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FERMI MOTION IN PION DEUTERON SCATTERING AT HIGH ENERGIES

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A B S T R A C T

The Fermi motion plays an important role in resonant multiple scattering. Its inclusion restores the agreement between experiments and Glauber theory.

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High precision measurements of the total cross-sections for the scattering of π^+ and π^- mesons by deuterons have recently been reported [1]. The measurements were performed at intervals of about 20 MeV, in the range 350-2400 MeV kinetic energy for the mesons. The cross-sections for the scattering of π^- were found to be about 1 mb higher than those of π^+. A preliminary comparison with the multiple scattering theory of Glauber [2] was also reported [1),3)]. The conclusions drawn were that this theory was unable to describe the experimental data. Since then there has been considerable doubt on the validity of the Glauber expansion in the resonance region.

In our opinion the comparison with theory was done in an unfortunate way. The main contribution, normally about 96%, to the theoretical cross-section comes from single scattering terms. It is independent of any specific multiple scattering theory. Therefore, the term which has to be tested contributes only 4%. The failure of the theory was entirely blamed on this small correction term. The dangerous point in this kind of approach is that although the relative experimental errors are very small the total normalization is somewhat uncertain. In view of the smallness of the correction term a slightly incorrect normalization could easily produce queer results. It is therefore not surprising to find that the contradictions were most embarrassing in a region where the correction term is unusually small, only 2% instead of 4%. Here the correction term turned out to have an incorrect sign!

We have reinvestigated the whole problem and note that a renormalization of the experimental data by about 1.7% brings theory and experiment into a spectacularly good agreement. Whether this interpretation is correct or not has to be born out by future experiments. Incidentally, the renormalization is of the same order of magnitude as the difference between the results obtained in [1] and those of [4] and [5], as can be seen in Fig. 1.

In the multiple scattering theory of Glauber, the mesons are scattered by the individual nucleons in the nucleus. The scattering amplitude for the scattering by a bound nucleon is assumed to be the same as for a free nucleon. In the deuteron case the theory is very simple; only single and double scattering terms are relevant. Therefore we write the total cross-section as a sum of two terms,

$$\sigma_{\pi D} = \sigma_s + \sigma_d \tag{1}$$

As was already noted above the single scattering term σ_s is independent of any specific multiple scattering theory. A test of the Glauber theory is therefore necessarily a test of the contribution from the double scattering term σ_d. In the region above 700 MeV the contribution from this term is normally only about 4%. A decisive test therefore demands accurate measurements, and certainly also an accurate evaluation of the main contribution, the single scattering term. We also want to remark that there are processes which are not included in the Glauber theory, i.e., pion absorption by the deuteron. In the energy region we are considering such effects are negligible.

In the conventional high energy approach to multiple scattering theory the momentum of the struck nucleon is neglected as compared to the momentum of the impinging meson. In our case this cannot be done. There are a number of promiment resonances in the pion-nucleon system in the energy range we are considering, producing rapidly varying cross-sections. Therefore the Fermi motion inside the deuteron must properly be taken into account. As is well known the deuteron wave function contains an S wave part $\varphi_S(\bar{p})$ and a D wave part $\varphi_D(\bar{p})$. If we consider scattering by unpolarized deuterons the appropriate generalization of the single scattering term turns out to be

$$\sigma_s = \int d^3p \left\{ |\varphi_s(\bar{p})|^2 + |\varphi_D(\bar{p})|^2 \right\} \cdot \frac{v(\bar{k},\bar{p})}{v(\bar{k},0)} \cdot \left\{ \sigma_{\pi^+ p}(\bar{k},\bar{p}) + \sigma_{\pi^- p}(\bar{k},\bar{p}) \right\} \tag{2}$$

Here $v(\bar{k},\bar{p})$ denotes the relative velocity between a meson of momentum $\bar{k}$ and a proton of momentum $\bar{p}$, and $\sigma(\bar{k},\bar{p})$ denotes the total cross-section in the same configuration. The appearance of the additional velocity dependent factor is due to the fact that the impinging meson has a different relative velocity with respect to the deuteron itself $v(\bar{k},o)$ and the nucleon inside the deuteron $v(\bar{k},\bar{p})$. We have also used invariance under rotations in isotopic spin space to relate πn cross-sections to πp cross-sections. The formula is the same for the different charge states of the meson since the deuteron has isotopic spin $T=0$. A more detailed discussion of the formula is found in [6].

We will now calculate the single scattering term. No free parameters are involved. As wave functions for the deuteron we have used standard Hulthén and Gartenhaus functions as given by Moravcsik [7]. In position space the Gartenhaus function is given by

$$\varphi_S(\bar{x}) = \text{const}\cdot\frac{1}{r}\left(1-e^{-2.5r}\right)\cdot\left(1-e^{-1.59r}\right)\left(e^{-0.232r}-e^{-1.90r}\right) \tag{3a}$$

$$\varphi_D(\bar{x}) = \text{const}\cdot\frac{1}{r}\begin{cases} 0.658\,r^3 & , 0<r<0.63 \\ 2.34\,r^3 e^{-2r} & , 0.63<r<2.10 \\ 0.147\,e^{-0.256r}+0.810\,e^{-0.577r} & , 2.10<r \end{cases} \tag{3b}$$

The units used are fermi. Furthermore the D wave part is normalized to give 6.7% D state contribution. We want to point out that the wave functions given in [7] are not sufficiently accurately normalized for our purposes. The normalization error in the wave function (3a) is almost 2%. We have therefore renormalized them but keeping a D wave part of 6.7%.

When the Gartenhaus function is used in (2) we get the result shown in Fig. 2. In the same figure we have simultaneously displayed the coherent sum, which gives the cross-section when the Fermi motion is neglected. Two features are immediately visible. In the high energy region where the pion-nucleon cross-sections are smooth the Fermi motion has almost no importance on the deuteron cross-section. On the other hand we see that it is certainly very important in the resonance region, and that it is more important at high energies than at low energies. The effect on the maximum at 900 MeV is much bigger than on the minimum at 500 MeV. It is easy to understand qualitatively the effect of the Fermi motion. Assume that the π meson has an energy ω in the laboratory system of the deuteron and that it hits a nucleon of momentum $\bar{p}$. In the laboratory system of the nucleon we then find an energy

$$\omega' = \omega + \omega \frac{\bar{p}^2}{M^2 + \sqrt{\bar{p}^2 + M^2}} - \bar{k} \cdot \bar{p}/M \quad , \qquad (4)$$

where M is the nucleon mass. The last term in this equation produces an energy spread

$$\Gamma_{fermi} \simeq 2k \left(\frac{\langle \bar{p}^2 \rangle}{3M^2} \right)^{1/2} ,$$

whereas the middle term produces an energy shift

$$\Delta E_{fermi} \simeq \omega \frac{\langle \bar{p}^2 \rangle}{2M^2} .$$

Here $\langle \bar{p}^2 \rangle$ is the expectation value of the squared momentum in the deuteron state. It follows that both the smearing and the shift are more important at high energies than at low energies. In general the Fermi broadening is of importance whenever the half width Γ is of the same order of magnitude as Γ_{Fermi}.

We have also investigated the specific effects produced by the D state on the total cross-section. Thus, in Fig. 3 we have displayed the difference between the results obtained when both S and D wave parts of the deuteron wave function are used in (2) and when only the S wave part is used. In both cases, of course, the total wave function is normalized to unity. It is clearly seen that the inclusion of the D wave accentuates the importance of the Fermi motion. This is due to the much broader momentum distribution in the D state. It is even too broad to allow a simple quantitative estimate of the resulting effect.

We have also studied the effect of a Hulthén wave function for the S wave. This function is given by

$$\varphi_S(\bar{x}) = \text{const} \cdot \frac{1}{r} \cdot \left(e^{-0.232 r} - e^{-1.202 r} \right) , \tag{5}$$

where we are still using fermi as units. The corresponding momentum space wave function is a bit broader than the Gartenhaus counterpart. It is found that the cross-section obtained from the Hulthén function never differs more than 0.15 mb from the one obtained from the Gartenhaus function. Thus, the single scattering term is rather insensitive to the wave function used, in sharp contrast to the double scattering term.

We now turn to the double scattering contribution σ_d in Eq. (1). In the usual high energy approach, where the momenta of the nucleons are neglected and where the intermediate scatterings are restricted to the forward direction, this term is given by [8]

$$\sigma_d = -\frac{1}{4\pi} \langle \frac{1}{r^2} \rangle \left\{ \sigma_+ \sigma_- (1 - \gamma_+ \gamma_-) - \frac{1}{4} \left[(\sigma_+ - \sigma_-)^2 - (\gamma_+ \sigma_+ - \gamma_- \sigma_-)^2 \right] \right\} \tag{6}$$

Here $\langle 1/r^2 \rangle$ is the expectation value of $1/r^2$ in the deuteron state. For the Gartenhaus wave function, when the D wave is included, we have

$$\langle \frac{1}{r^2} \rangle = 0.308 \ \text{fm}^{-2} . \tag{7}$$

Furthermore

$$\gamma_\pm = \text{Re}\, f_\pm / \text{Im}\, f_\pm , \tag{8}$$

where $f_\pm$ are the elastic forward scattering amplitudes for the scattering of π^+ and π^- respectively by protons. All quantities in (6) are evaluated in the laboratory system for the proton. Since the nucleon momenta have been neglected this system coincides with the pion deuteron laboratory system. The actual numerical values for the real parts were obtained from forward dispersion relations [9].

In our treatment discussed above, the double scattering term has been treated in an approximative way. We have thus neglected Fermi motion, angular dependence of the amplitudes and spin flip amplitudes. The Fermi motion is easily included when non-forward intermediate scatterings are neglected. A calculation shows that only small changes are produced. Our conclusions reported above do not change. The inclusion of non-forward intermediate scatterings is a bit more delicate. A detailed evaluation of the resulting changes will be undertaken. We do not expect this effect to be significant.

We have reported on a study of the Glauber multiple scattering theory and its relation to recent measurements, especially those of Bugg et al. [1]. In these measurements the relative errors are very small, but the total normalization is somewhat uncertain. We stress once more the fact that all normalization questions must be definitely settled before a meaningful comparison with theory can be done. Thus, comparing existing data [1),4),5)] and theory suggests that a small renormalization of some of the data may be necessary. Admitting this renormalization the agreement with theory is very good for energies above 700 MeV. Below 700 MeV the situation is still unclear. In the high energy region very small fluctuations still remain after the renormalization. They seem to be connected with the single scattering term rather than with the double scattering term. They could easily be removed by admitting a slightly broader momentum distribution for the deuteron, but at present we see no justification for this explanation. We have found no necessity either for off-mass shell corrections or for corrections due to $\pi\pi$ scattering in the modified meson cloud of the bound system.

We would like to acknowledge discussions with Professors R.J. Glauber, B.E.Y. Svensson and D.V. Bugg. We would also like to thank Dr. R. Strauss for his kind permission to use his values for the real parts.

R E F E R E N C E S

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FIGURE CAPTIONS

Figure 1 :

Theoretical and experimental values for the total cross-section as a function of the kinetic energy of the pion. The dashed line is the mean value of $\pi^+ D$ and $\pi^- D$ as measured by Bugg et al.

Figure 2 :

The single scattering contribution to the total cross-section. The solid line is the coherent sum and the dashed line the coherent sum folded with Fermi motion.

Figure 3 :

The effect of the D wave on the single scattering term. The quantity plotted is the difference between the results obtained when the deuteron wave function contains both S and D waves and when it contains only the S wave.

Figure 4 :

Comparison between theory and the measurements of Bugg et al. The quantity plotted is

$$\frac{\sigma_{exp} - \sigma_{th}}{\sigma_{th}} \times 100$$

and the bars indicate the experimental systematic errors. Fig. a shows the result for $\pi^- D$ scattering and Fig. b the results for $\pi^+ D$ scattering.

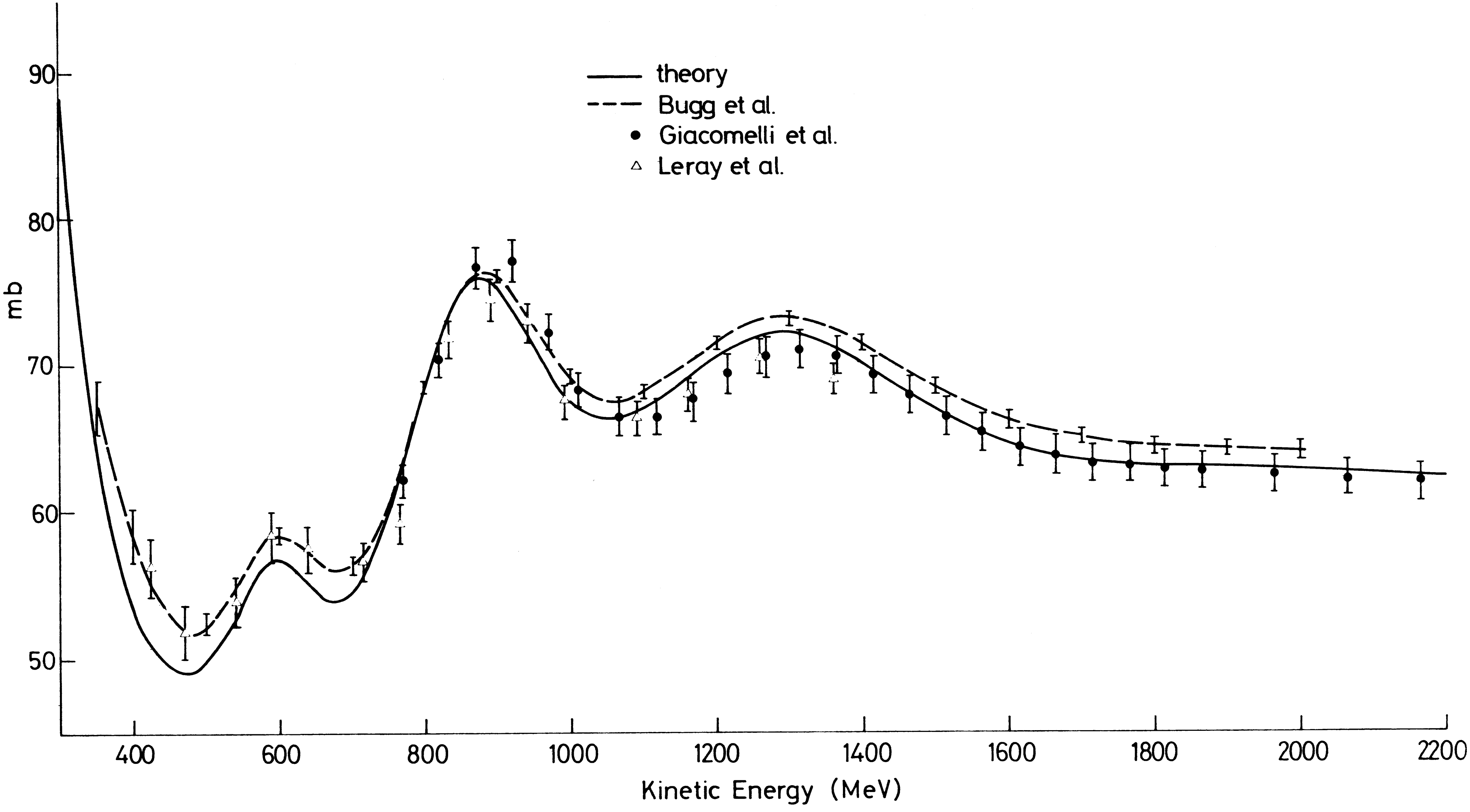
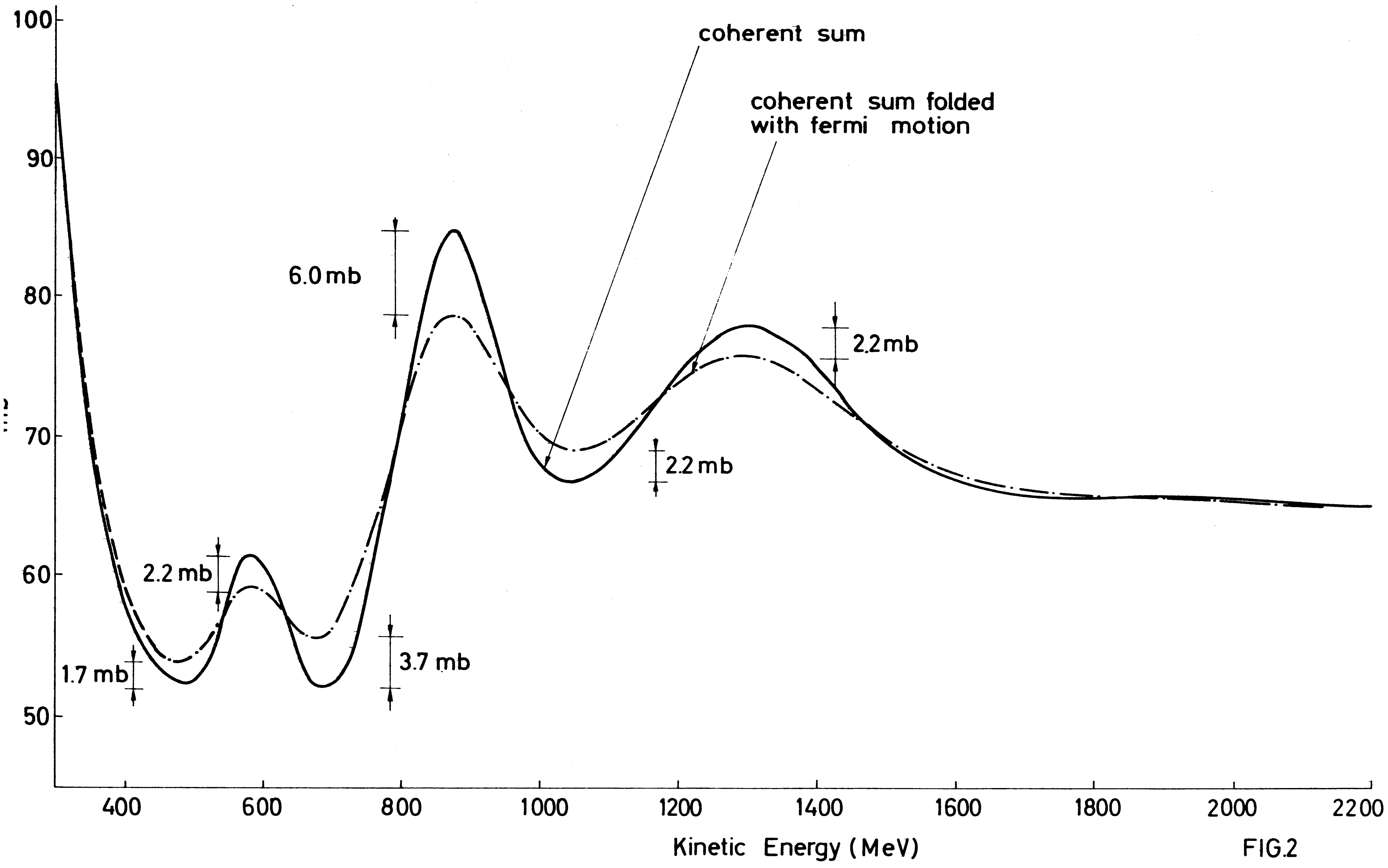

theory
Bugg et al.
Giacomelli et al.
Leray et al.
mb
90
80
70
60
50
400
600
800
1000
1200
1400
1600
1800
2000
2200
Kinetic Energy (MeV)

FIG. 1



100
90
80
70
60
50
coherent sum
coherent sum folded
with fermi motion
6.0 mb
2.2 mb
2.2 mb
2.2 mb
1.7 mb
3.7 mb
400
600
800
1000
1200
1400
1600
1800
2000
2200
Kinetic Energy (MeV)

FIG.2

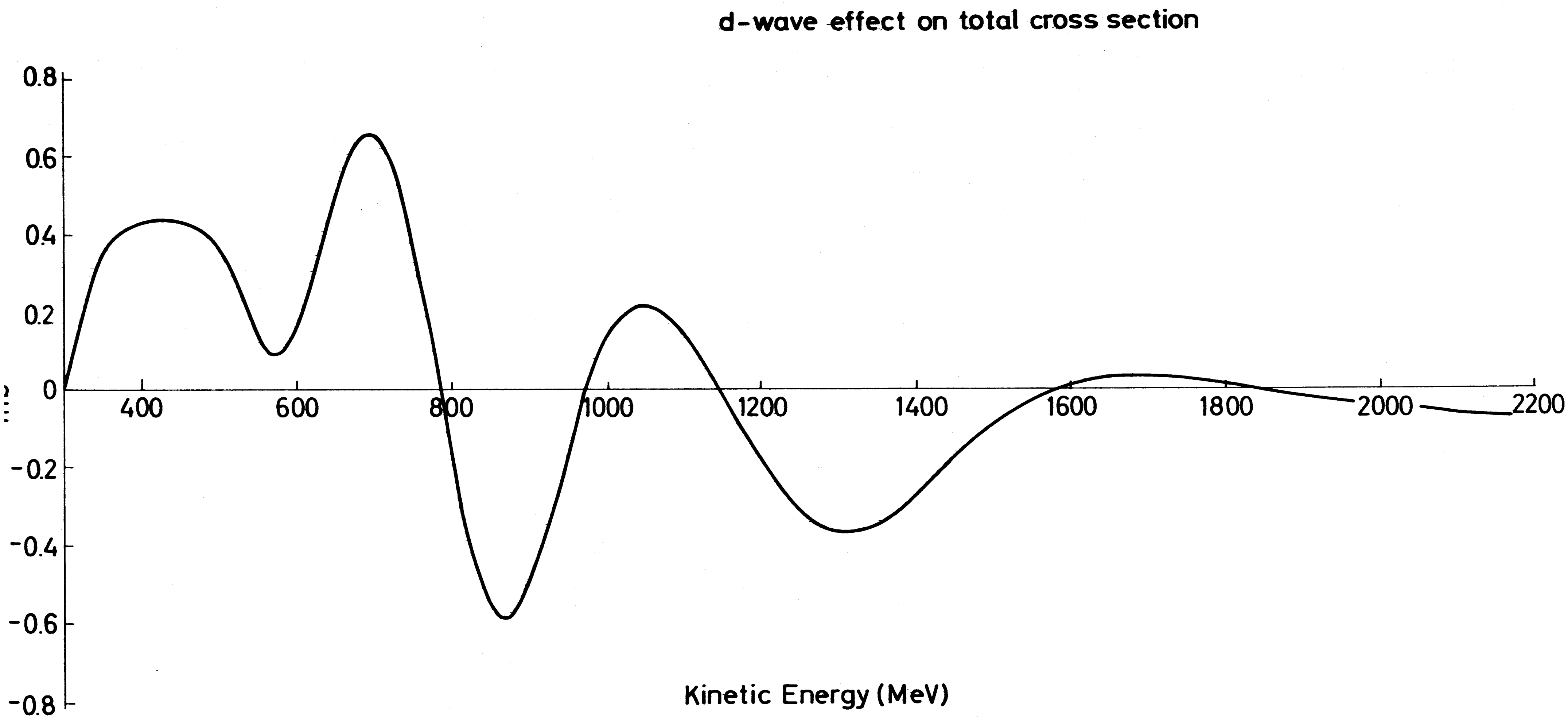
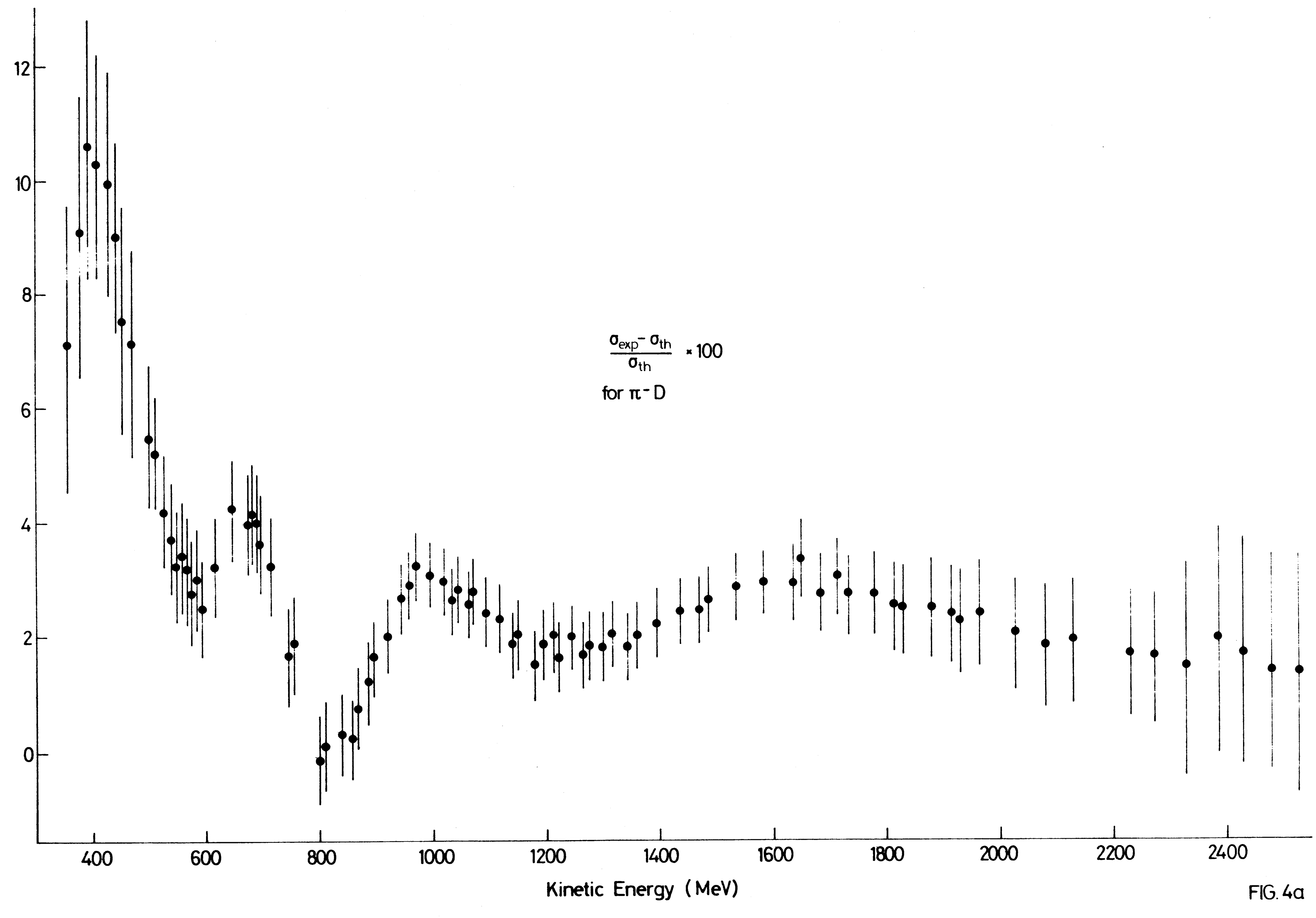

d-wave effect on total cross section
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
400
600
800
1000
1200
1400
1600
1800
2000
2200
Kinetic Energy (MeV)

FIG.3



12
10
8
6
4
2
0
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
Kinetic Energy (MeV)
$\frac{\sigma_{exp} - \sigma_{th}}{\sigma_{th}} \times 100$
for π - D

FIG. 4a

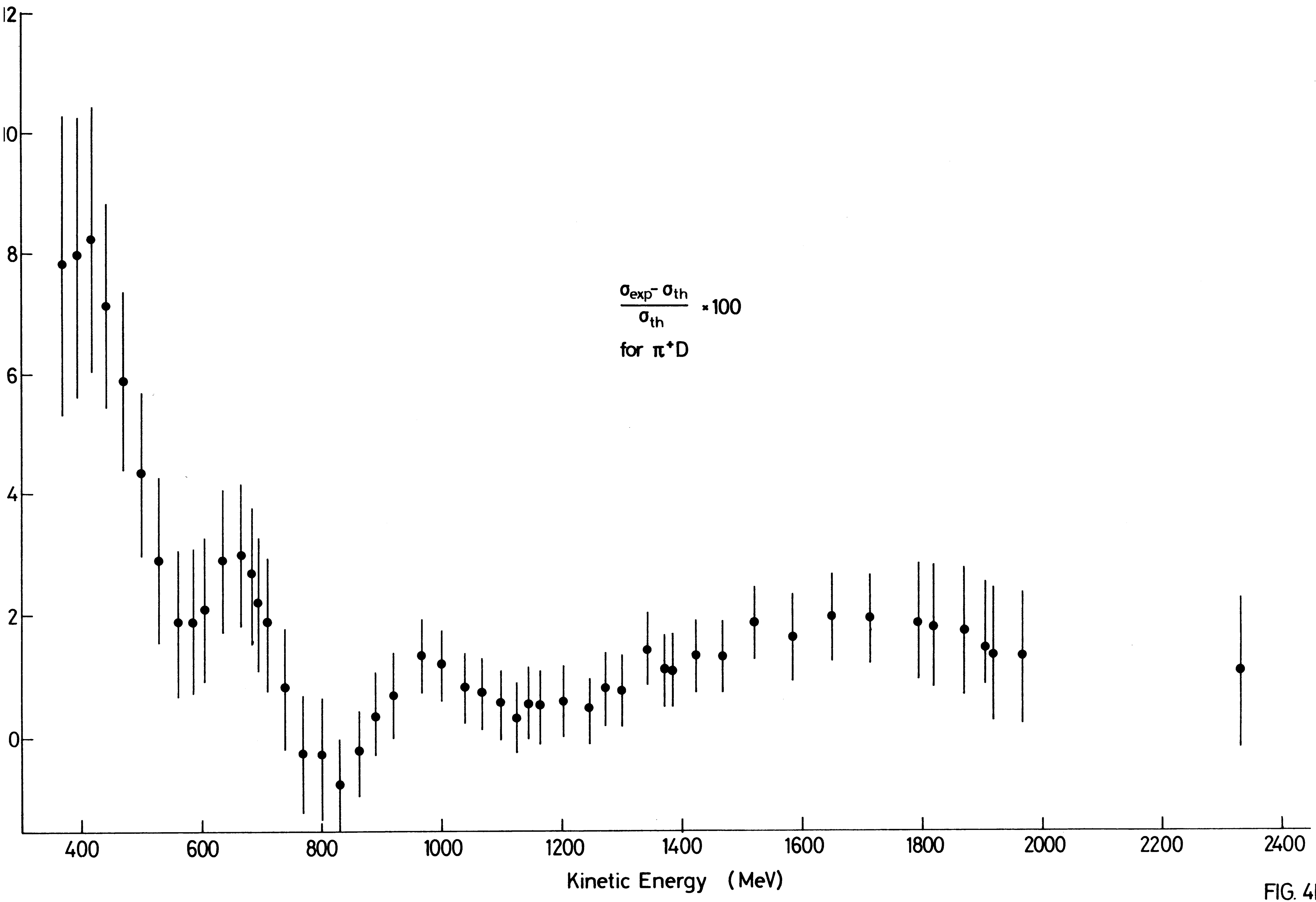

12
10
8
6
4
2
0
$\frac{\sigma_{exp} - \sigma_{th}}{\sigma_{th}} \times 100$
for $\pi^+ D$
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
Kinetic Energy (MeV)

FIG. 4b