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# RELATION BETWEEN DEEP INELASTIC SCATTERING AND ANNIHILATION

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## A B S T R A C T

Locality and spectrum properties are used to derive representations for the scaling functions of deep inelastic scattering and annihilation processes. A generalized crossing relation among these scaling functions emerges.

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The study of the structure of hadrons has largely profited from the investigations of deep inelastic lepton nucleon scattering. In these processes the question of a possible connection between deep inelastic electron nucleon scattering

$$e + p \rightarrow e' + \text{"anything"}$$

and the deep electron positron annihilation process

$$e + \bar{e}' \rightarrow p + \text{"anything"}$$

has become important. Drell, Levy and Yan [1] proposed crossing relations involving the structure functions $W$ and $\bar{W}$ for scattering and annihilation, respectively. In a model of scalar currents and particles, which we choose for the discussion in this note for reasons of simplicity, the crossing relation of these authors reads

$$\bar{W}(q^2, u) = - W(q^2, u) . \tag{1a}$$

An immediate consequence of this is the crossing relation for the corresponding scaling functions $F$ and $\bar{F}$

$$\bar{F}(\omega) = - F(-\omega) \tag{1b}$$

in the annihilation region $-1 \leq \omega \leq 0$, where the scaling variable is defined by $\omega = -2pq/q^2$. Recently a number of papers [2] has dealt with the problem of crossing for scaling functions in various models. The most general discussion starting from a light cone expansion has been given by Gatto and Preparata [3] who modify Eq. (1b) for the case where $\bar{F}(\omega)$ exhibits a cut in the annihilation region.

It is the purpose of this letter to propose representations for both scaling functions $F(\omega)$ and $\bar{F}(\omega)$ from explicitly exploiting locality and spectrum. As a result both scaling functions are determined by one and the same double spectral function, which itself is entirely given in terms of final hadron states. An immediate consequence of these representations is a generalized crossing relation among $F$ and $\bar{F}$.

We start from the connected part of the virtual non-forward "Compton amplitude" for scalar particles and currents

$$T^{ret}(q^2, Q^2, s, t, u) = i\int d^4x\, e^{iqx}\, \theta(x_0) \langle p' | [J(x), J(0)] | p \rangle_c \tag{2}$$

which we represent by an ansatz which takes fully into account locality, physical spectrum and crossing symmetry *)

$$T^{ret}(q^2, Q^2, s, t, u) =$$
$$\frac{1}{\pi^3} \int_{q_t^2}^{\infty} dq'^2 \int_{q_t^2}^{\infty} dQ'^2 \int_{s_t}^{\infty} ds' \frac{g(q'^2, Q'^2, s', t)}{(q'^2 - q_+^2)(Q'^2 - Q_+^2)} \left[ \frac{1}{s' - s_+} + \frac{1}{s' - u_+} \right] . \tag{3}$$

Here the integrations extend over normal discontinuities **). We have used the following notation $(p^2 = p'^2 = M^2)$, $Q = (q+p'-p)$, $s = (q+p')^2$, $u = (q-p)^2$, $t = (p'-p)^2$, the subscript $+$ refers to a small positive imaginary part of $q_0$ in the corresponding variable. Representation (3) can be justified in a similar way as the representation for the three-point function in Ref. 4). By the same arguments the triple discontinuity occurring in (3) is given by

$$\theta(q_0)\, \theta(Q_0)\, \theta(q_0 + p_0')\, g(q^2, Q^2, s, t) =$$

$$\frac{1}{2^3} \int d^4x\, d^4y\, d^4z\, e^{iqx} e^{ip'y} e^{-ipz} \langle 0 | J(x)\, j(y)\, j(z)\, J(0) | 0 \rangle \tag{4}$$

with $j(y) = (\Box + m^2)\phi(y)$, $\phi(y)$ being the interpolating field of the scalar particles. For $g$ we have the obvious crossing property

$$g(q^2, Q^2, s, t) = g(Q^2, q^2, s, t) . \tag{5}$$

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*) For the sake of simplicity we do not incorporate single and double spectral contributions. Modifications of our results due to those additional terms will be given in a forthcoming publication.

**) For the discussion in this note we neglect all anomalous discontinuities.

The non-forward structure function A is proportional to the imaginary part of $T^{ret}$:

$$A(q^2, Q^2, s, t, u) = \frac{1}{\pi} \operatorname{Im} T^{ret} =$$

$$\frac{1}{2\pi} \int d^4x \, e^{iqx} \langle p' | [J(x), J(0)] | p \rangle_c = \frac{1}{2\pi i} \operatorname{disc}_{q_0} T^{ret} . \qquad (6)$$

As a direct consequence of the causal representation we get

$$\pi^3 A(q^2, Q^2, s, t, u) =$$

$$\Big\{ \epsilon(q_0)\, \theta(q^2 - q_t^2) \int_{q_t^2}^{\infty} dQ'^2 \int_{s_t}^{\infty} ds' \; \frac{g(q^2, Q'^2, s', t)}{Q'^2 - Q^2} \left[ \frac{1}{s' - s} + \frac{1}{s' - u} \right]$$

$$-\pi^2 \epsilon(q_0)\, \epsilon(Q_0)\, \theta(q^2 - q_t^2)\, \theta(Q^2 - q_t^2) \big[ \epsilon(q_0 + p_0')\, \theta(s - s_t)\, g(q^2, Q^2, s, t) + \epsilon(q_0 - p_0)\, \theta(u - s_t)\, g(q^2, Q^2, u, t) \big] \Big\}$$

$$+ \Big\{ \epsilon(Q_0)\, \theta(Q^2 - q_t^2) \int_{q_t^2}^{\infty} dq'^2 \int_{s_t}^{\infty} ds' \; \frac{g(q'^2, Q^2, s', t)}{q'^2 - q^2} \left[ \frac{1}{s' - s} + \frac{1}{s' - u} \right] \qquad (7)$$

$$+\pi^2 \epsilon(q_0)\, \epsilon(Q_0)\, \theta(q^2 - q_t^2)\, \theta(Q^2 - q_t^2) \big[ \epsilon(q_0 + p_0')\, \theta(s - s_t)\, g(q^2, Q^2, s, t) + \epsilon(q_0 - p_0)\, \theta(u - s_t)\, g(q^2, Q^2, u, t) \big] \Big\}$$

$$+ \Big\{ \epsilon(q_0 + p_0')\, \theta(s - s_t) \int_{q_t^2}^{\infty} dq'^2 \int_{q_t^2}^{\infty} dQ'^2 \; \frac{g(q'^2, Q'^2, s, t)}{(q'^2 - q^2)(Q'^2 - Q^2)}$$

$$- \pi^2 \epsilon(q_0)\, \epsilon(Q_0)\, \theta(q^2 - q_t^2)\, \theta(Q^2 - Q_t^2)\, \epsilon(q_0 + p_0')\, \theta(s - s_t)\, g(q^2, Q^2, s, t) \Big\}$$

$$+ \Big\{ \epsilon(q_0 - p_0)\, \theta(u - s_t) \int_{q_t^2}^{\infty} dq'^2 \int_{q_t^2}^{\infty} dQ'^2 \; \frac{g(q'^2, Q'^2, u, t)}{(q'^2 - q^2)(Q'^2 - Q^2)}$$

$$- \pi^2 \epsilon(q_0)\, \epsilon(Q_0)\, \theta(q^2 - q_t^2)\, \theta(Q^2 - q_t^2)\, \epsilon(q_0 - p_0)\, \theta(u - s_t)\, g(q^2, Q^2, u, t) \Big\} .$$

The four terms in curly brackets are the discontinuities in the four invariant variables $q^2$, $Q^2$, s and u, respectively. They correspond to the four diagrams a)-d) of Fig. 1.

The term represented by the third curly bracket in Eq. (7) restricted to forward direction describes (via the optical theorem) the process $e+p \to e'+$"anything". Therefore, the structure function W for this process is given by

$$W(q^2,s) = \epsilon(s+M^2-q^2)\,\theta(s-s_t)\,\frac{1}{\pi^3}\int_{q_t^2}^{\infty} dq'^2 \int_{q_t^2}^{\infty} dQ'^2 \,\frac{g(q'^2,Q'^2,s,0)}{(q'^2-q^2)(Q'^2-Q^2)} \,. \tag{8}$$

We emphasize that the scattering process is kinematically constraint to $q^2<0$, i.e., to a region where A has no $q^2$ discontinuity, so that $i\epsilon$ prescriptions in this variable are irrelevant. This is no longer true for the annihilation process $e+\bar{e}' \to p+$"anything" given by the u discontinuity of the Compton amplitude. In this process we have $q^2>0$ so that for $q^2>q_t^2$ we have a $q^2$ discontinuity. This $q^2$ discontinuity is actually a forward direction limit of both the $q^2$ and $Q^2$ discontinuities. From Fig. 2 which pictures the annihilation process it is obvious that the $i\epsilon$ prescriptions have to be opposite in the variables $q^2$ and $Q^2$. This argument shows clearly that the annihilation process has to be obtained as a limit of the non-forward Compton amplitude. Thus, we have for the structure function $\bar{W}$ of the annihilation process

$$\bar{W}(q^2,u) = -\epsilon(u+M^2-q^2)\,\theta(u-s_t)\left[\frac{1}{\pi^3}\int_{q_t^2}^{\infty} dq'^2 \int_{q_t^2}^{\infty} dQ'^2 \,\frac{g(q'^2,Q'^2,u,0)}{(q'^2-q^2)(Q'^2-q^2)} + \frac{1}{\pi}\,\theta(q^2-q_t^2)\,g(q^2,q^2,u,0)\right]. \tag{9}$$

Already at this point, i.e., for finite virtual photon mass $q^2$, we can establish a <u>generalized crossing relation for structure functions</u>

$$\overline{W}(q^2, u) = - \mathrm{Re}\, W(q^2,u) - \frac{2}{\pi} \epsilon(u+M^2-q^2)\,\theta(u-s_t)\,\theta(q^2-q_t^2)\, g(q^2,q^2,u,0). \tag{10}$$

Here, a few remarks are in order:

1) this relation connects the structure functions for scattering and annihilation processes even for definite final hadron states, e.g., $e+p \to e'+p+\pi$ is related to $e+\bar{e}' \to p+\bar{p}+\pi$;

2) the generalized crossing relation replaces the naîve crossing relation (1a) which is suggested by the kinematics of the two processes;

3) the positivity of the cross-sections related to the structure functions is guaranteed by the expression (4) for the spectral function $g(q^2,Q^2,s,t)$;

4) the signs occurring in Eq. (10) depend on the Bose statistics of the scalar particles. They turn into plus signs for Fermi statistics.

The transition to the deep inelastic region (i.e., Bjorken limit) of the structure functions $W$ and $\overline{W}$ can now be studied in detail. To this end we assume that the triple discontinuity $g(q^2,Q^2,s,0)$ itself shows scaling behaviour of the following form

$$\lim_{\substack{s \to \infty \\ \frac{q^2}{s}, \frac{Q^2}{s} \text{ fixed}}} s^n g(q^2,Q^2,s,0) = \psi\left(\frac{q^2}{s}, \frac{Q^2}{s}\right). \tag{11}$$

The crossing relation (5) implies

$$\psi(x,y) = \psi(y,x). \tag{12}$$

Obviously, our approach is independent of any assumptions about leading light cone behaviour, nevertheless the power $n$ can be related to the power of the leading light cone singularity, e.g., $n=1$ corresponds to a behaviour of the form $\epsilon(x_0)\,\delta(x^2)$ (free field case), $n=0$ to $x_\rho\, \partial^\rho\, [\epsilon(x_0)\,\delta(x^2)]$ [current divergences in a free quark model [5]].

Introducing the scaling assumption (11) into Eqs. (8) and (10) we obtain the relations

$$\mathcal{F}(\omega)=\frac{1}{(\omega-1)^n}\,\frac{1}{\pi^3}\int_0^\infty dx\int_0^\infty dy\,\frac{\psi(x,y)}{\left(x+\frac{1}{\omega-1}\right)\left(y+\frac{1}{\omega-1}\right)}\;,\;(1\leq\omega<\infty), \tag{13}$$

and

$$\bar{\mathcal{F}}(\omega)=(-1)^n\,\mathcal{R}e\,\mathcal{F}(-\omega)+\frac{2}{\pi(1+\omega)^n}\,\psi\left(\frac{1}{1+\omega},\frac{1}{1+\omega}\right),\;(-1\leq\omega\leq 0), \tag{14}$$

where the scaling functions $F(\omega)$ and $\bar{F}(\omega)$ for scattering and annihilation, respectively, are defined by

$$\mathcal{F}(\omega)=\lim_{\substack{(-q^2)\to\infty\\ \omega\ \text{fixed}}}(-q^2)^n\,\epsilon(-q^2\cdot\omega)\,W(q^2,s)\;, \tag{15}$$

and

$$\bar{\mathcal{F}}(\omega)=\lim_{\substack{q^2\to\infty\\ \omega\ \text{fixed}}}(q^2)^n\,\epsilon(-q^2\cdot\omega)\,\bar{W}(q^2,u)\;. \tag{16}$$

We should like to comment these results with some remarks.

1) Scaling of a given type in the deep inelastic scattering region, characterized by the power $n$ in Eq. (11), implies the same type of scaling in the deep annihilation region. This is a direct consequence of the restriction to triple discontinuities in ansatz (3).

2) Equation (14) is the generalized crossing relation for scaling functions correcting Eq. (1b). The double spectral function $\psi(x,y)$ can directly be constructed from Eqs. (11) and (4), thus it is entirely given in terms of physical intermediate states, thereby taking into account the physical spectrum in all variables. Positivity constraints on $\psi(x,x)$ are obviously satisfied because of Eq. (4). Finally, we point out that $\psi(x,y)$ determines completely $F(\omega)$ and $\bar{F}(\omega)$.

3) Gatto and Preparata [3] discussed a relation of similar structure as Eq. (14). The difference to our work lies in two points
i) they start from a light cone expansion for the commutator, ii) their correction term occurring in the relation for $\bar{F}(\omega)$ is not given in a constructive form and only indirectly related to $F(\omega)$ by delicate limit procedures.

4) The representation (13) for the deep inelastic scaling function $F(\omega)$ shows that $F(\omega)$ has a cut for $-\infty < \omega \leq 1$, arising from normal discontinuities in the mass variables of the virtual photons. The deep inelastic scattering region lies along the real axis $1 \leq \omega < \infty$, i.e., to the right of the cut. According to the generalized crossing relation (14) the information about the annihilation structure function is given by the real part of the analytic continuation of $F(\omega)$ onto its cut between $0 \leq \omega \leq 1$ plus the term containing the double spectral function $\psi$ at equal arguments.

5) In models which do not exhibit a cut in $F(\omega)$ in the interval $0 \leq \omega \leq 1$ the generalized crossing relation (14) reduces to Eq. (1b), originally proposed by Drell, Levy and Yan [1]. In this case, the integrations in the representation (13) extend only over the intervals $0 \leq x,y \leq 1$.

6) Positivity constraints on $F(\omega)$ and $\bar{F}(\omega)$ are satisfied because of those fulfilled for $W$ and $\bar{W}$.

As a last point supporting our ansatz we should like to mention that the commutator $A(q^2,Q^2,s,t,u)$, Eqs. (6) and (7), taken to its Bjorken limit vanishes in forward direction in the interval $-1 < \omega < 1$, i.e., the sum of all discontinuities in Eq. (7) adds up to zero in this region. The same feature occurs in light cone models and canonical field theory.

For predictions of scaling functions one has to start from a model for the triple discontinuity $g(q^2,Q^2,s,0)$, Eq. (4). The scaling functions will then be given in all regions. Moreover, the approach to scaling, i.e., the non-leading terms in the Bjorken limit, is completely determined from a study of $W(q^2,s)$ and $\bar{W}(q^2,u)$ given by Eqs. (8) and (9).

In a forthcoming paper we will give a resonance model for the scaling functions in the deep inelastic scattering and deep annihilation region.

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## FIGURE CAPTIONS

Figure 1 :

Discontinuities of the virtual non-forward Compton amplitude

a) in the variable $q^2$;
b) in the variable $Q^2$;
c) in the variable $s$;
d) in the variable $u$.

Figure 2 :

Diagrams for the annihilation cross-section.

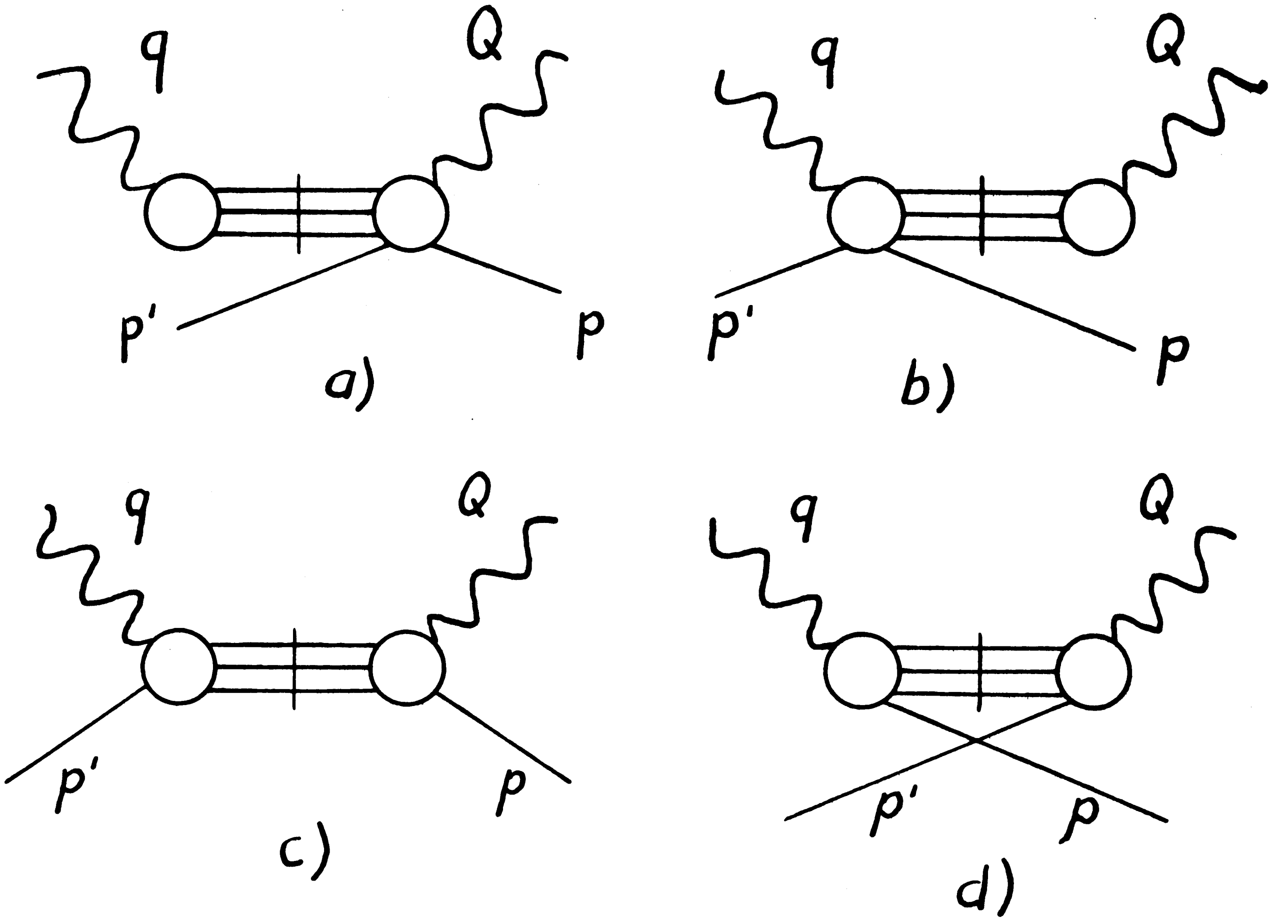


FIG. 1

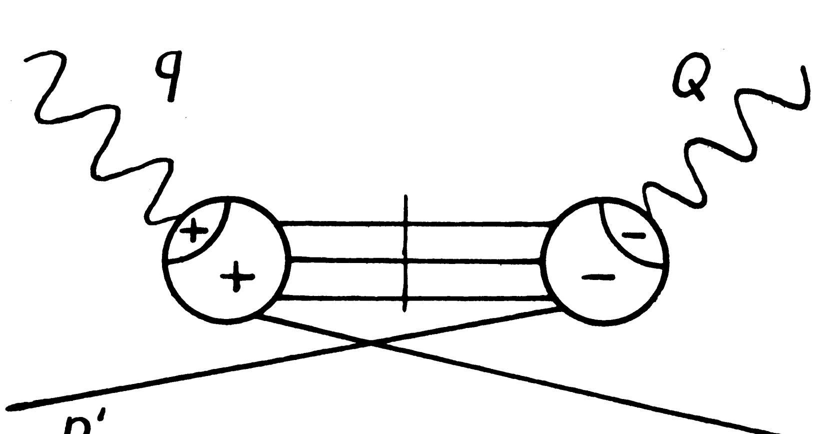


FIG. 2