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FACTORISATION OF TOTAL CROSS-SECTIONS AT HIGH ENERGY

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A B S T R A C T

We show that, within a consistent picture of a Pomeron pole of intercept one which evades the decoupling arguments, total cross-sections should factorise in the energy regime where three and more Pomeron cuts are negligible.

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There has recently been considerable discussion on the consistency of an intercept one Pomeron pole with unitarity. It has been claimed [1] that such a pole should decouple from elastic processes at $t = 0$, and, therefore, from total cross-sections. On the other hand, Gribov [2] has shown how the pole may only decouple from inelastic processes. This mechanism also predicts that all total cross-sections should approach the same constant, which means that we are far from asymptopia in the present energy regime. However, the successful fitting of the rising pp cross-section at ISR energies with a simple Pomeron pole and two-Pomeron cut [3],[4], suggests that this is not the case.

In another paper [5], we present a consistent picture of a Pomeron pole of intercept one, together with its cuts, which consistently evades the decoupling arguments. The only important vanishing is that of the triple-Pomeron coupling at zero external masses. We view this essentially as a t channel constraint, that the Pomeron cuts should not modify too much the trajectory of the pole. In the framework of Gribov's Reggeon calculus [6] this can be seen as the condition that the renormalised skeleton graph expansion converges at $t = 0$. We differ from Gribov in that we envisage a specific dynamical mechanism acting within the bare Reggeon calculus to produce the zero, along the lines of Bronzan's analysis [7].

The main results of Ref. 5) are:

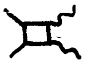
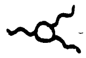
i) the Pomeron has no serious decouplings and in particular does not decouple from total cross-sections;

ii) total cross-sections should have the asymptotic form

$$\sigma_{ab} = g_a g_b \left[1 - \frac{\gamma^2}{\ln s} \right] + O\left(\frac{1}{(\ln s)^2} \right) \qquad (1)$$

where g_a, g_b depend only on the external particles a and b, and γ is a universal number. Thus in the energy regime where multi-Pomeron cuts are negligible, which we seem to be approaching in pp scattering at ISR energies, we expect total cross-sections to factorize and tend to their asymptotic limits proportionally.

We proceed to discuss briefly how these results come about. Bronzan [7] envisages the triple-Pomeron coupling appearing as the result of summing the bare diagrams [*] of Fig. 1, which satisfy the integral equation of Fig. 2. The exchanged Pomerons could be replaced by a general one and two-Pomeron irreducible kernel. Both the two-Pomeron cut and the singular potential induced by the Pomeron exchange force the solution of the integral equation to have the triple-Pomeron zero. In the specific model of Bronzan [7] the solution has several undesirable features, notably the generation of an infinite number of other poles by the ladder sum, which accumulate at $j = 1$ when $t = 0$. However, we assume that in a more realistic model with softened vertices such difficulties disappear, and that the triple-Pomeron coupling has a simple linear zero [**].

In evaluating the elastic amplitude in a Reggeon calculus model, one begins with the bare vertices illustrated in Fig. 3. These will always appear additively as shown. To these must be added all two-Pomeron iterations, with and without the pole, to give a sum of diagrams like Fig. 4, where, in Bronzan's model, the square bubble would represent the sum of diagrams like Fig. 5. On taking the discontinuity across the two-Pomeron cut, this sum can be re-arranged to give Fig. 6, where the new renormalized vertices and propagator are defined in Figs. 7 and 8, and the crosses indicate that the Pomerons are taken on mass shell. It is clear that  vanishes at $t = 0$ by exactly the same mechanism as the triple Pomeron coupling  . Thus the only surviving terms in the amplitude at $t = 0$ are those illustrated in Fig. 9, where we have also included the renormalised pole term. At $t = 0$ the first diagram gives the term $g_a g_b$ in (1) and the second has a logarithmic j plane cut giving the $O(1/\ln s)$ term.

At this stage we should point out the essential difference between our result and that of Gribov and Migdal [6] who do not have total cross-sections factorising in the manner of Eq. (1). These authors regard

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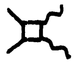
*) Both the propagators and vertex functions in these diagrams are bare with respect to Pomeron cut renormalisations, but because this renormalisation of the propagator is weak the bare intercept is already one.

**) In Ref. 5) we give a more general analysis of such a model using two-Pomeron unitarity.

the triple-Pomeron zero as a self-consistency condition which appears even before two-Pomeron iterations. Thus the triple-Pomeron coupling appearing in Fig. 3 already has the zero, making the two terms of the same order of magnitude at $t=0$. The effect of the two-Pomeron iterations of Fig. 4 is merely to impose a constraint on the renormalized four-Pomeron coupling. Thus  does not vanish at $t=0$, which spoils the factorization. In our approach, where the triple-Pomeron zero comes about via two-Pomeron iterations, the second term in Fig. 3 completely swamps the first at $t=0$. We shall now indicate how such a picture of the origin of the triple-Pomeron zero saves the Pomeron from further serious decouplings.

The most stringent of the decoupling arguments is that of Jones et al. [8] who argue via the inclusive sum rules that the contribution on the right-hand side of Fig. 10 must vanish at zero external masses, and thus the Reggeon-Pomeron particle vertex must vanish when the Pomeron mass is zero. However, in terms of Bronzan's model of Fig. 1 it is clear that there are other contributions which exactly cancel the pure pole contribution. The contribution in Fig. 10 comes from the first term in Fig. 1, since this "bare" vertex is already in fact renormalised by Reggeon-Reggeon cuts. However, we can cut a vertex in the second term in a similar way as shown in Fig. 11. Since we are concentrating on the region of phase space where all other produced particles are well separated from d in rapidity, we obtain the third diagram in Fig. 11. For $\bar{t}<0$ this diagram actually leads that in Fig. 10 because of the Reggeon-Pomeron cut. However even at $\bar{t}=0$ it is of the same order as that in Fig. 10 because of an enhancement due to a collision with the triangle singularity implicit in Fig. 11. The further diagrams in Fig. 1 will give similar contributions which, together with the Jones et al., contribution, must all cancel to give the triple-Pomeron zero on integrating over particle d. Thus we see how the same mechanism which avoids the decoupling results also implies Eq. (1).

We conclude with some further remarks and extensions.

i) Since the rise of the total cross-section is determined by the second diagram in Fig. 9, we can make a connection between this rise and large missing mass production, as is done by several other authors [9] who have a non-vanishing triple-Pomeron coupling. This is because the cut diagram in Fig. 9 is determined by the slope of Gribov's vertex at $t=0$. The large missing mass production is determined by the slope of the inclusive vertex. While these two slopes are not identical, we expect them to be of the same order of magnitude.

ii) In the most complete fit to the ISR data [3], the coupling  was left as a free parameter at $t = 0$, and it was found to be surprisingly small, smaller than that suggested by the absorption model. This supports our claim that it is in fact zero, and all low missing mass states are cancelled in the fixed-pole residue [10] which determines the magnitude of the $(\ell n\, s)^{-1}$ term in (1).

iii) Equation (1) implies that the sign of the two-Pomeron cut, which has only been proved negative for processes elastic in the t channel [10], is in fact negative for all two-body processes.

iv) Barring complications of spin, our result should extend to quantum number exchange, and Reggeon-Pomeron cuts should couple only through the Regge pole at $t = 0$.

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FIGURES

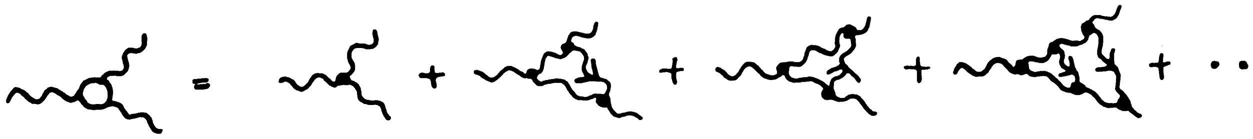

Fig 1

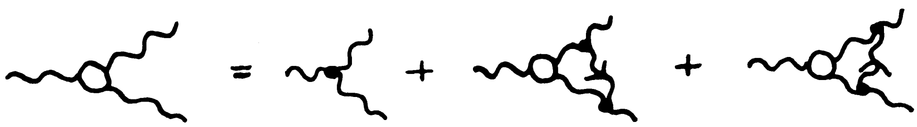

Fig 2

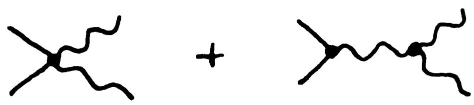

Fig 3

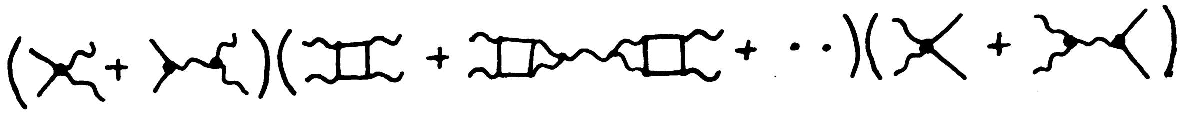

Fig 4

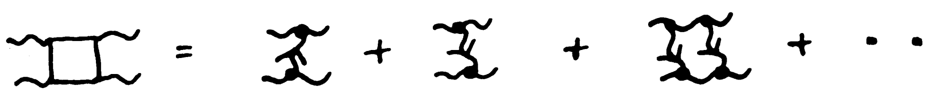

Fig 5

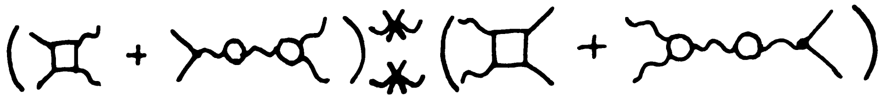

Fig 6

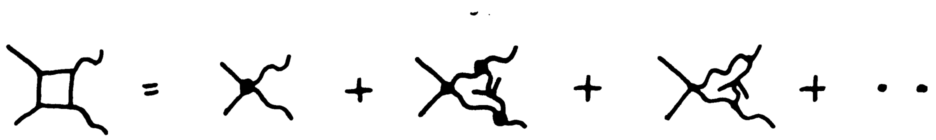

Fig 7

Fig 8

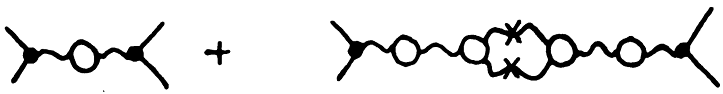

Fig 9

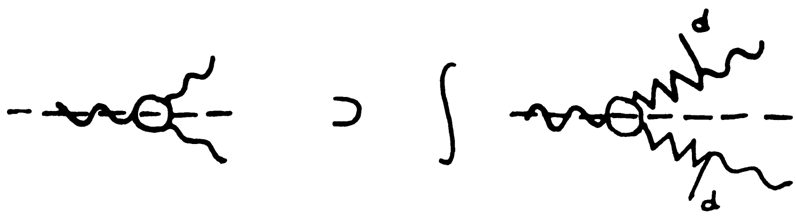

Fig 10

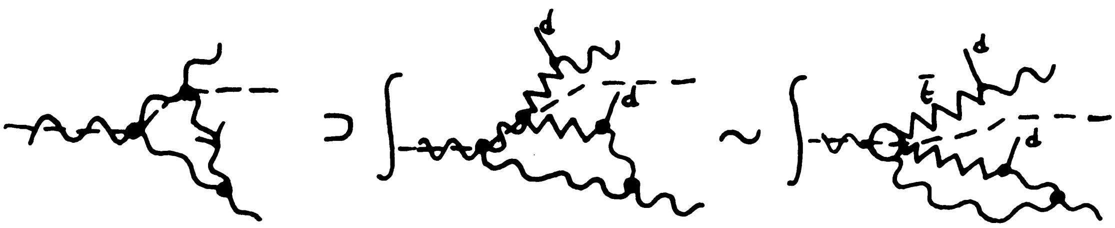

Fig 11