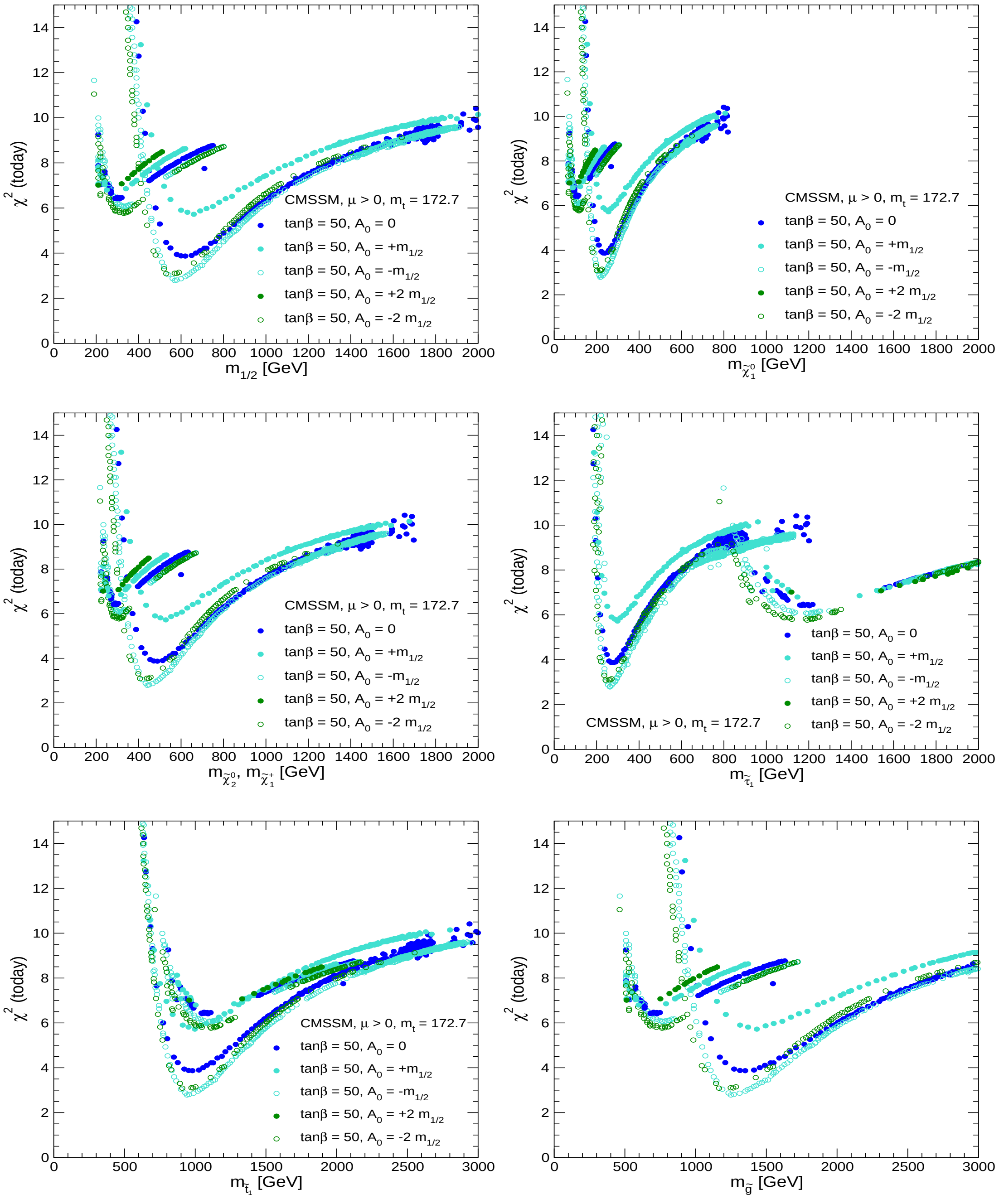

Fig. 8: *As in Fig. 7, but now for* $\tan\beta = 50$.

density down into the range allowed by WMAP. These points have a $\Delta\chi^2$ of at least 3.5, so most of them are excluded at the 90% C.L.

Taken at face value, the preferred ranges for the sparticle masses shown in Figs. 7 and 8 are quite encouraging for both the LHC and the ILC. The gluino and squarks lie comfortably within the early LHC discovery range, and several electroweakly-interacting sparticles would be accessible to ILC(500) (the ILC running at $\sqrt{s} = 500$ GeV). This is the case, in particular, for the $\tilde{\chi}^0_1$, the $\tilde{\tau}_1$, and possibly the $\tilde{\chi}^0_2$ and the $\tilde{\chi}^\pm_1$. The best-fit CMSSM point is quite similar to the benchmark point SPS 1a [47], which is close to point B of [48] and has been shown to offer good experimental prospects for both the LHC and ILC [49].

The minimum values of χ^2 are 2.5 for $\tan\beta = 10$ and 2.8 for $\tan\beta = 50$, found for $m_{1/2} \sim 320, 570$ GeV and $A_0 = +m_{1/2}, -m_{1/2}$, respectively, revealing no preference for either large or small $\tan\beta$ [9]. This also holds for intermediate $\tan\beta$ values, see [3] for details.

4. Future evolution

In view of the possible future evolution of both the central value of m_t and its experimental uncertainty δm_t, we have analyzed the behaviour of the global χ^2 function for $166 \text{ GeV} < m_t < 179 \text{ GeV}$ and $1.5 \text{ GeV} < \delta m_t < 3.0 \text{ GeV}$ for the case of $\tan\beta = 10$ (assuming that the experimental results and theoretical predictions for the precision observables are otherwise unchanged), as seen in the left panel of Fig. 9. We see that the minimum value of χ^2 is almost independent of the uncertainty δm_t, but increases noticeably as the assumed central value of m_t decreases. This effect is not strong when m_t decreases from 178.0 GeV to 172.7 GeV, but does become significant for $m_t < 170 \text{ GeV}$. This effect is not independent of the known preference of the ensemble of precision electroweak data for $m_t \sim 175 \text{ GeV}$ within the SM [22, 23], to which the observables M_W and $\sin^2\theta_{\text{eff}}$ used here make important contributions. On the other hand, as already commented, within the CMSSM there is the additional effect that the best fit values of $m_{1/2}$ for very low m_t result in M_h values that are excluded by the LEP Higgs searches [16, 17] and have a very large $\chi^2_{M_h}$, resulting in an increase of the lowest possible χ^2 value for a given top-quark mass value. This effect also increases the value of $m_{1/2}$ where the χ^2 function is minimized. On the other hand, the right panel in Fig. 9 demonstrates that the 90% C.L. upper limit on $m_{1/2}$ shows only a small variation, less than $\sim 10\%$ for m_t in the preferred range above 170 GeV [10]. Finally, we note that the upper limit on $m_{1/2}$ is essentially independent of δm_t for the preferred range $m_t \gtrsim 170 \text{ GeV}$. Thus for the latest experimental value, $m_t = 172.5 \pm 2.3 \text{ GeV}$ the results for the preferred $m_{1/2}$ range remain essentially unchanged as compared to our analysis here.

It is striking that the preference noted earlier for relatively low values of $m_{1/2}$ remains almost unaltered after the change in m_t and the change in the treatment of the LEP lower limit on M_h. There seems to be little chance at present of evading the preference for small $m_{1/2}$ hinted by the present measurements of M_W, $\sin^2\theta_{\text{eff}}$ and $(g-2)_\mu$, at least within the CMSSM framework. It should be noted that the preference for a relatively low SUSY scale is correlated with the top mass value lying in the interval $170 \text{ GeV} \lesssim m_t \lesssim 180 \text{ GeV}$.

5. Conclusions

Precision electroweak data and rare processes have some sensitivity to the loop corrections that might be induced by supersymmetric particles. Present data exhibit some preference for a relatively low scale of soft supersymmetry breaking: $m_{1/2} \sim 300 \ldots 600 \text{ GeV}$. This preference is largely driven by $(g-2)_\mu$, with some support from measurements of M_W and $\sin^2\theta_{\text{eff}}$. Here we have presented a re-evaluation in the light of new measurements of m_t and M_W, and a more complete treatment of the information provided by the bound from the LEP direct searches for the Higgs boson. The preference for $m_{1/2} \sim 300 \ldots 600 \text{ GeV}$ is maintained in the CMSSM [11].

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[9] In our previous analysis, we found a slight preference for $\tan\beta = 10$ over $\tan\beta = 50$. This preference has now been counterbalanced by the increased pressure exerted by the Higgs mass constraint.

[10] The plot has been obtained by putting a smooth polynomial through the otherwise slightly irregular points.

[11] A more complete discussion, also including models with non-universal Higgs masses or gravitino dark matter, is given in [3].

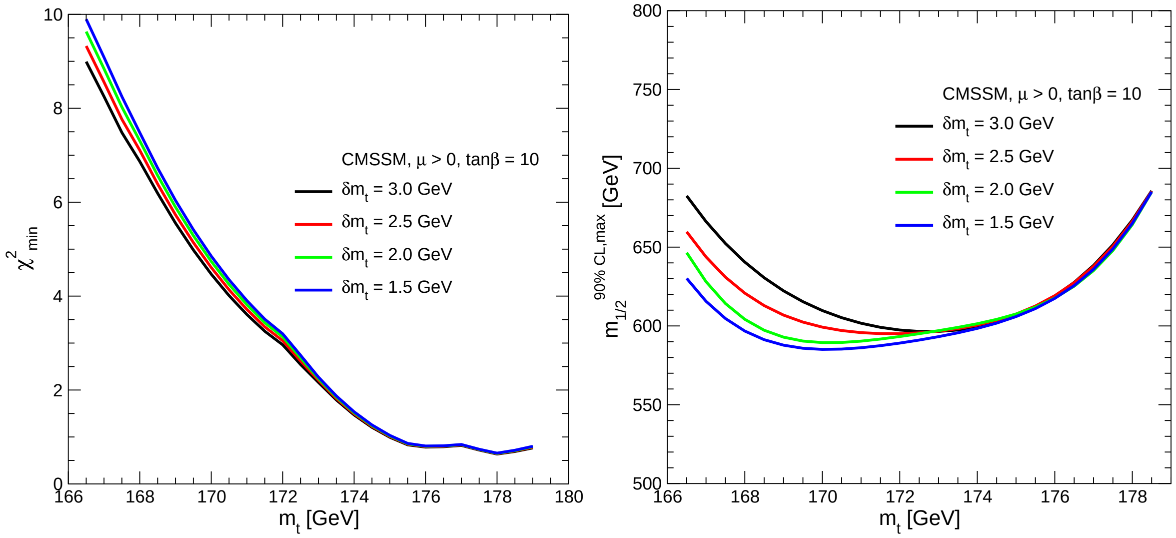

Fig. 9: *The dependence of (a) the minimum value of the* χ^2 *distribution,* $\chi^2_{\rm min}$, *and (b) the 90% C.L. upper limit for* $m_{1/2}$ *on* m_t *and its experimental error* δm_t, *keeping the experimental values and theoretical predictions for the other precision observables unchanged.*

The ranges of $m_{1/2}$ that are preferred would correspond to gluinos and other sparticles being light enough to be produced readily at the LHC. Many sparticles would also be observable at the ILC in the preferred CMSSM parameter space. In this respect the measurement of M_W is increasing in importance, particularly in the light of the recent evolution of the preferred value of m_t. Future measurements of M_W and m_t at the Tevatron will be particularly important in this regard.

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References

[1] J. Ellis, S. Heinemeyer, K. Olive and G. Weiglein, *JHEP* **0502** 013, hep-ph/0411216.

[2] J. Ellis, S. Heinemeyer, K. Olive and G. Weiglein, hep-ph/0508169.

[3] J. Ellis, S. Heinemeyer, K. Olive and G. Weiglein, to appear in *JHEP*, hep-ph/0602220.

[4] H. Nilles, *Phys. Rep.* **110** (1984) 1.

[5] H. Haber and G. Kane, *Phys. Rep.* **117**, (1985) 75;
R. Barbieri, *Riv. Nuovo Cim.* **11**, (1988) 1.

[6] T. Appelquist and J. Carazzone, *Phys. Rev.* **D 11** (1975) 2856;
A. Dobado, M. Herrero and S. Peñaranda, *Eur. Phys. J.* **C 7** (1999) 313, hep-ph/9710313,

Eur. Phys. J. **C 12** (2000) 673, hep-ph/9903211; *Eur. Phys. J.* **C 17** (2000) 487, hep-ph/0002134.

[7] S. Heinemeyer, W. Hollik and G. Weiglein, *Phys. Rept.* **425** (2006) 265, hep-ph/0412214.

[8] J. Ellis, K. Olive, Y. Santoso and V. Spanos, *Phys. Lett.* **B 565** (2003) 176, hep-ph/0303043.

[9] H. Baer and C. Balazs, *JCAP* **0305** (2003) 006, hep-ph/0303114;
A. Lahanas and D. Nanopoulos, *Phys. Lett.* **B 568** (2003) 55, hep-ph/0303130;
U. Chattopadhyay, A. Corsetti and P. Nath, *Phys. Rev.* **D 68** (2003) 035005, hep-ph/0303201;
C. Munoz, hep-ph/0309346.

[10] C. Bennett *et al.*, *Astrophys. J. Suppl.* **148** (2003) 1, astro-ph/0302207;
D. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **148** (2003) 175, astro-ph/0302209.

[11] D. Spergel *et al.*, astro-ph/0603449;
U. Seljak, A. Slosar and P. McDonald, astro-ph/0604335.

[12] V. Abazov *et al.* [D0 Collaboration], *Nature* **429** (2004) 638, hep-ex/0406031;
P. Azzi *et al.* [CDF Collaboration, D0 Collaboration], hep-ex/0404010.

[13] J. Aguilar-Saavedra *et al.*, hep-ph/0511344.

[14] CDF Collaboration, D0 Collaboration and Tevatron Electroweak Working Group, hep-ex/0507091.

[15] Tevatron Electroweak Working Group, hep-ex/0603039.

[16] LEP Higgs working group, *Phys. Lett.* **B 565** (2003) 61, hep-ex/0306033.

[17] ALEPH, DELPHI, L3, OPAL Collaborations and LEP Working Group for Higgs boson searches, hep-ex/0602042.

[18] J. Ellis, K. Olive, Y. Santoso and V. Spanos, *Phys. Rev.* **D 69** (2004) 095004, hep-ph/0310356;
B. Allanach and C. Lester, *Phys. Rev.* **D 73** (2006) 015013, hep-ph/0507283;
B. Allanach, *Phys. Lett.* **B 635** (2006) 123, hep-ph/0601089;
R. de Austri, R. Trotta and L. Roszkowski, hep-ph/0602028.

[19] J. Haestier, S. Heinemeyer, D. Stöckinger and G. Weiglein, *JHEP* **0512** (2005) 027, hep-ph/0508139; hep-ph/0506259.

[20] S. Heinemeyer, W. Hollik, D. Stöckinger, A.M. Weber and G. Weiglein, hep-ph/0604147.

[21] S. Heinemeyer, S. Kraml, W. Porod and G. Weiglein, *JHEP* **0309** (2003) 075, hep-ph/0306181;
S. Heinemeyer and G. Weiglein, hep-ph/0508168.

[22] The ALEPH, DELPHI, L3, OPAL, SLD Collaborations, the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, hep-ex/0509008;
The ALEPH, DELPHI, L3 and OPAL Collaborations, the LEP Electroweak Working Group, hep-ex/0511027.

[23] F. Spano, talk given at *Rencontres de Moriond, Electroweak Interactions and Unified Theories*, La Thuile, Italy, March 2006, see also:
`lepewwg.web.cern.ch/LEPEWWG/Welcome.html`.

[24] A. Höcker, hep-ph/0410081.

[25] A. Czarnecki and W. Marciano, *Phys. Rev.* **D 64** (2001) 013014, hep-ph/0102122.

[26] M. Knecht, hep-ph/0307239;
M. Passera, hep-ph/0509372.

[27] G. Bennett *et al.*, [The Muon g-2 Collaboration], *Phys. Rev. Lett.* **92** (2004) 161802, hep-ex/0401008.

[28] G. Bennett *et al.*, [The Muon g-2 Collaboration], hep-ex/0602035.

[29] A. Aloisio *et al.* [KLOE Collaboration], hep-ex/0407048.

[30] R. Akhmetshin *et al.* [CMD-2 Collaboration], *Phys. Lett.* **B 578** (2004) 285, hep-ex/0308008.

[31] M. Achasov *et al.* [SND Collaboration], *J.E.T.P.* **101** (2005) 1053, hep-ex/0506076.

[32] M. Achasov, on behalf of the SND2 Collaboration, talk at the *International Workshop on e^+e^- Collisions from ϕ to ψ*, BINP, Novosibirsk, February 27 to March 2, 2006.

[33] H. Asatrian, A. Hovhannisyan, C. Greub, T. Hurth and V. Poghosyan, hep-ph/0512097.

[34] K. Adel and Y. Yao, *Phys. Rev.* **D 49** (1994) 4945, hep-ph/9308349;
C. Greub, T. Hurth and D. Wyler, *Phys. Lett.* **B 380** (1996) 385, hep-ph/9602281; *Phys. Rev.* **D 54** (1996) 3350, hep-ph/9603404;
A. Ali, talk given at ICHEP04, Beijing, August 2004, to appear in the proceedings, see: `ichep04.ihep.ac.cn/db/paper.php`.

[35] P. Gambino and M. Misiak, *Nucl. Phys.* **B 611** (2001) 338, hep-ph/0104034.

[36] K. Chetyrkin, M. Misiak and M. Münz, *Phys. Lett.* **B 400**, (1997) 206, [Erratum-ibid. **B 425** (1998) 414] hep-ph/9612313.

[37] T. Hurth, E. Lunghi and W. Porod, *Nucl. Phys.* **B 704** (2005) 56, hep-ph/0312260.

[38] M. Neubert, *Eur. Phys. J.* **C 40** (2005) 165, hep-ph/0408179.

[39] R. Barate *et al.* [ALEPH Collaboration], *Phys. Lett.* **B 429** (1998) 169;
S. Chen *et al.* [CLEO Collaboration], *Phys. Rev. Lett.* **87** (2001) 251807, hep-ex/0108032;
P. Koppenburg *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **93** (2004) 061803, hep-ex/0403004;

K. Abe *et al.* [Belle Collaboration], *Phys. Lett.* **B 511** (2001) 151, hep-ex/0103042;
B. Aubert *et al.* [BABAR Collaboration], hep-ex/0207074; hep-ex/0207076;
see also `www.slac.stanford.edu/xorg/hfag/` .

[40] E. Barberio *et al.* [The Heavy Flavor Averaging Group], hep-ex/0603003.

[41] S. Heinemeyer, W. Hollik and G. Weiglein, *Comp. Phys. Comm.* **124** 2000 76, hep-ph/9812320; *Eur. Phys. J.* **C 9** (1999) 343, hep-ph/9812472. The codes are accessible via `www.feynhiggs.de` .

[42] S. Heinemeyer, to appear in *Int. J. Mod. Phys.* **A**, hep-ph/0407244;
T. Hahn, S. Heinemeyer, W. Hollik and G. Weiglein, hep-ph/0507009;
M. Frank, T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *in preparation*.

[43] G. Degrassi, S. Heinemeyer, W. Hollik, P. Slavich, G. Weiglein, *Eur. Phys. J.* **C 28** (2003) 133, hep-ph/0212020.

[44] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, *Eur. Phys. J.* **C 39** (2005) 465, hep-ph/0411114; hep-ph/0506254.

[45] S. Heinemeyer, W. Hollik and G. Weiglein, *JHEP* **0006** (2000) 009, hep-ph/9909540.

[46] J. Feng, K. Matchev and T. Moroi, *Phys. Rev. Lett.* **84** (2000) 2322, hep-ph/9908309; *Phys. Rev.* **D 61** (2000) 075005, hep-ph/9909334;
J. Feng, K. Matchev and F. Wilczek, *Phys. Lett.* **B 482** (2000) 388, hep-ph/0004043;
J. Feng and K. Matchev, *Phys. Rev.* **D 63** (2001) 095003, hep-ph/0011356.

[47] B. Allanach *et al.*, *Eur. Phys. J.* **C 25** (2002) 113, hep-ph/0202233.

[48] M. Battaglia *et al.*, *Eur. Phys. J.* **C 22**, 535 (2001), hep-ph/0106204;
M. Battaglia, A. De Roeck, J. Ellis, F. Gianotti, K. Olive and L. Pape, *Eur. Phys. J.* **C 33** (2004) 273, hep-ph/0306219.

[49] G. Weiglein *et al.* [LHC / ILC Study Group], *Phys. Rept.* **426** (2006) 47, hep-ph/0410364.