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ATLAS Luminosity Detector

Simplified computations on thermo-mechanical straining of the fibre planes

Introduction

The current baseline design for absolute luminosity measurement at ATLAS consists of scintillating fibre detectors inserted in Roman Pots. Detection of scattered protons with a precision of $\sigma_x=\sigma_y=0.03$ mm is achieved by arranging 'U' and 'V' layers (±45 degr.) of scintillating fibres (0.5 mm square) in a staggered way. The arrangement leads to an effective fibre pitch of 0.05 mm.

The spatial precision depends on many factors, among them being possible geometrical distortions of the fibre planes due to temperature variations during operation, or temperature differences between the construction / alignment phase and the operation in the LHC tunnel.

In this report we present first a simple analytical estimate of the expected effects. A finite element study provides a more detailed picture of the distortions.

On the hardware

A double stack of polystyrene (PS) scintillating fibres on an alumina (aluminium oxide ceramic ; Al_2O_3) substrate is proposed. The fibres are of quasi-square cross-section of side 0.5mm. The 64-fibre arrangement has the appearance of a flat cable of thickness 0.5mm. The rectangular alumina substrate is 0.17mm thick. This substrate is covered with a PS "flat cable" under +45degr. on one side, and with another flat cable under -45degr. on the other side. The portion of the surface not covered by a PS flat cable is covered by an alumina filling plate, of thickness 0.5mm. The result is a sandwich of uniform thickness, 1.17mm over-all.

The materials used are very different. Related to a possible temperature excursion of the glued assembly, and the induced mechanical straining as a consequence thereof, following questions have been raised :

- Will such straining put the fibres at risk of failure ?
- Order of magnitude of out-of-plane deformation (bending/torsion/warping) of the sandwich ?

For alumina, we will assume it to be isotropic, with tensile elastic modulus E=300GPa (and Poisson coefficient 0.3) and thermal expansion coefficient $\alpha=7\ 10^{-6}K^{-1}$.

Bulk, isotropic PS, as found in injection-moulded products, has on average E=3.3GPa and $\alpha=80\ 10^{-6}K^{-1}$.

1. In-plane effects : mechanical strain induced in the fibres

Given the overwhelming stiffness of the ceramic, one can rougly state that some 7 10^{-5} of mechanical strain is imposed in the PS , per degree of thermal excursion. An excursion of about 14 degrees is thus needed to induce a strain of 1 per mille in the PS.

The poly-methyl-methacrylate (PMMA) fibre liner occupies a very small volume fraction, and is hence neglected in the stiffness bookkeeping. The induced mechanical strains (not stresses) would be the same as the ones in the PS [1] .

The fibres, initially under 45deg. , are bent towards the vertical axis somewhat further "downstream". The bending radius of the innermost fibre is 40mm. The resulting bending strain, being the ratio between half the fibre thickness and the bending radius, is about 6 per mille. This is a *significant* value.

The polymer chains in the PS fibre are claimed to be (somewhat) oriented. This will result in a greater elastic modulus and a smaller thermal expansion [2] , we do not know to which extent. The bending strain reasoning does not alter. The thermal excursion discussion may be altered somewhat, in the sense of smaller straining in the PS.

The induced mechanical straining in the PS/PMMA due to fibre bending is at least as significant as that produced by a (reasonable) thermal excursion of the sandwich plate.

The in-plane deflection as result of a thermal excursion of one degree, say at 50mm away from the anchorage point, are expected to be of the order of 3.5 10^{-4} mm , since the alumina's thermal dilatation will dominate as result of its high stiffness.

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1 Only those due to a thermal excursion. We cannot possibly guess those due to manufacturing, which can be important. It is equally difficult to guess the straining induced by the "optical cement" used to glue the fibres into a "flat cable".

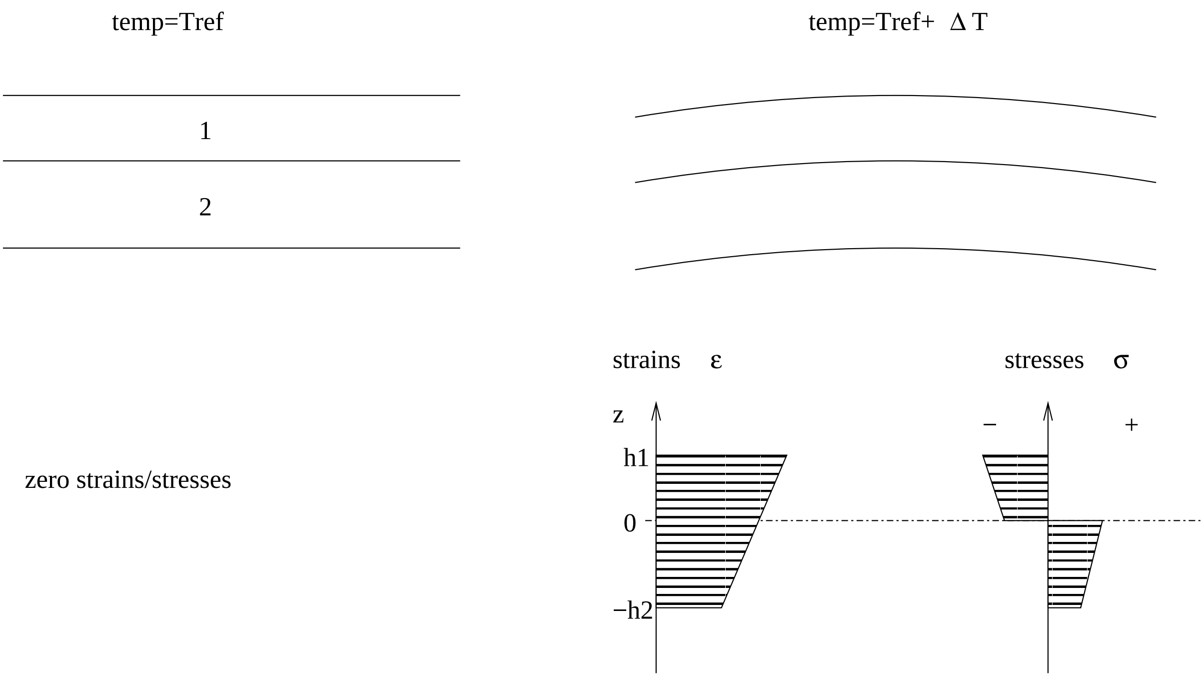
2 ALONG ; the material becomes orthotropic

2. Out-of-plane effects : sandwich deformation

The driving forces for such deformation, per unit temperature excursion, are lay-up asymmetry and difference in thermal expansion coefficients. Obviously, the lay-up thicknesses and the elastic moduli have their word to say as well. Therefore, we will examine two variants in order to account for our uncertainty on PS chain orientation :

- Variant A : neglect orientation, assume bulk properties (E=3.3GPa)
- Variant B : assume E=33GPa along , but together with the unaltered α=80 $10^{-6}K^{-1}$ (pessimistic !)

A simple beam model can already help us establishing a few orders of magnitude.
The sketch below depicts the situation for $\alpha_1 > \alpha_2$ and positive ΔT .



Beam bending theory assumes that initially flat cross-sections remain flat when bent. Thus, the (total) strain can only vary linearly through thickness :

$$\varepsilon(z) = \varepsilon_0 + \kappa z$$

where ε_0 is the beam's specific elongation[3] and κ its curvature, as result of the thermal excursion.
The total strain is the superposition of the mechanical strain (due to stress) and the (free) thermal strain :

$$\varepsilon = \sigma / E + \alpha \Delta T$$

Consequently, the stress :

$\sigma(z) = E_1 [\varepsilon(z) - \alpha_1 \Delta T]$ for positive z

$\sigma(z) = E_2 [\varepsilon(z) - \alpha_2 \Delta T]$ for negative z

3 to a very good approximation. Rigourously, it shall be measured on the neutral line, which generally does not coincide with the z=0 line.

Both the total tensile force and the total bending moment shall vanish :

$$\int_{-h_2}^{h_1} \sigma(z)\, dz = 0 \qquad \int_{-h_2}^{h_1} \sigma(z)\, z\, dz = 0$$

The unknowns ε_0 and κ can be solved from these two equations.
The sagitta s of a curved beam with free length L :

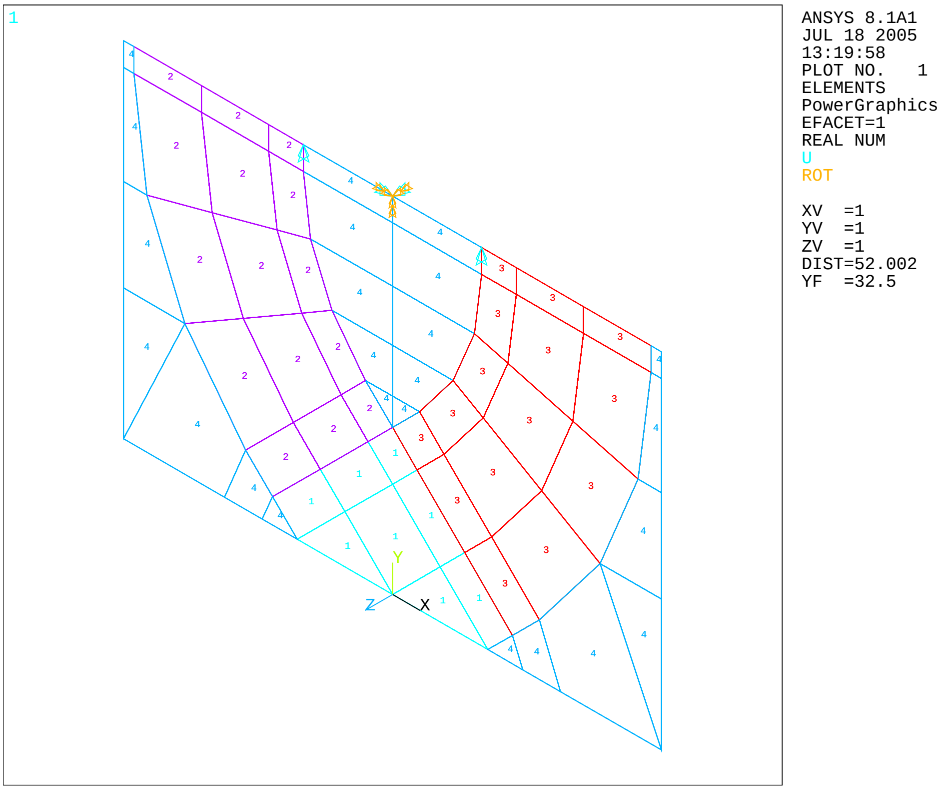
$$s = \frac{\kappa L^2}{2}$$

Take layer 1 to be PS , h_1=0.5mm , and layer 2 to be alumina , h_2=0.67mm (remember the filling plates). Assume furthermore a typical L=50mm . Then, per degree of excursion, we get s=0.01mm for variant A (E_1=3.3GPa) , and s=0.06mm for variant B (E_1=33GPa) .

Note that the *actual* geometry never really corresponds with the above. Hence, this simple beam model only helps in establishing a rough order of magnitude, nothing more.

3. Finite element multi-layer shell model

The model and the applied boundary conditions (model support) are shown in the plot below. Use has been made of multi-layer shell elements[4] with possibility of orthotropic layer material input. We have neglected the influence of the PMMA sheath, the optical cement, and the glue to the ceramic.



The finite element model. Coordinate system : x along (horizontal) , y across (vertical) , z perpendicular to plane. Plate fixation : top central location.

The “zones” 1, 2, 3 and 4 in the figure above correspond to the various configurations detailed in the table below.

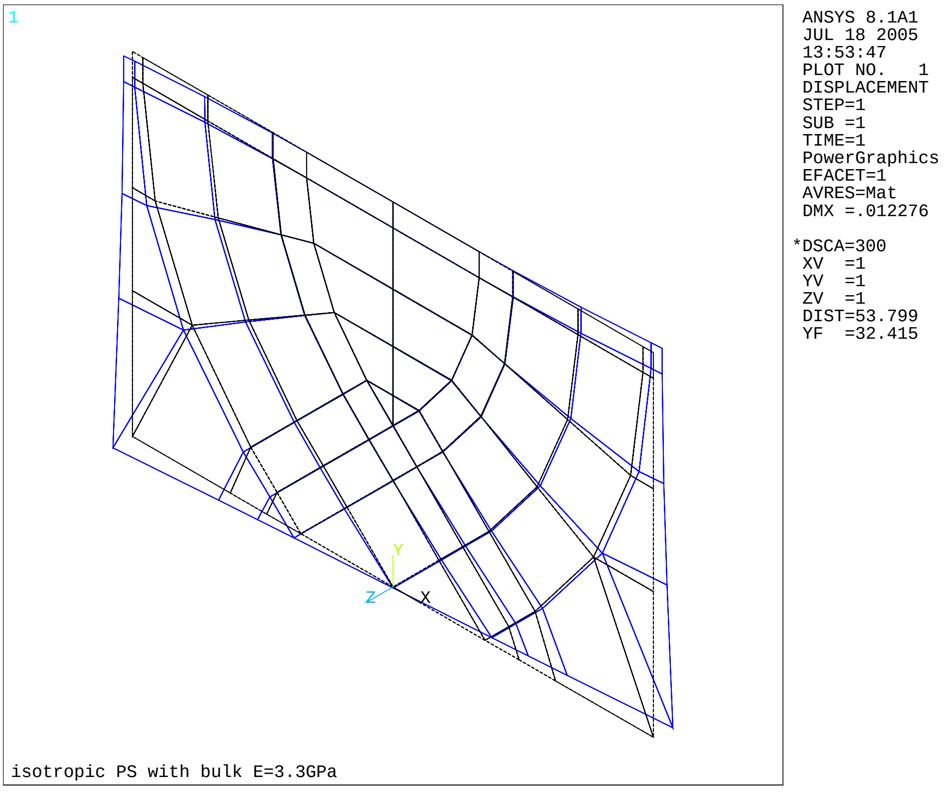
	bottom layer (0.5mm)	***central layer (0.17mm)***	***top layer (0.5mm)***
zone 1	PS	ceramic	PS
zone 2	PS	ceramic	ceramic
zone 3	ceramic	ceramic	PS
zone 4	ceramic	ceramic	ceramic

Below are the results for a +1degree excursion, first for variant A , then for variant B . The displacements are magnified by a factor of “DSCA” in the plots' legend (DMX is the model-wide maximum value of the magnitudes of the nodal displacements, in mm) , and the values of the displacements' contour plots are in mm .

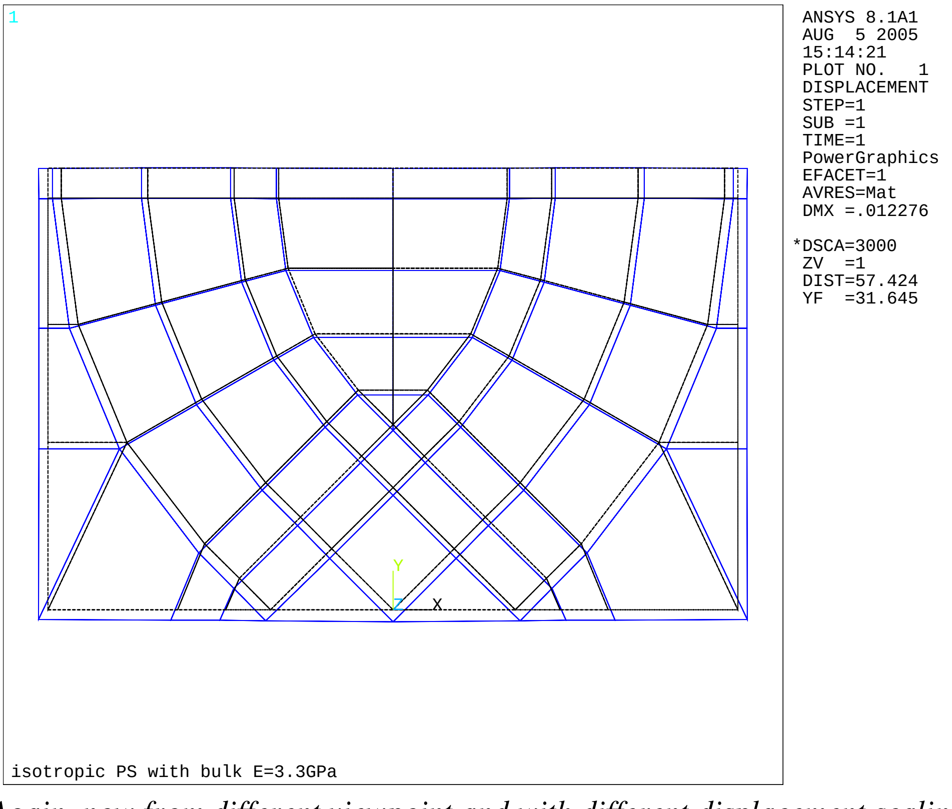
4 The elements are quadratic (8-node) . The simple plot algorithm gives a false, too coarse, impression.

3.1. Results for variant A : PS with bulk properties

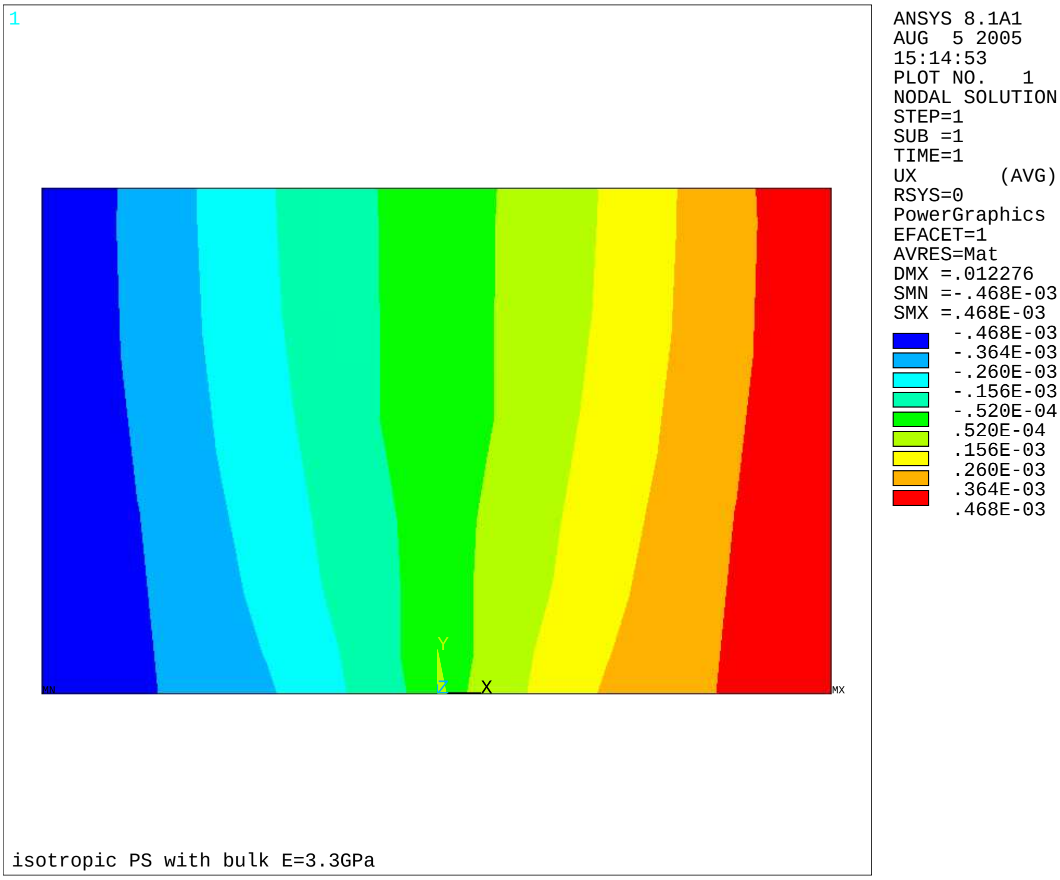
The in-plane deflections are nearly exactly those of homogeneous alumina. The out-of-plane deformation pattern is mainly a bending along x , and there seems to be some agreement with the simple beam model.



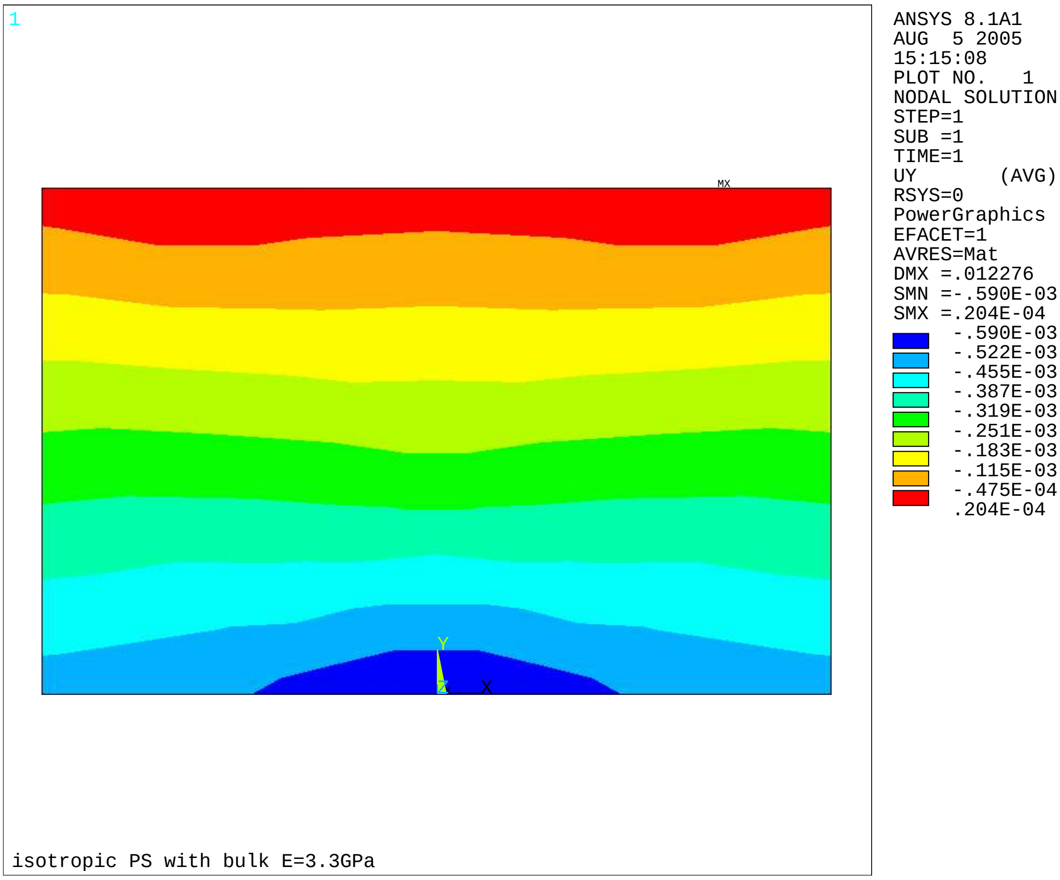
Deformed vs undeformed geometry



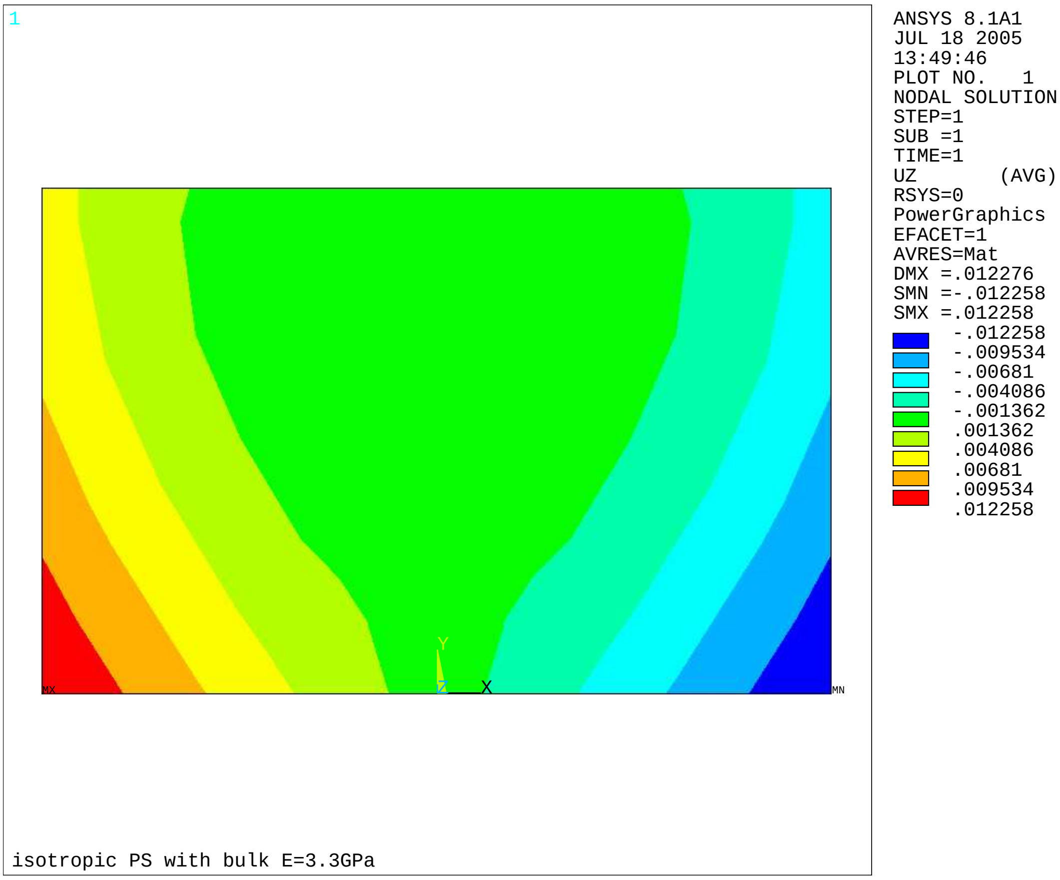
Again, now from different viewpoint and with different displacement scaling



x-displacements, in [mm]



y-displacements, in [mm]

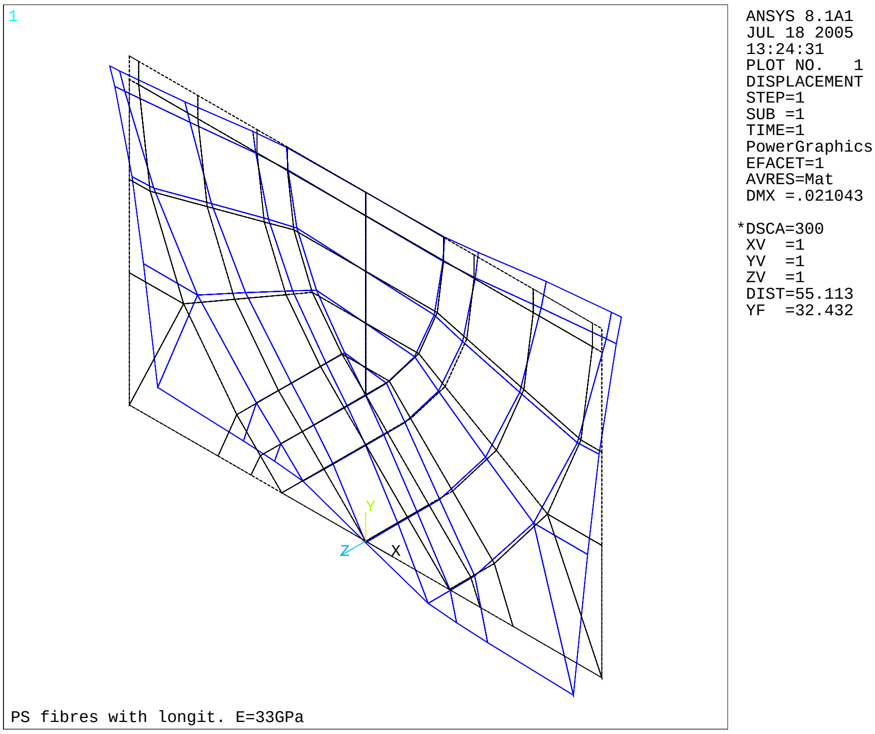


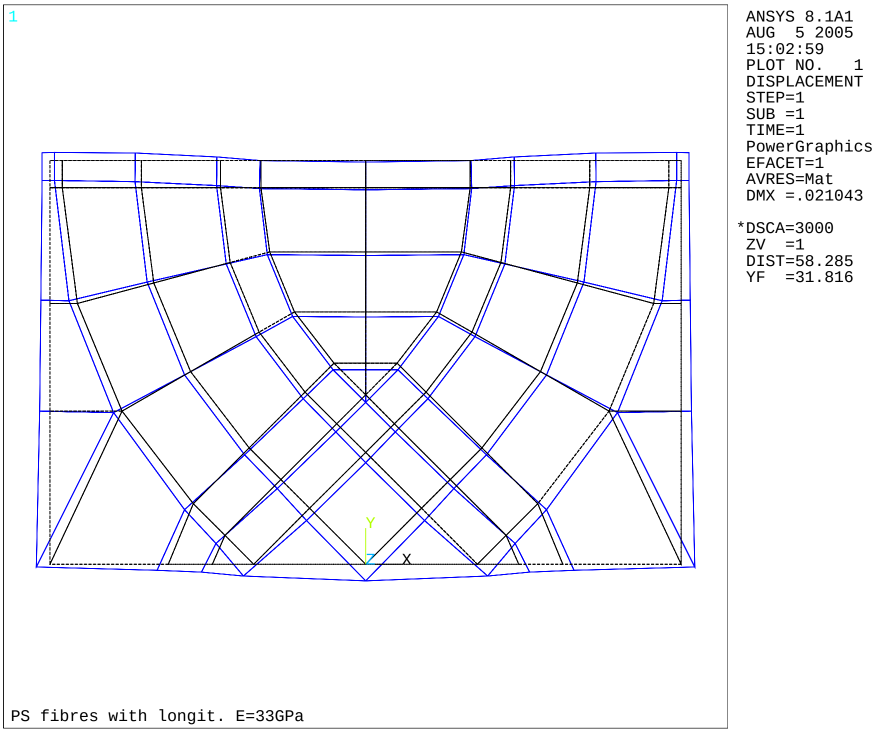
z-displacements, in [mm]

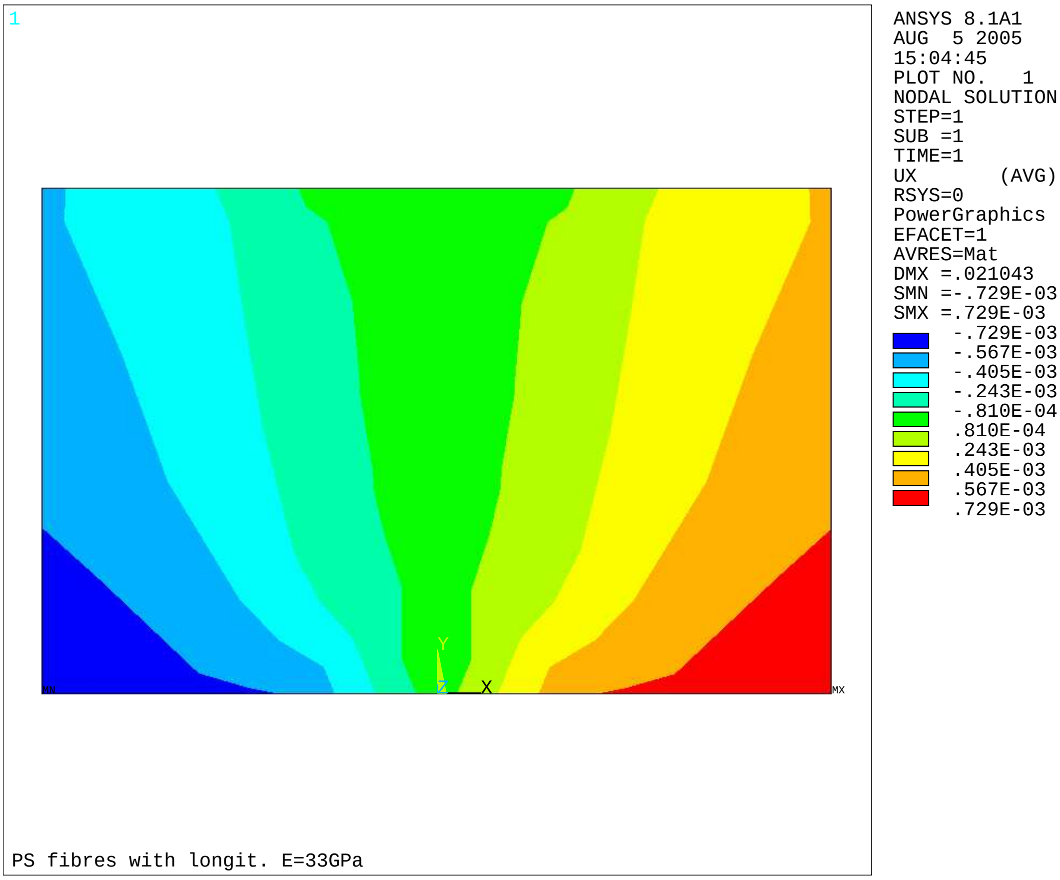
3.2. Results for variant B : PS with assumed E=33GPa along fibre axis

The high longitudinal stiffness of the PS causes the pushing and pulling to influence the in-plane deflection pattern. Concerning out-of-plane deformations, the discrepancy with the naive beam model from above is important. The super-stiff PS is mainly in global y direction. But for bending along y , on must not forget the stiffening of the filler plates *beside* the PS "flat cable". And this side stiffening with ceramic is 1.17mm thick, not 0.67 !

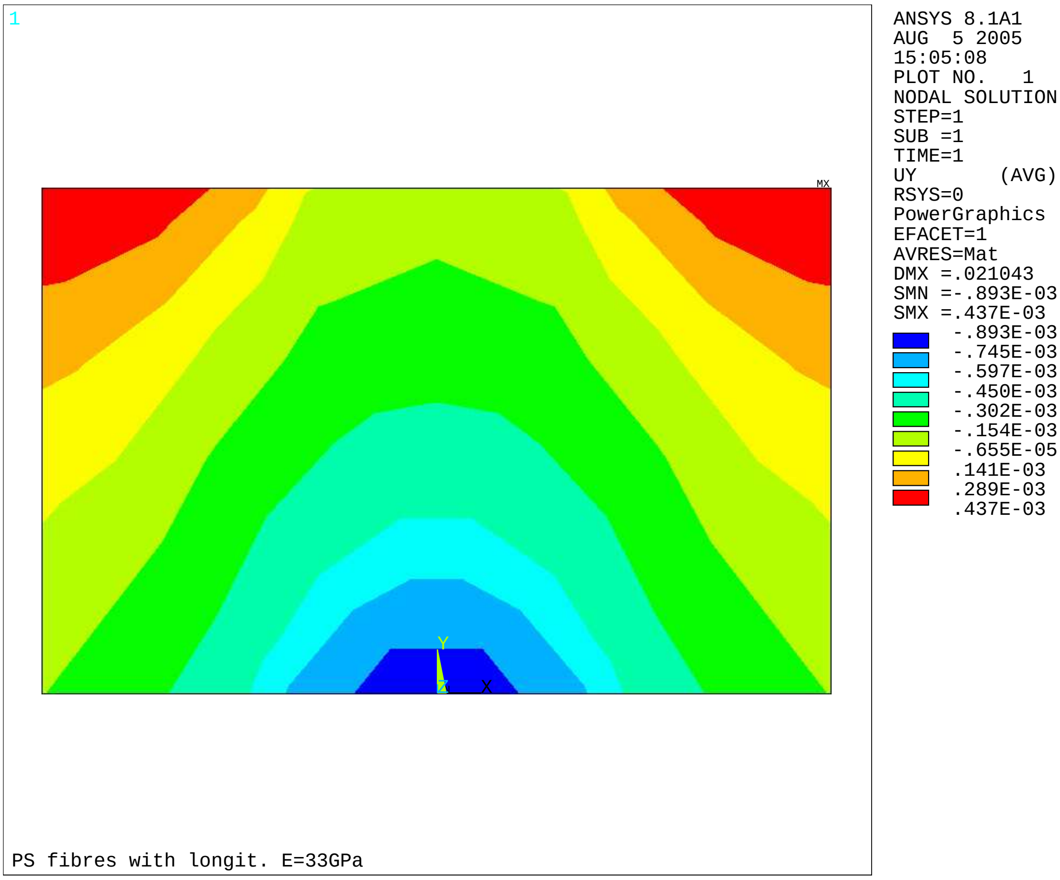
It is good to maintain all four ceramic filling plates per sandwich. Their stiffening is most useful to limit out-of-plane deformations.



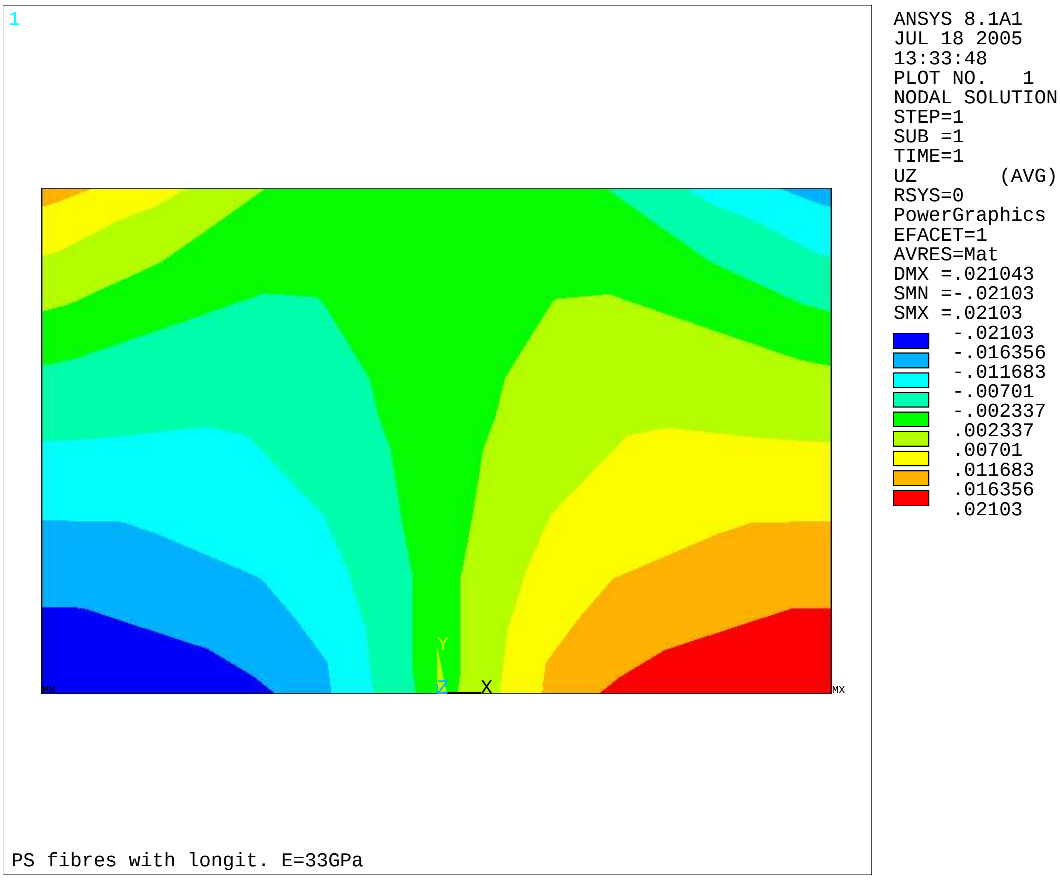




x-displacements, in [mm]



y-displacements, in [mm]



z-displacements, in [mm]

4. Conclusions

- There would have been no point looking for a substrate material with thermal expansion that near-to-matches these of PS and PMMA. Indeed, fibre straining, and thus risk of long-time failure, is dominated by fibre bending, not by a thermal excursion of reasonable magnitude. The chosen substrate material, alumina, has high stiffness and, unlike polymer materials, is a "true solid", in that it has little to no anelastic effects. This is important in view of time-stability of the array's geometry.

- The alumina filling plates are very beneficial to the bending/warping stiffness.

- The deformations of the proposed geometry as a result of a modest thermal excursion tend to be low and are not believed to threaten the device's functioning.