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THE AMOUNT OF DIFFRACTION DISSOCIATION IN PROTON-PROTON SCATTERING AT 24 GeV/c

P. Pirilä and P.V. Ruuskanen *)

CERN - Geneva

A B S T R A C T

We estimate the amount of diffractive single and double dissociation in proton-proton scattering at 24 GeV/c in the context of the factorizable diffraction excitation model. We find that a realistic estimate for the maximal diffractive contribution is 1/3 of the total inelastic cross-section. The study of inclusive pion spectra shows that the possible early scaling cannot be attributed to production through diffractive excitations in any region of the phase space.

*) On leave of absence from the Department of Physics, University of Jyväskylä, Finland.

Address after 1 September 1972:
Department of Physics, University of Jyväskylä, Nisulankatu 78, Jyväskylä.

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I. INTRODUCTION

One of the present popular models of particle production is the diffraction excitation model [1),2),3)] (DEM) [*)]. Even though the model is supposed to be valid only at infinitely high energies, it has been often used to explain certain features of the data at presently common accelerator energies ($p_{lab} \lesssim 30$ GeV/c). This is especially the case with the Nova Model [3)] which tries to give a unified description of all the recently available experimental one and two-particle distributions in the range $p_{lab} = 10$-30 GeV/c. The successes in reproducing certain features of data with various versions of DEM, are claimed to manifest the importance or the dominance of diffractive production at these energies. Some predictions deal with gross features concerning all the data like the explanation of the flatness of the proton spectrum through the shape of the diffractive excitation spectrum [5)]. Others concern limited regions of phase space: e.g., the early scaling in the intermediate x range ($0.3 < x < 0.7$) is thought to be due to the energy independence of the excitation spectrum [3),6)].

The relevance of the arguments depends naturally on the extent of diffractive dominance. Otherwise, one can only conclude that diffraction and the relevant production mechanism, whatever it might be, have that same property or that the quantity in question is not a very sensitive test.

In the present work we try to estimate the maximum amount of diffractive production for proton-proton interactions at 24.0 GeV/c. In order to be able to do this directly from the data assuming only that diffraction is roughly energy independent, one would need data at different energies and for several exclusive channels. Such an analysis has been performed [7),8)] for some low multiplicity channels in πp scattering. For higher multiplicities the required data do not exist and the analysis would probably become too cumbersome to be practical. Our approach is much simpler and will strongly depend on the assumed properties of the diffraction.

The idea is simply to start from a region of phase space which most probably is dominated by diffraction, namely the region where both the missing mass M and the momentum transfer t of the proton are small, and to try to extrapolate to other parts of the phase space by using

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*) The name DEM is here used much in the same spirit as by Gottfried and Kofoed-Hansen in Ref. 4).

a model. Although the results thus obtained are strongly model dependent, they correspond to a consistent use of the stated properties.

To estimate the size of the diffractive single dissociation cross-section $2\sigma_1$, we essentially have to state the M dependence of the excitation spectrum. In our estimates we come to the conclusion that $2\sigma_1$ is at most 10 mb. It does not seem reasonable to assume that what remains of σ_{inel} would be dominated by the double dissociation cross-section σ_2. Moreover, we cannot point to any part of phase space where double dissociation dominates, and are, therefore, forced to make an extra assumption in order to normalize σ_2.

We will in fact take the differential cross-sections for the excitations to be factorizable [9]. In the case of low mass resonances produced through single dissociation the experimental results are consistent with factorization [8),10),11)]. Even assuming factorization the magnitude of σ_2 is harder to estimate because it is affected by the t dependence of the dissociation vertex which cannot be obtained directly from experiment for all masses.

To see the importance of diffractive particle production for the inclusive distributions, we calculate the pion spectra. Since we do not include kaons in the calculations, our results are somewhat too high, but they are still not high enough to fill in the spectra in any of the regions where we have data to compare with.

In Section II we describe the model and the method of calculation. In Section III, we explain how we have treated the data to put it into a form which is most suitable for the comparison with our calculations. In Section IV, results are given and their implications discussed.

II. THE MODEL

In the DEM the reaction mechanism is divided into two separate steps: the excitation of the incoming particle (or both of them) into a state with mass M, and its subsequent decay. We will call these heavy excited states fireballs without any reference to any particular theory of fireballs. To calculate physical quantities we have to specify how the fireballs are produced and how they decay. We use for actual numerical calculations Monte Carlo simulation of the individual events, which means in practice, that we have to specify the model to the last detail. Even though Monte Carlo methods are cumbersome, they guarantee that the numerical results correspond to the physical input of the model. At energies corresponding to incident momenta $p_{lab} \lesssim 30$ GeV/c we think that the approximations which must be made in other methods of calculation could affect the numerical results. In addition, when calculating different quantities, one has to make different approximations which means that the reliability of the results may vary depending on what has been calculated.

As stated in the Introduction, our purpose is primarily to study quantitatively the consequences of the ideas of DEM instead of trying to build a new model. However, since the details and the degree of specification of the DEM as used by different authors vary, and some of the versions might lead to different quantitative results, we will describe in detail the model as we have used it.

i) Production of Fireballs

We assume that the differential cross-section for one fireball production can be written in a factorizable form

$$\frac{d^2\sigma_1}{dM\,dt} = \frac{1}{16\pi}\, g^2 G^2(M)\, e^{(a+A(M))\cdot t} \qquad (1)$$

Here M is the mass of the fireball and t the momentum transfer in the process. In terms of Pomeron exchange $ge^{at/2}$ is the proton-proton-Pomeron vertex and $G(M)e^{A(M)t/2}$ the analogously defined proton-fireball-Pomeron vertex. The energy factor s^2 coming from Pomeron propagator is cancelled against the flux factor. The elastic differential cross-section is given by

$$\frac{d\sigma_{el}}{dt} = \frac{1}{16\pi}\, g^4 e^{2at} \qquad (2)$$

and the cross-section for simultaneous excitation of two fireballs with masses M_1 and M_2 by

$$\frac{d^3\sigma_2}{dM_1\,dM_2\,dt} = \frac{1}{16\pi}\, G^2(M_1)G^2(M_2)\, e^{(A(M_1)+A(M_2))\cdot t} \qquad (3)$$

The first important consequence of the factorized form is that there will be an effective cut-off both in $d\sigma_1/dM$ and $d^2\sigma_2/dM_1dM_2$ before the kinematical boundary due to the effect of t_{max} being different from zero. For the case of one fireball production this is rather well defined since the elastic slope a seems to be experimentally clearly larger than the inelastic slope $A(M)$ for large M. The parameter a which is well known then essentially determines the cut-off and the uncertainties in $A(M)$ have a small effect.

The second consequence of factorization is that the relative normalization of double and single dissociation is fixed. It is clear that factorization is not exact, but it should still give a rough estimate of the one-to-two ratio.

Actually, the function which we parametrize in order to specify the production strength is not $G^2(M)$ but $\rho(M)$ defined by *)

$$\frac{d\sigma_1}{dM} = \frac{g^2}{16\pi} \cdot \frac{G^2(M)}{a+A(M)}\, e^{t_{max}\cdot(a+A(M))} \equiv \rho(M)\, e^{t_{max}\cdot(a+A(M))} \tag{4}$$

The Monte Carlo calculations show that for values $M \lesssim 1.8$ GeV the missing mass spectrum of protons gets contribution only from the through going protons but not from protons coming from the fireballs. Because we want to estimate the maximal amount of diffraction we assume that the cross-section in this region is completely due to diffractive excitations. This assumption therefore equates $\rho(M)$ to the proton's missing mass spectrum for $M \leq 1.8$ GeV. For higher values of M the missing mass spectrum gets contributions from the excited systems too and cannot be used to determine $\rho(M)$.

As the only energy dependence in the model is in the growth of accessible phase space for the excitation of higher and higher masses (in addition to a possible shrinkage), $\rho(M)$ has to drop at least like M^{-1} in order to obey the unitarity bound. Since we will assume a decay mechanism in which the number of particles coming from the decay of a fireball grows linearly in M, we actually need a M^{-2} behaviour if we want the average multiplicity to grow like $\ell n\, s$ [3]. This behaviour can be argued also on the basis of duality [3] but it is not clear how relevant the duality arguments are for the production of high mass fireballs.

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*) We have ignored here the contribution from the lower bound of the t integration. It is correctly included in the numerical calculations.

Our simplest parametrization for the region $M > 1.8$ GeV is then the M^{-2} form normalized in such a way as to give, when extended through the low mass resonances, the experimentally observed cross-section in the interval between the two prominent resonance peaks at around 1.4 and 1.7 GeV. This, combined with an approximation of the low mass data by straight lines, is spectrum 1 in Fig. 1.

There is no experimental support for any strong high mass resonances, and the highest well-known diffractively produced resonance at 2.19 GeV [12] seems to be too weak even for an M^{-2} decrease [13]. We thus think that it is realistic to start the M^{-2} tail at 1.8 GeV as in spectrum 1. Since we try to establish an upper limit, we, however, consider also a possibility where we raise the normalization of the M^{-2} tail at $M = 2.5$ GeV as high as possible so that we saturate the missing mass spectrum when the protons coming from the excitations are added. This gives us the spectrum 2 in Fig. 1. There is no special reason for the value $M = 2.5$ GeV. We simply believe that this value is far enough above the threshold *) that if the M^{-2} behaviour is ever to have any meaning, it should majorize $\rho(M)$ at least from there on.

We then turn to the t dependence of the production. In a factorizable model, as mentioned already, the t dependence affects not only the shapes of distributions but also the magnitude of the cross-sections. For one-fireball production this is simply due to the exponential cut-off factor in Eq. (4). To see the situation in two-fireball production, we integrate Eq. (3), and write the result in terms of $\rho(M)$ **)

$$\frac{d^2\sigma_2}{dM_1 dM_2} = \frac{1}{2a\sigma_{el}}\,\rho(M_1)\rho(M_2)\,\frac{(a+A(M_1))(a+A(M_2))}{A(M_1)+A(M_2)}\; e^{t_{max}\cdot(A(M_1)+A(M_2))} \qquad (5)$$

So, in addition to the exponential factor from the upper limit t_{max}, we have also a weight factor depending on the slopes.

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*) Total cross-sections are typically rather smooth once the c.m. energy exceeds 2.3 GeV.

**) See Footnote on p. 4.

The elastic slope is well known and we have used the value $2a = 9.2\ \mathrm{GeV}^{-2}$. For the inelastic vertex, the inspection of the experimental data [14),15)] shows that except for the low mass region, $M \lesssim 1.8$ GeV, the slope is rather shallow $[A(M) \lesssim 1\ \mathrm{GeV}^{-2}]$ and could even go to zero with increasing excitation mass. Because of the $(A(M_1) + A(M_2))^{-1}$ factor in expression (5), the double fireball cross-section σ_2 depends crucially on the behaviour of $A(M)$. If $A(M)$ goes to zero too fast as M increases, σ_2 can even diverge with increasing energy. Here we do not try to draw any conclusions from the energy dependence since we do not know how to take properly into account the shrinkage which in the elastic scattering seems to persist [16)] at least up to the highest ISR energies. We cannot, however, let $A(M)$ be too small because the t distribution of the proton at large missing mass values then becomes too flat. The simple parametrization

$$A(M) = \frac{\alpha}{M - b}$$

with $\alpha = 0.75\ \mathrm{GeV}^{-1}$ and $b = 1$ GeV has been used in the numerical calculations. For higher missing mass values this leads to a similar t dependence as observed experimentally. At low masses the t dependence shows experimentally a break and a strong forward peak and there our parametrization reproduces the t distribution only in an average sense. We have also tried other parametrizations with essentially the same results.

ii) The Decay of Fireballs

The decay is assumed to be an isotropic chain decay independent of the production and of the mass of the fireball except close to the threshold. We will also assume that all the decay products are pions. Since the transverse motion of the fireballs does not contribute very much to the transverse momentum of the pions, the transverse momentum distributions directly give information concerning the decay distribution in the fireball rest frame. Strictly speaking, this is true only if fragmentation is the dominant mechanism but we anyway try to reproduce the shapes of the experimental p_T^2 distributions of pions. To do this we assume that the fireball decay distribution, as a function of the rest frame decay momentum, is a sum of two Gaussians. We do not have any physical motivation to use Gaussians. The reasons to use them are 1) Gaussians are easy to use from the computational point of view and 2) the sum of two Gaussians is a parametrization which is flexible enough for our purposes. The explicit parametrization which we use is

$$f(\vec{p}) = 0.6\left(\frac{15}{\pi}\right)^{3/2} e^{-15\vec{p}^2} + 0.4\left(\frac{4}{\pi}\right)^{3/2} e^{-4\vec{p}^2} \tag{6}$$

Since in our calculations at 24 GeV/c a large part of the fireballs have masses below 2.5 GeV (the effective upper limit due to the t_{max} effect is ~ 4 GeV) a considerable number of pions come from decays which are affected by the πN threshold. The assumption of the independence of the consecutive decays must break down when the previous decay leads to a mass close to the threshold since the probability of the last decay is then suppressed due to the smallness of the phase space which is available.

This suppression is introduced into our model by the following procedure: in our (Monte Carlo) calculation a decay of a fireball of mass M' that leads to a fireball with small mass M is accepted with probability $P \propto pe^{-ap^2}$, where p is the momentum available in the decay of the fireball with mass M to a nucleon and a pion. If the decay $M' \rightarrow M + \pi$ is not accepted it is replaced by a direct decay of M' to a nucleon and a pion. The maximum of P is normalized to one and the procedure is applied only when M is below the point which corresponds to this maximum. The factor p in P comes from the volume of the two-particle phase space. At this point we have replaced the two Gaussians of Eq. (6) by one because it simplifies the calculations essentially. We use the value $a = 10.6$, which is the weighted mean of 15 and 4 of Eq. (6).

We have checked that with this prescription the three-body decays below 2 GeV are similar to the three-body phase space and that the amount of two-body decays corresponds well to the amount of three-particle final states which we estimate from bubble chamber data [17] at $p_{lab} = 28.5$ GeV/c.

To determine the charges of the particles in the decay chain we assume that each fireball state is a mixture of $I = \frac{3}{2}$ and $I = \frac{1}{2}$ states with a ratio 2 which is the ratio of statistical weights $(2I + 1)$. The first state we take to be $I = \frac{1}{2}$, and let it decay with probability 2/3 to $I = \frac{3}{2}$ states and 1/3 to $I = \frac{1}{2}$ states. We also treat the final nucleon correctly as a $I = \frac{1}{2}$ state. The way charges are assigned in the decays of a high mass fireball is probably not very important since we have a long decay chain, but for low mass excitations the number of neutral pions and the neutron to proton ratio depends on the relative amount of $I = \frac{3}{2}$ and $I = \frac{1}{2}$ states.

iii) The Method of Calculation

As the physical model which we have adopted corresponds to a process that consists of several rather simple steps, it easily allows for a straightforward Monte Carlo simulation. We first determine randomly the mass of the fireball (or two masses for double excitations) with probability density $\rho(M)$, and the direction of its momentum from the probability density $e^{t(a+A(M))}$ for t $[e^{(A(M_1)+A(M_2))t}$ for double excitations$]$. Then we let the fireball (fireballs) decay as described in ii). After that we determine the charges of the particles, and finally we transform all the momenta to the c.m. system. With this method it is possible to take proper care of all the kinematical effects, like the t_{max} effect, and the recoil in the fireball decay. Each event exactly fulfils energy-momentum conservation.

At one point only do we find it necessary to abandon the idea of generating events with exactly that probability distribution which our model predicts: we take care of the factor $(a+A(M_1))(a+A(M_2))/(A(M_1)+A(M_2))$ of Eq. (5) by giving the corresponding weight to all two-fireball events. The almost complete omission of weights makes our program very efficient compared to most other Monte Carlo programs, (e.g., FOWL [18] when multiplicity corresponds to our mean multiplicity). In addition we can easily calculate any inclusive, semi-inclusive or exclusive distribution. The program can be used at least up to the energies corresponding to $p_{lab} \approx 200$ GeV/c. A similar program has been used by Adair [19] in his calculations.

III. THE DATA

As mentioned above, we get the most direct information concerning the production of baryonic fireballs from the missing mass distributions of the reaction $p+p \to p+$ anything. The best experiments for determining this distribution at $p_{lab} \gtrsim 20$ GeV/c are the counter experiments of Refs. 14) and 15) at 24 GeV/c. As can be seen from Fig. 2 these experiments cover almost completely that part of the M-t plane where the cross-section is non-negligible. The only exceptions are at t very near to t_{max} $(t > t_{max} - 0.1)$ and in the high missing mass large angle area $(M \gtrsim 5$ GeV, $t \lesssim -5$ GeV$^2)$.

In order to make reliable comparisons with our numerical calculations possible, we have interpolated (in some cases also extrapolated) and integrated the data, thus getting the experimental distributions presented in this paper. The proton distributions which we present have been determined directly from the measured points, whereas the pion spectra are integrated from the interpolated data as given by the experimental group [15].

For the interpolation and integration of the proton data we have used non-linear methods (three point formulae and exponential functions). The interpolation error is less than 1% except when $M > 5.8$ GeV, i.e., in the rapidly decreasing part of $d\sigma/dM$. As the data do not cover the whole physical region some extrapolations have been necessary to determine $d\sigma/dM$. We estimate the uncertainty of $d\sigma/dM$ due to these extrapolations to be ~6% at $M = 1.3$ GeV, 3% at $M = 1.5$ GeV, 2% at $M = 2.0$ GeV, less than 1% when 3 GeV $< M <$ 5 GeV, 1.5% at $M = 5.5$ GeV, and less than 1% when 5.7 GeV $< M <$ 5.8 GeV. Below 4.5 GeV the uncertainty is mainly due to the extrapolation to t_{max} (i.e., forward direction), and the error estimates are based on the assumption that there is no strong structure in the forward direction $t > t_{max} - 0.1$ GeV^2.

The x distributions of the protons have been determined by the same methods but no extrapolations have been necessary, interpolation errors are < 1%. The pion data are integrated by trapezoidal rule and the errors due to the integration are $\lesssim$ 2%.

The systematic errors in the original experiment are 3-5% dominating thus over the errors due to our treatment of the data almost everywhere. The points near the threshold ($M < 1.3$ GeV) are less reliable due to the contamination from elastic events. The statistical errors are negligible in integrated distributions. In addition there is an over-all normalization error of 12-15%.

IV. RESULTS AND CONCLUSIONS

In this Section we present the results of the numerical calculations and draw our conclusions. First we discuss the magnitude of the single and double dissociation cross-sections, and the possibility of saturating the total inelastic cross-section with fragmentation. We then study the inclusive pion spectra in order to see whether there are any regions of phase space where early scaling can be attributed to the energy independence of diffraction.

i) The Amount of Diffraction

Given the functions $\rho(M)$ and $A(M)$ we can determine $2\sigma_1$ and σ_2 simply by integrating the expressions (4) and (5) over the physical values of produced masses. In the case of the lower spectrum (spectrum 1) we get $2\sigma_1 = 7.8$ mb, $\sigma_2 = 1.8$ mb and $\sigma_d = 2\sigma_1 + \sigma_2 = 9.6$ mb. The results corresponding to spectrum 2 are $2\sigma_1 = 9.3$ mb, $\sigma_2 = 2.4$ mb and $\sigma_d = 11.7$ mb. Experimentally the total inelastic cross-section is 30 mb. To see how the calculated cross-sections are built up we have plotted in Fig. 3 the various contributions to the missing mass spectrum in case 2, as well as the experimental points. It is rather clear from this plot that with a factorizable model which necessarily introduces the t_{max} effect one cannot hope to fill in the spectrum and that the protons that are missing correspond to very high values of missing mass. The same effect is seen in Fig. 4 where we present the invariant cross-section as a function of x averaged over p_T^2 bins:

$$\langle f(x, p_T^2)\rangle_{\Delta p_T^2} = \frac{1}{\Delta p_T^2} \int dp_T^2 \, E \frac{d^3\sigma}{d\vec{p}^{\,3}} \tag{7}$$

Here our distributions are much too low at small $|x|$.

Having seen that most of the inelastic cross-sections at 24 GeV/c must be produced by other mechanism than diffractive excitations, it seems improbable that these mechanisms would have no contribution to $d\sigma/dM$ at $M \leq 2.5$ GeV. Thus our higher estimate is probably somewhat too high *).

In order to see the size of the effect due to the exponential cut-off in t_{max} we calculated $2\sigma_1$ for spectrum 2 without this cut-off. The result is 12.0 mb (compared to 9.3 mb with cut-off), so that even in this case we get at most 40% single diffraction in the total inelastic cross-section.

*) In addition we should note that the experimental data which we have used seem to have somewhat too high normalization, since the integration of $d\sigma/dM$ gives $\langle n_p\rangle \sigma_{inel} = 42.6$ mb. Using $\sigma_{inel} = 30$ mb this leads to $\langle n_p\rangle = 1.42$. The value given by Scandinavian Collaboration [20] at 19 GeV/c is 1.41 ignoring kaons, however. Assuming $K^-/\pi^- = 0.03$ and $K^+/\pi^+ = 0.085$ we get $\langle n_p\rangle = 1.33$.

The results of Gottfried and Kofoed-Hansen in Ref. 4) on the amount of diffraction from the normalization at the high energy limit are similar to ours. We also note that a study made by M. Uehara [21] for πp scattering at 16 GeV/c, using as a starting point the known diffractive contributions in exclusive channels, leads qualitatively to the same conclusion.

ii) Pion Production

Having found that the diffractive excitations do not dominate the whole inelastic cross-section, we are still left with the question, whether the pions produced with this mechanism could dominate the single particle distributions in some regions of phase space, as has been proposed [3),6),22)].

In Figs. 5 and 6 we present the calculated and experimental π^+ spectra. The quantity which has been plotted is the invariant cross-section averaged over the indicated bins as defined by Eq. (7) for x distributions, and in a similar way for p_T^2 distributions. The calculated spectra correspond to the higher $\rho(M)$ distribution (spectrum 2). It can be seen that the calculated spectra are everywhere well below the experimental points. The situation for negative pions is similar.

The only way of increasing the number of pions, when the function $\rho(M)$ is fixed, is to decrease the mean energy in the decay of fireballs. This will affect the p_T^2 spectra. As seen in Fig. 5, our parametrization, Eq. (6), fits the shapes rather well. We can thus produce only slightly more pions without getting too steep p_T^2 distributions. Our conclusion from Fig. 6 is that the pions produced by this mechanism do not dominate in the intermediate x region $(0.3 < x < 0.7)$ as proposed by Jacob and Slansky [3] and Berger [6]. The difference seems to be still larger in the wee-x region discussed by Hwa and Robertson [22], although the experimental points which we have used do not extend to this region.

To summarize: we have studied the amount of diffraction dissociation in proton-proton interactions at 24.0 GeV/c using a factorizable diffractive excitation model. Our results show that the maximal amount at this energy is 1/3 of the total inelastic cross-section. The investigation of the pion spectra shows, moreover, that the pion production is not dominated in any part of the phase space by diffraction dissociation.

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REFERENCES

1) R.K. Adair, Phys.Rev. 172, 1370 (1968).

2) R.C. Hwa, Phys.Rev. D1, 1790 (1970).

3) M. Jacob and R. Slansky, Phys.Rev. D5, 1847 (1972).

4) K. Gottfried and O. Kofoed-Hansen, Proton spectra according to the diffraction excitation model, CERN preprint TH.1514 (1972).

5) R.C. Hwa and C.S. Lam, Phys.Rev.Letters 27, 1098 (1971).

6) E.L. Berger, Fragmentation models for particle production, ANL/HEP 7220, Invited summary to Oxford Conference (April 5-7, 1972).

7) W. Kittel, S. Ratti and L. Van Hove, Nuclear Phys. B30, 333 (1971).

8) E. De Wolf, Y. Goldschmidt-Clermont, F. Grard and F. Verbeure, An LPS analysis of the reactions $K^+p \to K^+\pi^+\pi^-p$ and $K^+p \to K^0\pi^+\pi^0p$, CERN preprint CERN/D.Ph.II/Phys. 72-23 (1972).

9) L. Van Hove, Ann.Phys. 66, 449 (1971).

10) N.K. Yamdagni and S. Ljung, Phys.Letters 37B, 117 (1971).

11) P.G.O. Freund, Phys.Rev.Letters 21, 1375 (1968).

12) R.M. Edelstein, R.A. Corrigan, Jr., N.C. Hieu, T.J. McMahon, I. Nadelhaft, E.W. Anderson, E.J. Bleser, G.B. Collins, T. Fujii, J. Menes and F. Turkot, Phys.Rev. D5, 1073 (1972).

13) P.H. Frampton and P.V. Ruuskanen, Phys.Letters 38B, 78 (1972).

14) U. Amaldi, R. Biancastelli, C. Bosio, G. Matthiae, J.V. Allaby, A.N. Diddens, R.W. Dobinson, A. Klovning, J. Litt, L.S. Rochester, K. Schlüpmann and A.M. Wetherell, Phys.Letters 34B, 435 (1971).

15) J.V. Allaby, A.N. Diddens, R.W. Dobinson, A. Klovning, J. Litt, L.S. Rochester, K. Schlüpmann, A.M. Wetherell, U. Amaldi, R. Biancastelli, C. Bosio and G. Matthiae, Comparison of momentum spectra of secondary particles produced in proton-proton collisions at 14.2, 19.2 and 24.0 GeV/c. Contribution to the Oxford Conference (April 5-7, 1972).

16) G. Barbiellini, M. Bozzo, P. Darriulat, G. Diambrini Palazzi, G. de Zorgi, A. Feinberg, M.I. Ferrero, M. Holder, A. McFarland, G. Maderni, S. Orito, J. Pilcher, C. Rubbia, A. Santroni, G. Sette, A. Stande, P. Strolin and K. Tittel, Phys.Letters 39B, 663 (1972).

17) T.W. Morris, R.S. Panvini, F. Bomse, E. Moses, E. Salant, J. Waters and M. Webster, Analysis of low multiplicity pd interactions at 28.5 GeV/c, Contribution to the International Conference on High Energy Physics, Kiev (1970).

18) F. James, FOWL - A general Monte-Carlo phase space program, CERN program library (1970).

19) R.K. Adair, Phys.Rev. D5, 1105 (1972).

20) H. Boggild, E. Dahl-Jensen, K.H. Hansen, J. Johnstad, E. Lohse, M. Suk, L. Veje, V.J. Karimäki, K.V. Laurikainen, E. Riipinen, T. Jacobsen, S.O. Sørensen, J. Allan, G. Blomquist, O. Danielsen, G. Ekspong, L. Granstrøm, S.O. Holmgren, S. Nilsson, B.E. Ronne, U. Svedin and N.K. Yamdagni, Nuclear Phys. B27, 285 (1971).

21) M. Uehara, Role of diffractive excitation mechanism in multiple production of hadrons, Kyushu University preprint, Kyushu-72-HE-10 (July, 1972).

22) R.C. Hwa and D.C. Robertson, The question of black-body radiation in inclusive reaction in the wee-x region, University of Oregon preprint (March, 1972).

FIGURE CAPTIONS

Figure 1 :

The excitation spectra $\rho(M)$ which have been used in the calculations. Dots are the experimental points for $d\sigma/dM$ of proton. (See Section III.)

Figure 2 :

The points where the proton spectra have been measured. The continuous lines give the data points of Ref. 14) and there are typically 75 points on each line. The separated points are from Ref. 15).

Figure 3 :

The missing mass spectrum of the proton. For data points see Section III. The solid line is the calculated $d\sigma/dM$. Contributions from through going protons (-...-), protons coming from fireballs in the case of single excitations (---), and those corresponding to double excitations (-.-), are shown separately.

Figure 4 :

The invariant cross-section of protons as a function of x averaged over the indicated p_T^2 regions. The solid lines are the calculated distributions. Contributions from through going protons (-...-), protons coming from fireballs in the case of single excitations (---), and those corresponding to double excitations (-.-), are shown separately.

Figure 5 :

The invariant cross-section of positive pions as a function of p_T^2 averaged over the indicated x regions.

Figure 6 :

The invariant cross-section of positive pions as a function of x averaged over the indicated p_T^2 regions.

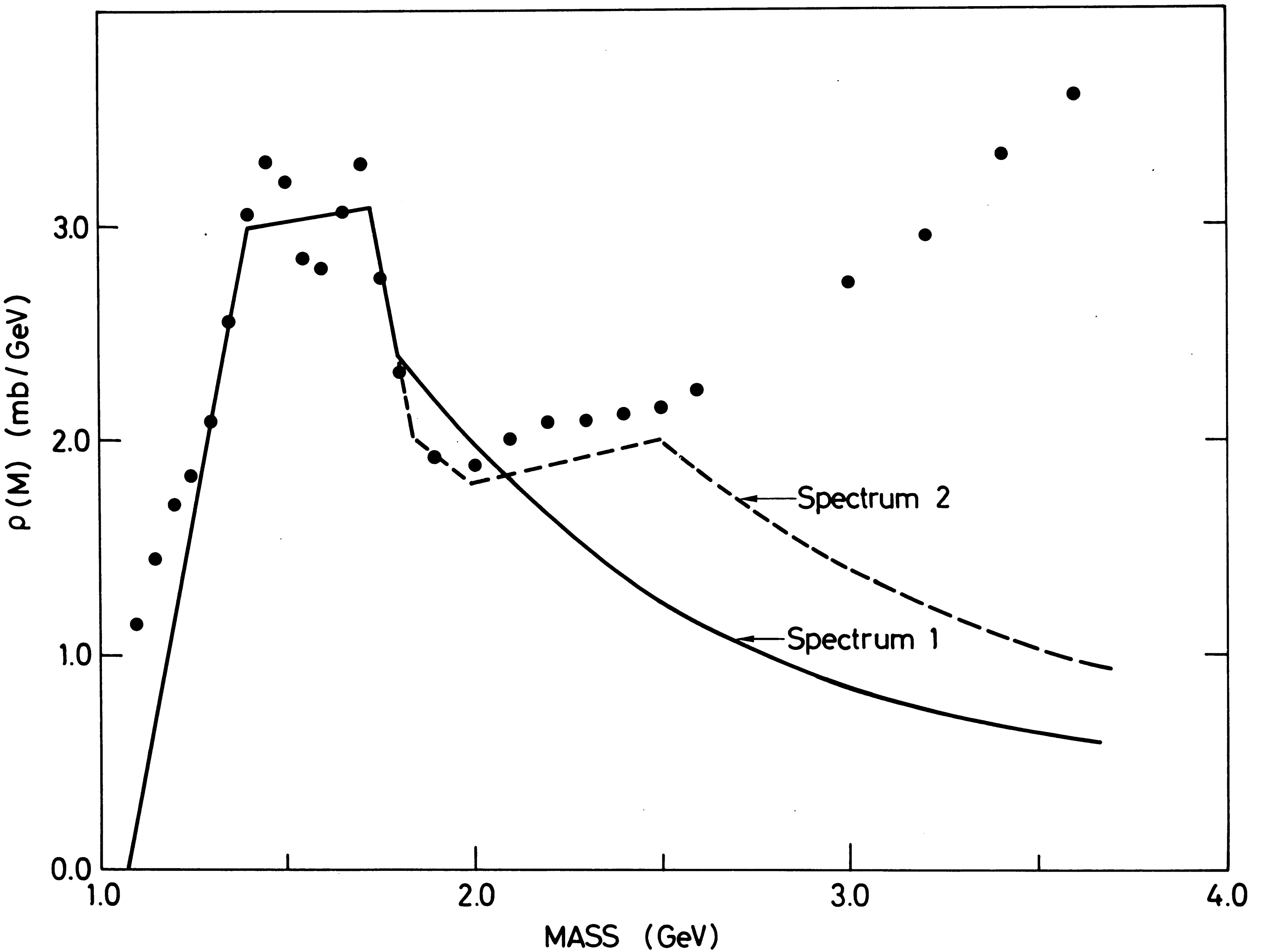


FIG.1

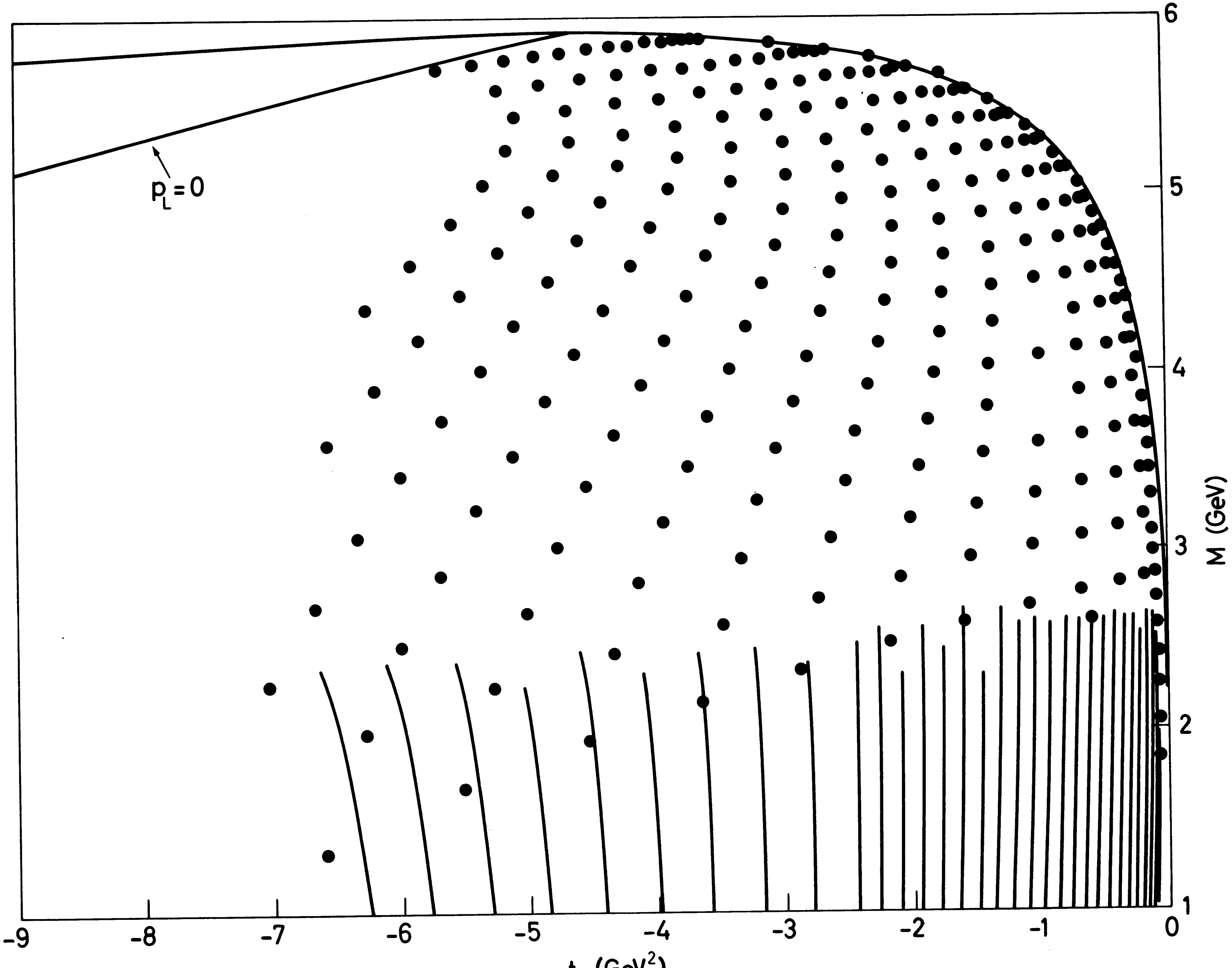


FIG. 2

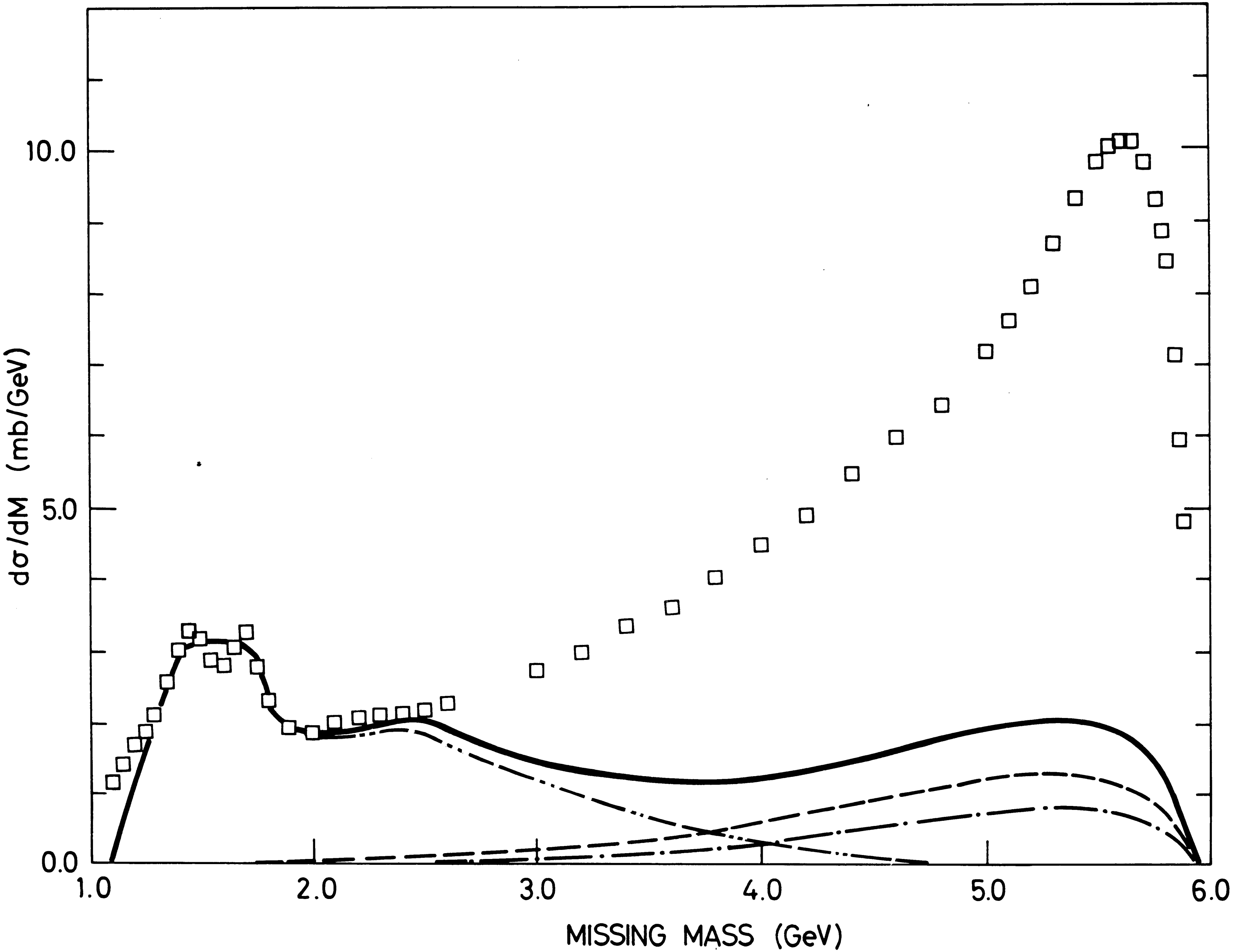
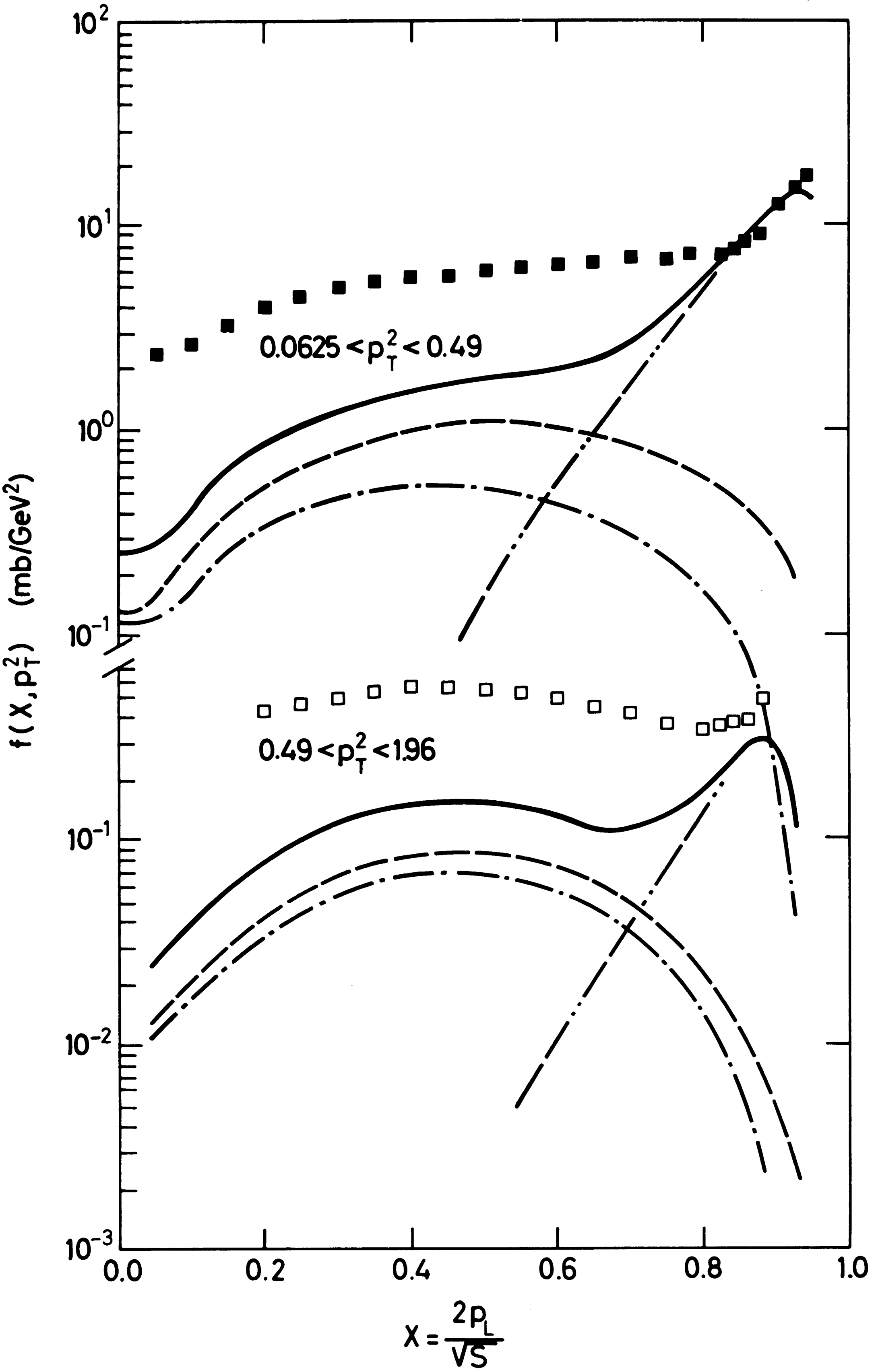
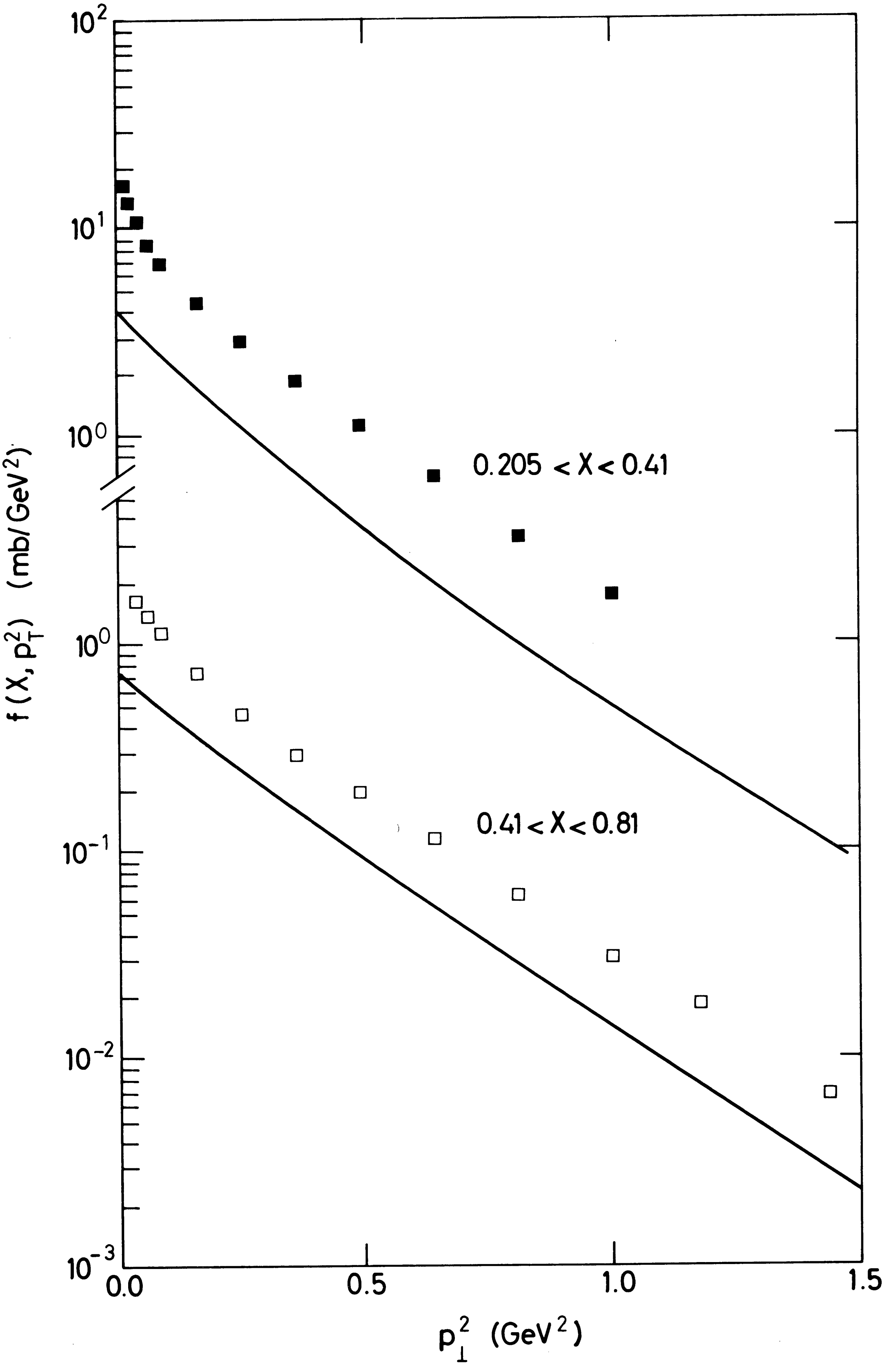


FIG.3



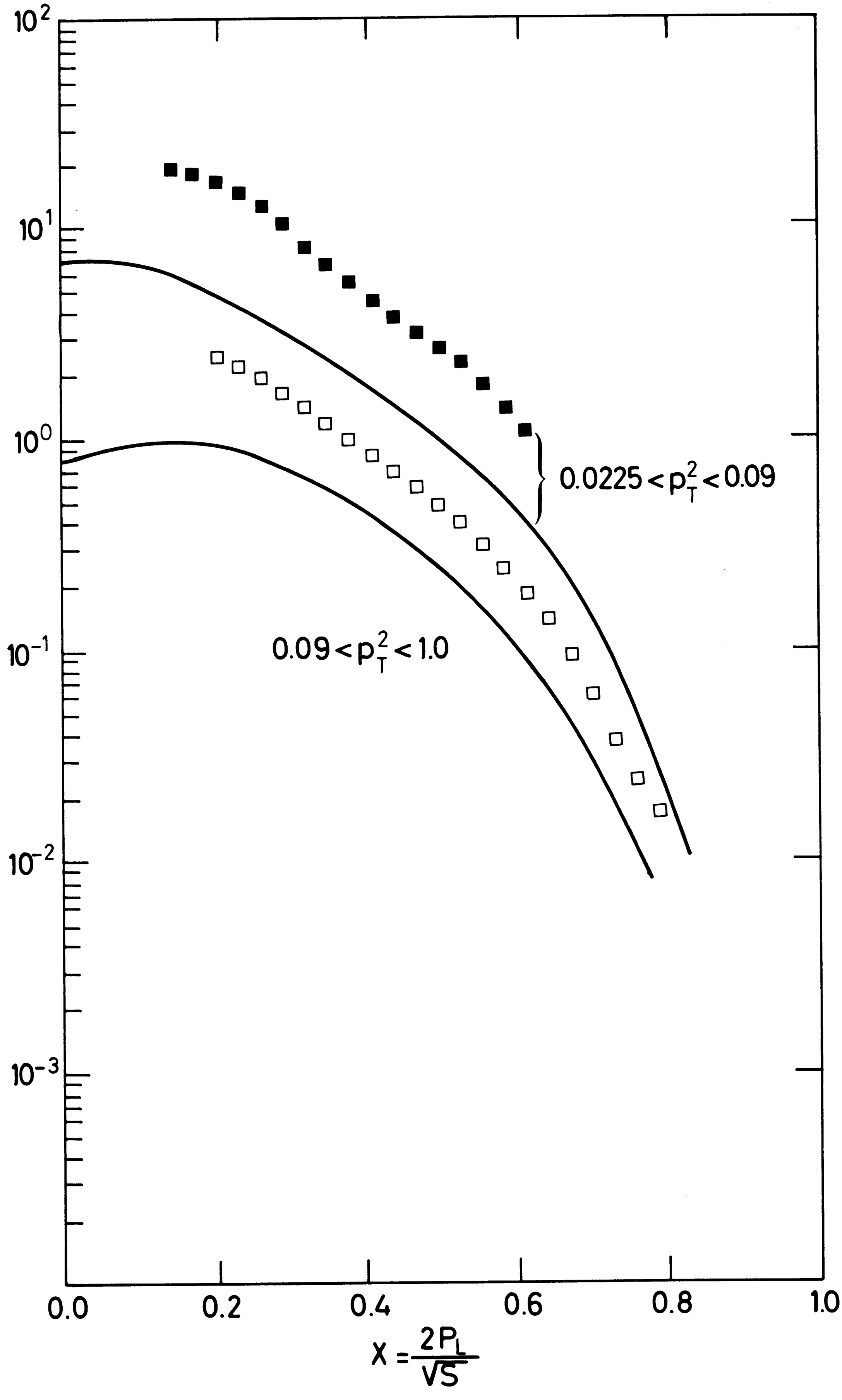
10^2
10^1
10^0
10^{-1}
10^{-1}
10^{-2}
10^{-3}
$f(X, p_T^2)$ (mb/GeV^2)
$0.0625 < p_T^2 < 0.49$
$0.49 < p_T^2 < 1.96$
0.0
0.2
0.4
0.6
0.8
1.0
$X = \frac{2p_L}{\sqrt{S}}$

FIG.4



f (X, p_T^2) (mb/GeV^2)
10^2
10^1
10^0
10^0
10^{-1}
10^{-2}
10^{-3}
0.205 < X < 0.41
0.41 < X < 0.81
0.0
0.5
1.0
1.5
$p_\perp^2$ (GeV^2)

FIG.5



10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
$0.0225 < p_T^2 < 0.09$
$0.09 < p_T^2 < 1.0$
0.0
0.2
0.4
0.6
0.8
1.0
$X = \frac{2P_L}{\sqrt{S}}$

FIG. 6