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QUANTITATIVE MEASURE OF CLUSTER FORMATION IN MULTIPARTICLE PRODUCTION

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A B S T R A C T

A dispersion parameter for individual events is defined and advocated as a quantitative measure of clustering of particles along the longitudinal rapidity axis in high energy multiparticle interactions. General arguments and Monte Carlo simulations indicate that the parameter should discriminate between fragmentation-type and multiperipheral-type models at $p_{lab} \gtrsim 100$ GeV/c.

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In the study of multiparticle final states in hadronic interactions, it remains an important challenge to specify simple variables which will help in the identification of salient physical characteristics. In this note we focus on a quantitative parametrization of the important concept of clustering of tracks in longitudinal rapidity space.

Roughly speaking, a group of particles will appear to form a cluster if its extension along the rapidity axis is small with respect to the total range of rapidity available. Apparent clustering has been observed in intermediate energy ($p_{lab} \lesssim 30$ GeV/c) experiments [1]. At the much higher energies obtained at the National Accelerator Laboratory (NAL) and at the CERN Intersecting Storage Rings (ISR), it is important to ascertain whether cluster formation persists. The study of high energy models in fact points to the presence or absence of dynamically induced clustering in individual events as a significant differentiating element [*] at high energies.

In this note we advocate the use of a many-particle inclusive dispersion parameter, which measures quantitatively the presence of clustering of tracks (observable particles). After defining dispersion, we discuss kinematic restrictions on its value and then the properties of current models in order to indicate what results may be expected.

We use longitudinal rapidity [**] as a single co-ordinate, having in mind an integration over transverse momenta p_T. For a given event, we consider a subset of N particles with rapidities y_j ; the subset may in fact be the entire event. Define average rapidity of the subset as

$$\bar{y} = \frac{1}{N} \sum_{j=1}^{N} y_j \qquad (1)$$

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*) Motivation and intuitive qualitative arguments are given in more detail in Refs. 2) and 3).

**) Rapidity $y = \frac{1}{2}\log[(E + p_L)/(E - p_L)]$ where E, p_L are energy and longitudinal momentum, respectively. We always use natural logarithms, not $\log_{10}$.

We calculate dispersion *) of the subset as

$$\delta = \left\{ \frac{1}{N-1} \sum_{j=1}^{N} (\bar{y} - y_j)^2 \right\}^{1/2} \tag{2}$$

In practice measurement of rapidities may not be necessary. As long as the relativistic approximations $y_{lab} \approx -\log(\frac{1}{2}\tan\theta_{lab})$ and/or $y_{cm} \approx -\log\tan(\theta_{cm}/2)$, respectively, are acceptable, it suffices to measure the polar angles of tracks with respect to the incident beam direction. In bubble chamber experiments, it is adequate for our purposes to examine tracks in one view of the chamber, obtaining the projected angles θ_{lab}^{proj}. In the absence of pronounced azimuthal structure of events, the dispersion can be estimated from

$$\log(\tan\theta_{lab}^{proj}) = \log(\tan\theta_{lab}) + \log(\cos\phi) \tag{3}$$

where azimuthal angle ϕ is distributed uniformly. In this note an argument in parentheses following δ indicates which variable is used [e.g., y, $\log(\tan\theta_{lab})$,...]. The subscript "ch" will be used when necessary to distinguish the dispersion of charged particles from that for all particles. The statistical property of dispersion as well as our simulations show that dispersion calculated for observable particles only is on the average a good measure of the "true" dispersion.

Interest in the dispersion parameter resides partially in the fact that a single isotropically decaying cluster (without correlation between decay products) gives a grouping of tracks whose density distribution leads to a universal shape, approximately Gaussian in y. In the cluster rest frame, a transformation of variables gives

$$d\sigma = \int_{|p|} f(|p|/\langle|p|\rangle)\, d^3p/E \propto \frac{dy}{\cosh^2 y}\, \frac{d\Delta\, e^{-\Delta/2}}{[\sinh\Delta]^{1/2}} \int_{\left(\frac{m\sinh y}{\langle|p|\rangle}\right)^2}^{\infty} dx^2 f(x) \tag{4}$$

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*) Presence of the factor $(N-1)^{-1}$ in Eq. (2) is recommended in statistical theory of unbiased estimators ; see, e.g., Ref. 4). Cosmic ray data [Cf. Ref. 3] suggest that values of $\langle\delta\rangle$ are significantly smaller (i.e., more clustering) than expected from naïve multiperipheral arguments. It is valuable to verify this result, which may be subject to the usual uncertainties of cosmic ray experiments.

where $\Delta = -\log(\cos\phi)$ and $0 \leq \Delta < \infty$. When the dependence on y in the dynamical integral factor on the right-hand side of Eq. (4) can be neglected, one gets $\delta(y) = 0.91$ and $\delta(\log \tan\theta_{lab}^{proj}) = 1.28$. The dynamical factor in general reduces the value of δ. However, $\langle |p| \rangle$ is fixed by the observed $\langle p_T \rangle$. For pions, $(m/\langle |p| \rangle)^2 \ll 1$. Thus the influence of the dynamical factor usually is unimportant, so that the above quoted values of δ provide a good reference for comparison with experimentally observed dispersions.

There are various ways in which values of dispersion can be computed from individual events, depending upon the way one selects the subset of particles. However, in this note we propose one very specific use of δ, related to the fact that features of certain clusters should allow important discrimination between models. In current specific fragmentation models, one postulates frequent formation at all energies of events in which, apart from a leading particle, a single isotropically decaying cluster (nova) [5] is produced *). By contrast, although permitted at finite energies, production of such isotropic clusters is an asymptotically negligible effect in all multiperipheral-like models [6]. [When using the term "multiperipheral-like", we have in mind models dominated to a large extent by longitudinal (one-dimensional) phase space, the usual concept.] This difference may be exploited to discriminate between the two extreme classes of models.

As remarked, there is particular interest in the class of events in which a leading particle is present, along with a cluster of other produced particles whose δ we want to determine. As an operational method for removing the leading particle in each event, we propose to eliminate the track with y_k satisfying

$$|y_k - \bar{y}| = \max\{|y_i - \bar{y}|,\ i = 1, \ldots N\} \qquad (5)$$

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*) We focus our attention on single cluster formation because when two clusters are present, it is hard to conclude whether they give evidence for a significant physical effect (e.g., fragmentation) or just result from a fluctuation of a basically uniform spectrum, and the fact that neutrals are not detected.

where, here, N is the total number of tracks in the event. For the remaining $n = N - 1$ tracks, we define

$$\bar{y}^1 = \frac{1}{n} \sum y_j$$
$$\delta^1 = \left\{ \frac{1}{n-1} \sum (\bar{y}^1 - y_j)^2 \right\}^{1/2} \tag{6}$$

The superscript (1) indicates that one particle is subtracted before calculation of average rapidity and dispersion. We propose that a simple scatter plot be made of $\delta^1(y)$ or of $\delta^1(\log \tan\theta^{proj}_{lab})$ vs. n from high energy data. Events in a band $\delta^1(y) \lesssim 0.9$ [or $\delta^1(\log \tan\theta^{proj}_{lab}) \lesssim 1.28$] correspond to production of a single isotropic cluster.

In order to show physical differences between models most clearly, we use somewhat extreme versions of current fragmentation and multiperipheral-like models. We discuss the expected variations of δ as a function of energy and multiplicity.

Consider first the Nova fragmentation picture [5]. Within this context, events with $\delta^1(y) \lesssim 0.9$ constitute at high enough energies a purified sample of single nova events. The usual n^{-2} behaviour of fragmentation models should be more apparent in these events since finite energy modifications are more serious for double than for single-nova events [7]. At sufficiently low energies ($p_{lab} \lesssim 30$ GeV/c), all events (in all models) necessarily give $\delta^1(y) \lesssim 0.9$, a result attributable to energy-momentum restrictions. To the extent that single nova formation provides an asymptotically finite fraction σ_1 of σ_{inel}, the nova model predicts slow decrease with energy of the population of the isotropy band, until this σ_1 limit is approached. For example, fits to σ_n vs. n at 200 GeV/c suggest that approximately 50% of σ_{inel} may lie in the isotropy band [7]. Another estimate, obtained from analysis of leading proton spectra at ISR energies [8], gives $\sigma_1 \approx 20\%$ of σ_{inel} at s = 1995 GeV/c.

It may turn out that no events with $\delta^1(y) \lesssim 0.9$ are found at very high (e.g., ISR) energies. This would be a significant result not only because of its consequences for the nova model. Indeed, isotropic decay is postulated (at least indirectly) in other models also [9], and is closely connected with the asserted linear rise of the average yield of decay pions from a fireball (nova) of mass M :

$$\langle m \rangle_M \propto M . \tag{7}$$

In contrast to Eq. (7) multiperipheral-like models lead to the average behaviour

$$\langle m \rangle_M \propto \log M , \tag{8}$$

where M denotes the mass of a group of secondaries. Correspondingly, in order that $< n > \propto \log s$ for all events, one should have

$$\frac{d\sigma}{dM^2} \propto \frac{1}{s} ; \tag{9}$$

i.e., a rapid fall with s of $d\sigma/dM^2$ (multi-Regge exchange would suggest, via triple Regge limit, a still faster fall with s at small M). The implications of the difference between (7) and (8) can be read from Fig. 1. In this sketch we indicate that $\sigma_n(M)$ is large only along a band given by $n \sim \log M$. What is important here is the large n behaviour at fixed M. Our sketch is no doubt inexact at small n, which is irrelevant for the following argument.

In order that $\delta(y) \lesssim 0.9$, $< p_L > \approx < p_T >$ in the cluster rest frame. Kinematics shows that this occurs for $n \gtrsim k M$ where $k \approx 1/< p_T >$. The line $n = k M$ is also drawn in Fig. 1. In multiperipheral-type models, events can fall above this line only if (i) M is small or (ii) with M large, $n \gg < n >_M$. Thus, Eq. (9) and the fact that σ_n vs. n falls at large n as $(n!)^{-1}$ in multiperipheral [6] or independent emission [10] type of models show that this breed of models gives a vanishingly small cross-section with $\delta(y) \lesssim 0.9$ as s grows. General considerations also lead to the prediction

$$\langle \delta \rangle_M \propto \log M \tag{10}$$

in all models dominated by longitudinal phase space [10].

In order to convert the above qualitative discussion into a set of at least semi-quantitative statements for multiperipheral-like models, we have performed Monte Carlo simulations based on a model in which particles are emitted independently (π model), according to a squared matrix element which factors in p_T^2. The longitudinal momenta of the final nucleons were weighted to reproduce an approximately flat longitudinal momentum spectrum (we do not try to reproduce the peak near $x = 1$ in the ISR inclusive proton spectrum [8] since this peak is most naturally interpreted as diffraction dissociation). Charges were assigned to particles according to statistical isospin factors [11]. The experimental over-all charged particle multiplicity distribution was reproduced essentially by free adjustment of the "coupling constant" of the independent emission squared matrix element for each value of multiplicity. Effort was made, within the scope of this approach, to reproduce all known aspects of single-particle inclusive distributions.

Roughly, the fraction of events with $\delta^1_{ch}(y) < 1$ we obtained is :

$\approx$ 90 to 100 %	for	$p_{lab} \lesssim$ 30 GeV/c
$\approx$ 10 %	for	p_{lab} = 200 GeV/c
$\lesssim$ 1 %	for	ISR energies

This result is expected from Eq. (9), under the assumption that only small cluster masses are responsible for small values of δ .

Features of the scatter plot of $\delta^1_{ch}(y)$ vs. n_{ch} at 200 GeV/c can be read from Fig. 2a. At each n_{ch} we obtain a roughly Gaussian distribution of cross-section vs. $\delta^1_{ch}(y)$, shown explicitly for $n_{ch} = 8$ in Fig. 3. In Fig. 2a, mean values and standard dispersion of these Gaussians are given for all n_{ch}. It will be noted that by 200 GeV/c the two extreme pictures (single nova dominance and independent pion emission) are distinguishable through their different predictions for dispersion. The expected asymptotic logarithmic increase of $< \delta(y) >$ [cf. Eqs. (9) and (10)] is obtained for $p_{lab} \gtrsim 80$ GeV/c. For $n_{ch} \gtrsim 10$, $< \delta^1_{ch}(y) > \approx 1.4$ at 200 GeV/c, and ≈ 2 at ISR energies. Dispersions in $\log \tan(\theta_{cm}/2)$ are somewhat larger : at ISR energies we get $< \delta^1_{ch}(\log \tan\theta_{cm}/2) > \approx 2.3$.

As stated above, our multiperipheral-like model estimates are based on a model in which pions are emitted independently. This as an idealization and we are aware of its limitations. On the other hand, we want to explore consequences of a model dominated by longitudinal phase space, but containing as little other dynamics as possible. We performed a series of calculations (lack of space prevents us from describing them here in detail) to estimate the flexibility of the predictions. The following effects were taken into account :

i) modifications of p_T distributions of pions ;

ii) modifications of the leading particle p_L distributions ;

iii) pairing effects (resonance formation).

As far as values of δ are concerned, we found that the limit of flexibility of the model is at the level of 10%, which is not a small variation, since δ is a sensitive parameter. Pairing seems to be the most important effect. We quote the results of a calculation where production of pions proceeds via intermediate independent emission of ρ mesons (ρ model). The resulting $< \delta^1_{ch}(y) >$ are denoted by $\times$ on Fig. 2a. In Fig. 2b, the dependence of $< \delta^1_{ch}(\log \tan\theta^{proj}_{lab}) >$ vs. n_{ch} at 200 GeV/c is shown. In the range $n_{ch} \gtrsim 10$, $< \delta^1_{ch}(y) > \approx 1.23$ and ≈ 1.75 at 200 GeV/c and ISR energies, respectively. The value of $< \delta^1_{ch}(\log \tan\theta_{cm}/2) >$ at ISR energy is ≈ 2.

In summary, we claim that the dispersion parameter δ is valuable as a quantitative measure of clustering present in individual multiparticle events. Our model calculations suggest that for momenta $\gtrsim 100$ GeV/c, δ has considerable power for discrimination between models. From a model independent point of view, ascertaining the quantitative extent of cluster formation in individual events at ISR and NAL energies is very important. For $p_{lab} \gtrsim 60$ GeV/c, the correlation parameter f_2, obtained from the over-all charged prong cross-sections, is observed to become increasingly positive, suggesting the presence of positive two-particle correlations at the inclusive level [12]. At present, it is not clear to what extent this correlation is true of <u>individual events</u> or whether it comes primarily from a less interesting effect, such as the interplay of two mechanisms each of which gives essentially no correlation at the level of individual events [e.g., the sum of two Poisson distributions for σ_n vs. n obviously gives a positive f_2 growing as $(\log s)^2$]. Measurements of δ should help to resolve this issue.

It is interesting to remark that following our suggestion [2)] preliminary experimental determinations of δ have been made at 200 GeV/c [13)] and for several ISR energies [14)] (particular conditions of these experiments partly motivated our choice of numerical examples in this note). These very preliminary results suggest significant clustering effects at 200 GeV/c. However, the values of $< \delta^{1}_{ch}(\log \tan\theta^{proj}_{lab}) >$ seem to fall within the limits of flexibility of the multiperipheral-like models (e.g., the ρ model). The picture which emerges from the ISR data seems to us to be multiperipheral-like.

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FIGURE CAPTIONS

Figure 1 The behaviour of cross-section $\sigma_n(M)$ as a function of multiplicity (n) and cluster mass (M) in multiperipheral-like models. Only events near and above the straight line $n = k M$ correspond to isotropic clusters. Important low multiplicity tails of $\sigma_n(M)$ are responsible for large dispersion of the values of δ.

Figure 2 a) Scatter plot of $\delta^1_{ch}(y)$ vs. n_{ch} at 200 GeV/c :

● π model,

✕ ρ model.

Indicated standard dispersions of the values of $\delta^1_{ch}(y)$ correspond to the π model. They are similar in magnitude for the ρ model, but are not shown to avoid confusion. The dashed line gives the mean dispersion expected for a single isotropic cluster.

b) Same as a) but for $\delta^1_{ch}(\log \tan\theta^{proj}_{lab})$ and in the framework of ρ model.

Figure 3 Cross-section vs. dispersion for $n_{ch} = 8$ at 200 GeV/c from the π model (cross-hatched) compared with mean dispersion from single nova formation (dashed line).

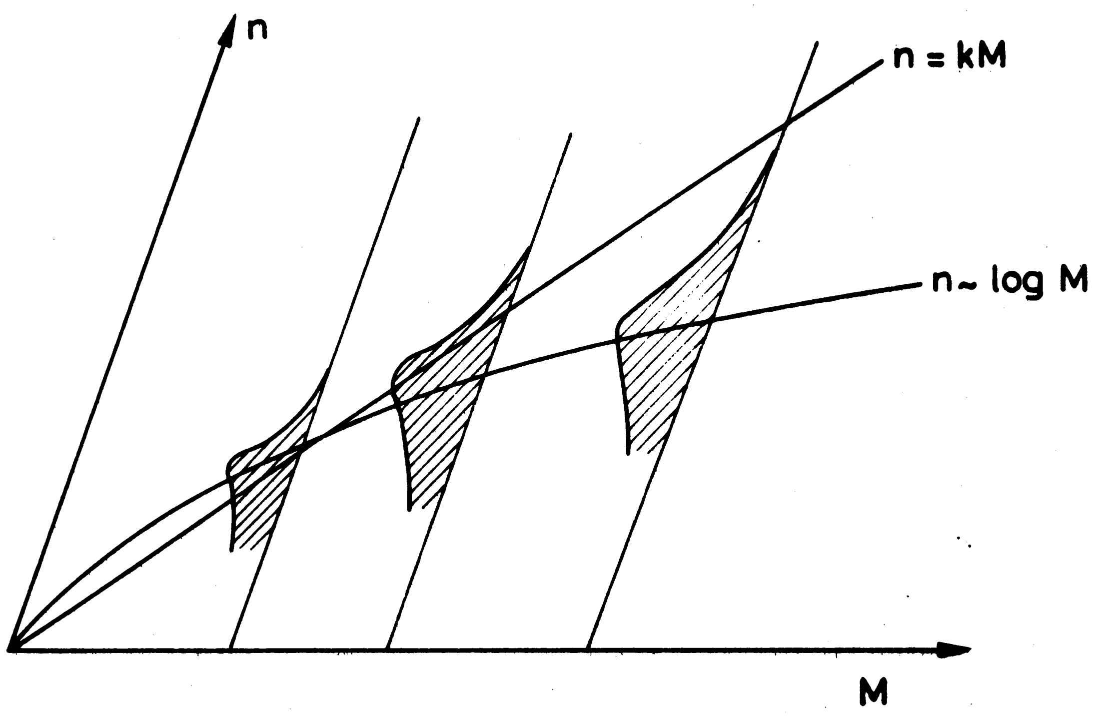
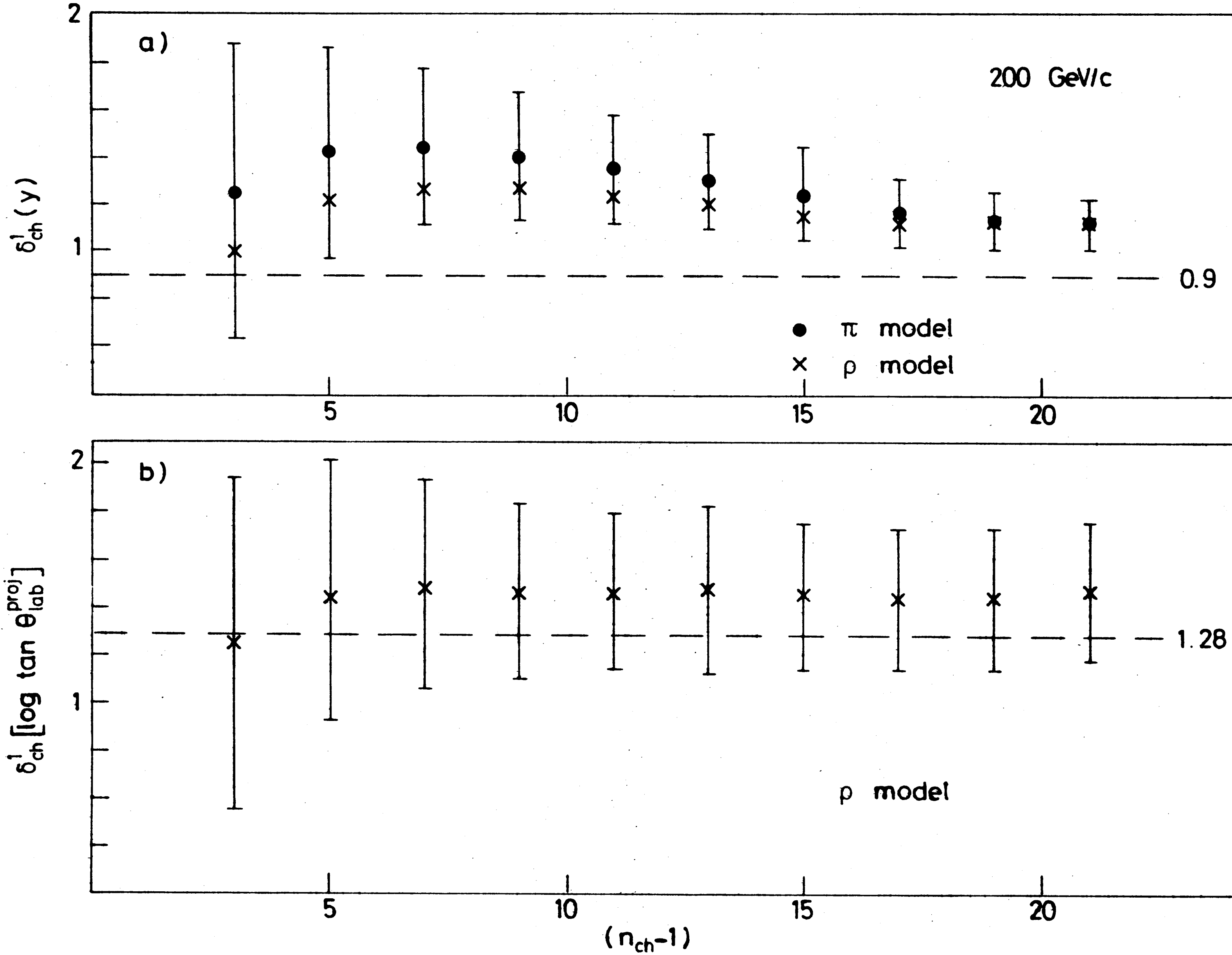
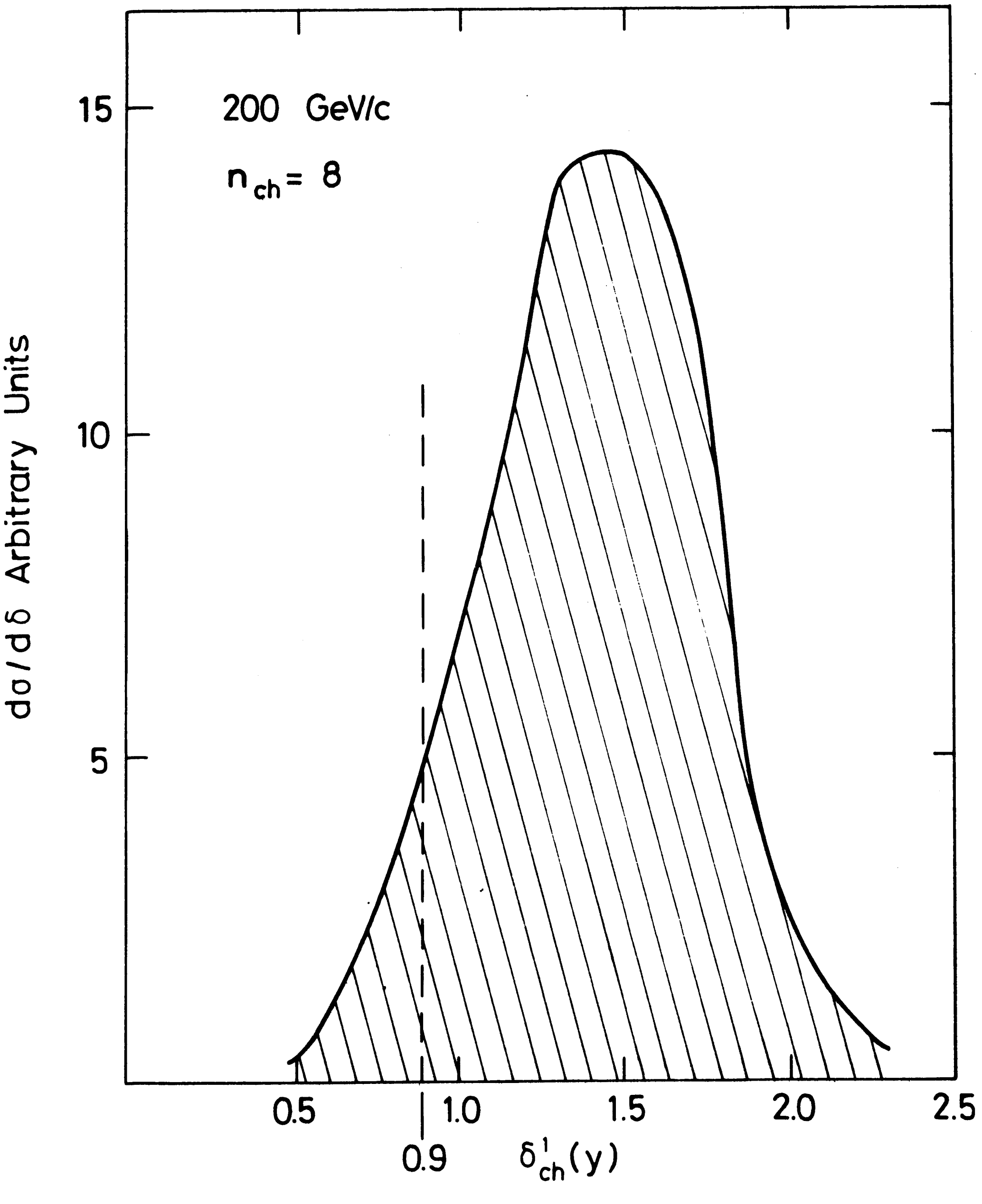


Fig. 1



a)
b)
200 GeV/c
π model
ρ model
ρ model
0.9
1.28
2
1
5
10
15
20
$\delta^1_{ch}(y)$
$\delta^1_{ch}[\log \tan \theta^{proj}_{lab}]$
$(n_{ch}-1)$

Fig. 2



200 GeV/c
n ch = 8
dσ/dδ Arbitrary Units
15
10
5
0.5
1.0
1.5
2.0
2.5
0.9
δ¹ch(y)

Fig. 3