Ref.TH.1716-CERN

CONSTRUCTION OF POMERON STATES IN THE ZERO-WIDTH APPROXIMATION

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A B S T R A C T

We present a kinematical construction for the Pomeron in models of relativistic composite particles in the zero-width approximation.

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Ref.TH.1716-CERN

27 July 1973

In a previous publication [1] we have stressed the kinematical approach to building quantum relativistic models of composite particles in the zero width approximation. The essential feature was the construction of the generators of the Poincaré group corresponding to a null plane quantization in field theory [2], that is one when the Hamiltonian formalism is implemented by preparing the system on a wave front [3]. In null plane co-ordinates, the generators are given by

$$\begin{aligned}
P^+ &= \frac{1}{p^-}(M^2 + \vec{p}^{\,2}) \\
P^- &= p^- \\
\vec{P} &= \vec{p} \\
M^{12} &= x^1 p^2 - x^2 p^1 + T^3 \\
M^{-+} &= -\tfrac{1}{2}\{x^+, p^-\} \\
M^{i-} &= x^i p^- \qquad\qquad (i=1,2) \\
M^{i+} &= \tfrac{1}{2}\{x^i, P^+\} - x^+ p^i - \frac{2\varepsilon^{ij}}{p^-}(p_j T^3 + T^j)
\end{aligned} \tag{1}$$

Here x^+, $\vec{x}$ label the three co-ordinates of the centre-of-mass of the composite particle. They are of course conjugate to the momenta

$$\begin{aligned}
&[x^+, p^i] = [x^+, x^i] = [x^i, x^j] = [p^i, p^j] = 0 \\
&[x^+, p^-] = -2i \quad , \quad [x^i, p^j] = i\delta^{ij}
\end{aligned} \tag{2}$$

The remaining operators M^2, T_3 and $\vec{T}$ all commute with the centre-of-mass variables and can therefore be understood as being constructed out of "relative co-ordinates". However, they must obey

$$[T^3, T^i] = i\varepsilon^{ij} T^j \tag{3a}$$

$$[T^1, T^2] = i M^2 T^3 \tag{3b}$$

while M^2, being a Casimir operator, commutes with everything. If one can construct these operators in terms of canonical variables acting in a positive definite Hilbert space, the problem is solved. To make contact with nature, one must require that the states of the system lie on linearly rising Regge trajectories. This means that the eigenvalues of M^2 have to be evenly spaced, which brings in harmonic oscillator representations for these operators. Under these requirements, it has not been proved possible to build simple representations for T_3, $\vec{T}$ and M^2. The simplest ones are those constructed in dual models and their algebras close only by adding ad hoc degrees of freedom or by enlarging the number of operators which correspond to 26 and 10 space-time dimensions, depending on the model [4]. For the moment, we will assume we have a satisfactory representation for these operators and we will draw certain conclusions imposed by the kinematics.

Specifically, we consider the decay of a composite particle (in the sense defined above) into two others. We shall label the Poincaré group generators by an additional subscript denoting the particle (a, b or c). For simplicity we operate in the momentum representation and in the rest frame of particle "a". We make an additional rotation so that the decaying particles move in the z direction.

The constraints are

$$\frac{1}{p_a^-} M_a^2 = \frac{1}{p_b^-} M_b^2 + \frac{1}{p_c^-} M_c^2 \tag{4a}$$

$$M_a = p_a^- = p_b^- + p_c^- \tag{4b}$$

$$T_a^3 = T_b^3 + T_c^3 \tag{4c}$$

Note that the last equation can be understood to express one of the little group generators for particle a in terms of those of particles b and c. It is then tempting to ask if the same simple additive rule can be extended to the remaining generators $\vec{T}_a$. Let us set

$$\vec{T}_a = \alpha(\vec{T}_b + \vec{T}_c) \tag{5}$$

where d is some constant to be determined later. Using the expression for T_a^3, it is clear that $\vec{T}_a$ transforms as a two-vector under it. Further we have

$$[T_a^1, T_a^2] = i d^2 (M_b^2 T_b^3 + M_c^2 T_c^3) \tag{6}$$

Then T_a^1 and T_a^2 can serve as little group generators only if we restrict ourselves to the subspace

$$d^2 M_b^2 = d^2 M_c^2 = M_a^2 \tag{7}$$

This is kinematically allowed only if $p_b^- = p_c^-$ which corresponds to the case of particle a "dividing up" like a cell into two equal parts into its rest frame. There, we have, from momentum conservation

$$M_a^2 = 2 M_b^2 + 2 M_c^2 \tag{8}$$

which fixes

$$d = 2 \tag{9}$$

To see what our construction corresponds to, let us assume that the states of particles b and c lie on straight line Regge trajectories with intercepts $\alpha_b(0)$ and $\alpha_c(0)$ respectively, and the same slope α'. For simplicity consider a state of $|b>$ of highest SU_2 weight, i.e.,

$$T_b^{1+i2} |b> = 0 \tag{10}$$

Then we have

$$T_b^3 |b> = (\alpha_b(0) + \alpha' M_b^2) |b> \tag{11}$$

Similarly, let $|c>$ be a state which obeys

$$T_c^{1+i2} |c> = 0 \tag{12a}$$

$$T_c^3 |c> = (\alpha_c(0) + \alpha' M_c^2) |c> \tag{12b}$$

Now form the composite state $|b>|c>$, which obeys therefore

$$T_a^{1+i2} |b>|c> = 0 \tag{13a}$$

$$T_a^3 |b>|c> = \left[\alpha_c(0) + \alpha_b(0) + \frac{\alpha'}{2} M_a^2\right] |b>|c> \tag{13b}$$

Hence we obtain the result that the states of particle a lie also on Regge trajectories but (in the case of identical b and c) with twice the intercept and half the slope. This is precisely the case of the Pomeron in nature with b and c having the leading ρ trajectory with $\alpha(0) = \alpha_\rho(0) = \frac{1}{2}$.

It is clear that our argument is purely kinematical and it must be implemented with $\alpha_a(0) \leq 1$ in order to satisfy unitarity - but this is a dynamical requirement which is beyond the scope of this paper.

We proceed to illustrate our contribution with the only models that attempt to build relativistic composite particles, the dual models. In these, the little group operators attain their simplest form when the number of spacelike dimensions attains certainly critical values, although it is possible to build such little groups for any dimension less than the critical one [5]. For simplicity, we will consider the little groups in the critical number of dimensions since it does not alter our argument. In such cases, the little groups have generators T_i and T_{ij} $i,j = 1,2,\ldots\ d_c-2$, which obey

$$\begin{aligned} [T_{ij}, T_{kl}] &= i(\delta_{ik}T_{jl} + \delta_{jl}T_{ik} - \delta_{il}T_{jk} - \delta_{jk}T_{il}) \\ [T_{ij}, T_l] &= i(\delta_{il}T_j - \delta_{jl}T_i) \\ [T_i, T_j] &= i M^2 T_{ij} \end{aligned} \tag{14}$$

and our construction generalizes trivially.

In the case of mesonic particles, the leading trajectory is always at intercept $\alpha(0) = 1$, which means that our constructed particle has intercept at $\alpha(0) = 2$. Further, investigation of the spectrum for our construction reveals it to be the same as that of the Pomeron for these models as well as that encountered in the Virasoro-Shapiro models [6]. Clearly our constructed Pomeron has no negative norm states if the constituent (composite) particles are themselves ghost-free [7]. In the fermion dual model, the leading trajectory has intercept at $\frac{1}{2}$ [8], which makes our construction have intercept at $\alpha(0) = 1$ - it is, of course, a boson. It corresponds to the fermion-antifermion annihilation channel and, like the photon in quantum electrodynamics has odd charge conjugation. However, in these models, charge conjugation becomes the "G parity" in the Neveu-Schwarz sector of the model. Hence our construction could be identified as an "odd G parity" Pomeron with intercept 1 [9].

In conclusion, we have shown on very general grounds that, in the zero width approximation, a particle to be identified with the Pomeron appeared quite naturally with half the slope and twice the intercept of the corresponding Reggeon. We stress that our construction is not unique and only refers to a particular kinematic situation

ACKNOWLEDGEMENTS

The author thanks Professor D. Olive for reading the manuscript and enlightening him on the odd G parity Pomeron during various discussions.

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One can see this by looking at the expressions for the T_i in the little group - they annihilate the ground state while T_{ij} do not because of the $\gamma_i \gamma_j$ term. Therefore, the only consistent solution is for the mass of the ground state to be zero.

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