Archives

Ref.TH.1472-CERN

AMPLITUDE ANALYSIS FROM DECAY CORRELATIONS : APPLICATION TO $\pi^- p \to K^{*0} \Lambda$ at 3.9 GeV/c

M. Abramovich, A.C. Irving*), A.D. Martin*) and C. Michael
CERN - Geneva

A B S T R A C T

Decay correlation data for $\pi^- p \to K^* \Lambda$ at 3.9 GeV/c are analyzed to determine the amplitude structure. We emphasize combinations of observables invariant under rotations between s and t channel frames.

---

*) On leave of absence from the University of Durham.

Ref.TH.1472-CERN
7 March 1972

A considerable amount of information can be extracted from the bubble chamber data for the quasi two-body reactions $\pi^- p \to K^*(\Lambda, \Sigma)$ and $K^- p \to (\omega, \rho, \phi)(\Lambda, \Sigma)$. To illustrate this we analyze the data for $\pi^- p \to K^{*0} \Lambda$ at 3.9 GeV/c and show how the angular correlations of the decays of the K^{*0} and Λ may be used to determine, apart from two phase factors, the six amplitudes for the production process.

For vector meson production four of the six amplitudes arise from unnatural parity exchange. These processes are thus important for gaining insight into unnatural parity exchange mechanisms. The most interesting such process [1] is $\pi^- p \to \rho^0 n$. However, a model independent determination of the $\pi^- p \to \rho^0 n$ amplitudes will require the use of a polarized target. Here we stress that the decay angular correlations for the hypercharge exchange reactions (e.g., $\pi^- p \to K^{*0} \Lambda$) contain the same amount of information without the use of polarized targets.

Before presenting the amplitude determination we give a conventional moment analysis of the angular correlations. In terms of statistical tensors the joint decay probability distribution is of the form [2]

$$W(\theta_1, \phi_1; \theta_2 \phi_2) = \sum_{J_1 M_1 J_2 M_2} F_1(J_1)\, F_2(J_2)\, T^{J_1\, J_2}_{M_1\, M_2}\; Y^{J_1}_{M_1}(\theta_1, \phi_1)^* \, Y^{J_2}_{M_2}(\theta_2, \phi_2)^* \quad (1)$$

where the F_i are known constants. For our process θ_1, ϕ_1 are the angles specifying the proton from the Λ decay $(J_1 \leq 1)$ and θ_2, ϕ_2 those for the K from the K^* decay $(J_2 \leq 2)$. One may choose the two systems of axes as either (i) helicity-type axes in which the z axes are along particle momenta and the y axes of both particles are normal to the reaction plane, or (ii) transversity-type axes in which the z axes are chosen normal to the reaction plane and the x axes *) are along particle momenta. These alternatives are related simply by relabelling the axes : $(x_T, y_T, z_T) = (z_H, x_H, y_H)$. To complete the definition of axes we must specify a frame for the particle

---

*) Sometimes these are taken to be y axes, see for example Kotanski [3], and then $(x_T, -y_T, z_T) = (x_H, z_H, y_H)$ where the minus sign is inserted to maintain a right-handed set of axes.

momenta. The most useful choices for z_H (or x_T) are (i) along the particle momentum in the s channel centre-of-mass frame, that is $z_H(\Lambda) = -z_H(K^*)$, or (ii) along the incoming meson (baryon) momentum in the decaying meson (baryon) rest frame. These alternatives, which are often called the s channel (or helicity) frame and the t channel (or Gottfried-Jackson) frame, are related by rotation through an angle $\chi(s,t)$ about the normal to the reaction plane. Observable quantities which are independent of such rotations about the normal for both baryon and meson frames, we call frame invariants. Besides their obvious use as consistency checks, we will consider a set of invariants which have direct physical interpretation.

Transversity statistical tensors transform under rotations about the normals in the following simple way

$$T^{J_1 J_2 \prime}_{M_1 M_2} = exp(-i M_1 \chi_1 - i M_2 \chi_2) \; T^{J_1 J_2}_{M_1 M_2} \tag{2}$$

and so offer an easy way of counting the number of frame invariants. Here χ_1, χ_2 are the rotation angles of the axes for the Λ and K^* decays respectively. Now parity conservation in the production process and in the K^* decay require that transversity tensors with $M_1 + M_2$ odd or with J_2 odd must vanish. This together with the Hermiticity condition

$$\left(T^{J_1 J_2}_{M_1 M_2} \right)^* = (-1)^{M_1 + M_2} \; T^{J_1 J_2}_{-M_1 \, -M_2} \tag{3}$$

leaves the following measurable tensors : T^{oo}_{oo}, T^{1o}_{oo}, T^{o2}_{oo}, T^{12}_{oo}, T^{12}_{11}, T^{o2}_{o2}, T^{12}_{o2}, T^{12}_{-11}. Hermiticity requires tensors with $M_1 = M_2 = 0$ to be real and so there are altogether 12 measurable quantities (we shall see that 10 are independent). Now the moduli of transversity tensors are unchanged by rotations [see Eq. (2)] and thus there are eight frame invariants.

In the top half of Table I, we give the helicity statistical tensors, expressed in terms of double density matrices, determined from the decay correlations in $\pi^- p \rightarrow K^{*o} \Lambda$ at 3.9 GeV/c. In this paper we normalize the cross-section (the twelfth measurable) to 1. The data *) are those of Ref. 5) where the

---

*) The events were selected by imposing the following cut on the $(K^+ \pi^-)$ effective mass $0.868 < M(K^+ \pi^-) < 0.920$ GeV. The Λ polarization quoted by Aguilar-Benitez et al. [4] was obtained by imposing a slightly different cut on these events.

cross-section is given. No S wave contribution in the K^* region was necessary to reproduce the available decay distributions and reflections from $\pi^- p \to K^+ Y_1^{*-}$ are suppressed since this reaction has double charge exchange.

The connection between helicity amplitudes and the helicity tensors of Table I is complicated. However, Byers and Yang [6] advocated using a Cartesian, rather than a spherical basis for the joint decay distribution of Eq. (1) since it leads to a relatively simple expansion in terms of amplitudes :

$$W = F_0 + \alpha_\Lambda (\lambda' F_1 + \mu' F_2 + \nu' F_3)$$

where α_Λ is the Λ decay asymmetry and

$$F_i = F_{i1}\lambda^2 + F_{i2}\mu^2 + F_{i3}\nu^2 + F_{i4}\lambda\mu + F_{i5}\lambda\nu + F_{i6}\mu\nu$$

(λ', μ', ν') and (λ, μ, ν) are the direction cosines of the proton from the Λ decay and the K from the K^* decay in their respective transversity frames. In terms of transversity amplitudes the 12 non-vanishing *) F_{ij} are

$$\begin{aligned}
F_{01} &= |b_+|^2 + |b_-|^2 & F_{04} &= 2\,\mathrm{Re}(b_+ c_+^* + b_- c_-^*) \\
F_{02} &= |c_+|^2 + |c_-|^2 & F_{34} &= -2\,\mathrm{Re}(b_+ c_+^* - b_- c_-^*) \\
F_{03} &= |a_+|^2 + |a_-|^2 & F_{15} &= 2\,\mathrm{Re}(a_+ b_+^* + a_-^* b_-) \\
F_{31} &= -|b_+|^2 + |b_-|^2 & F_{25} &= -2\,\mathrm{Im}(a_+ b_+^* + a_-^* b_-) \\
F_{32} &= -|c_+|^2 + |c_-|^2 & F_{16} &= 2\,\mathrm{Re}(a_+ c_+^* + a_-^* c_-) \\
F_{33} &= |a_+|^2 - |a_-|^2 & F_{26} &= -2\,\mathrm{Im}(a_+ c_+^* + a_-^* c_-)
\end{aligned} \tag{4}$$

with

$$a_\pm = T^0_{\pm\pm}\,, \quad -\frac{1}{\sqrt{2}}(b_\pm - i c_\pm) = T^1_{\mp\pm}\,, \quad \frac{1}{\sqrt{2}}(b_\pm + i c_\pm) = T^{-1}_{\mp\pm} \tag{5}$$

---

*) We normalize $F_{01} + F_{02} + F_{03} = 1$, use the Basel convention for the direction of the normal, and the Jacob and Wick phase factor $(-1)^{s-\lambda}$ for the baryon helicity states.

where the suffices on the transversity amplitudes $T^{K^*}_{\Lambda N}$ denote the spin projections along the normal. We see that the 12 observable moments determine 10 real numbers [6]; those in the first column determine all the magnitudes of, and those in the second column the relative phases within, the two sets of amplitudes (a_+, b_+, c_+) and (a_-, b_-, c_-). We call this the moment method for determining the amplitudes.

The helicity amplitudes are linearly related to these amplitudes

$$\begin{aligned} H^1_{++} + H^1_{--} &= i\,(a_+ + a_-) \\ H^1_{+-} - H^1_{-+} &= -\,(a_+ - a_-) \end{aligned} \qquad (6)$$

$$\begin{aligned} H^0_{++} &= \frac{1}{\sqrt{2}}(b_+ + b_-) \qquad & H^1_{++} - H^1_{--} &= -(c_+ + c_-) \\ H^0_{+-} &= +\frac{i}{\sqrt{2}}(b_+ - b_-) \qquad & H^1_{+-} + H^1_{-+} &= -i(c_+ - c_-) \end{aligned} \qquad (7)$$

At high energies the amplitudes are associated with definite exchange parity : $a_\pm$ describe natural parity exchange [e.g., $K^*(1^-)$, $K^{**}(2^+)$], whereas $b_\pm$ and $c_\pm$ respectively describe the production of helicity 0 and 1 K^*'s by unnatural parity exchange [e.g., $K(0^-)$, $K_B(1^+)$, $K_A(1^+)$ and $K_Z(2^-)$]. Angular momentum conservation implies a $(-t')^{n/2}$ behaviour for $H^m_{\lambda\mu}$ with $n = |m - \lambda + \mu|$.

Since transversity amplitudes only suffer a phase change under rotations about the normal it is straightforward to identify measurable frame invariant combinations. We show a choice of nine such quantities of which eight are independent

$$\begin{aligned} \sigma_N &= |a_+|^2 + |a_-|^2 = F_{03} \\ \sigma_U &= |b_+|^2 + |b_-|^2 + |c_+|^2 + |c_-|^2 = F_{01} + F_{02} \\ P_N \sigma_N &= |a_+|^2 - |a_-|^2 = F_{33} \\ P_U \sigma_U &= |b_-|^2 - |b_+|^2 + |c_-|^2 - |c_+|^2 = F_{31} + F_{32} \\ \Delta_\pm^2 n_\pm^2 &= 4\,\mathrm{Im}^2(b_\pm c_\pm^*) = 4 f_1^\pm f_2^\pm - (f_4^\pm)^2 \\ \Delta_0^2 \sigma_U^2 &= 4(|b_+|^2+|b_-|^2)(|c_+|^2+|c_-|^2) - 4\,\mathrm{Re}^2(b_+c_+^* + b_-c_-^*) = 4F_{01}F_{02} - F_{04}^2 \\ I_\pm^2 p_\pm^2 &= 4|a_+(b_+ \pm i c_+)^* + a_-^*(b_- \mp i c_-)|^2 = (F_{15} \mp F_{26})^2 + (F_{25} \pm F_{16})^2 \end{aligned} \qquad (8)$$

where $f_i^{\pm} = \frac{1}{2}(F_{oi} \mp F_{3i})$. The normalization factors, chosen so that the invariants Δ_o, $\Delta_{\pm}$ and $I_{\pm}$ lie between 0 and 1, are

$$n_{\pm} = \frac{1}{2}(1 \mp P_U)\,\sigma_U$$

$$p_{\pm} = \sigma_N + \sigma_U \pm (\Delta_+ n_+ - \Delta_- n_-)$$

At high energies (i.e., to order 1/s) $\sigma_{N,U}$ and $P_{N,U}$ are the cross-sections and polarizations arising from natural and unnatural parity exchanges. The further separation of the σ_U into helicity zero ($\sigma_U^o = F_{o1}, P_U^o\sigma_U^o = F_{31}$) and helicity one ($\sigma_U^1 = F_{o2}$, $P_U^1\sigma_U^1 = F_{32}$) K^* production is not frame invariant. The invariants $\Delta_{\pm}$ have a similar structure to polarizations and measure the phase between the unnatural parity exchange amplitudes associated with helicity 0 and 1 K^* production.

It is straightforward to express the invariants in terms of helicity moments, and those depending only on the meson decay are easily recognizable

$$\sigma_N = \rho_{11} + \rho_{1-1} \qquad \sigma_U = \rho_{00} + \rho_{11} - \rho_{1-1}$$

$$\Delta_o\,\sigma_U = 2\left[\rho_{00}(\rho_{11} - \rho_{1-1}) - 2(\mathrm{Re}\,\rho_{10})^2\right]^{1/2}$$

where, as before, $\sigma_N + \sigma_U$ is normalized to 1. Δ_o is well determined experimentally and has an interpretation as the unnatural parity exchange contribution to the polarization of the vector meson averaged over baryon spin. From moments alone the signs of $\Delta_{\pm}$ and Δ_o cannot be determined directly. Δ_o is zero when meson helicity 0 and 1 amplitudes are coherent in phase and baryon spin structure and Δ_o is 1 for incoherent amplitudes of equal strength.

The numerical values of the invariants ($\sigma_{N,U}$, $P_{N,U}$, Δ_o) for $\pi^- p \to K^{*o}\Lambda$ at 3.9 GeV/c are shown in the lower half of Table I together with the values of σ_U^o, σ_U^1, P_U^o and P_U^1. $\Delta_{\pm}$ and $I_{\pm}$ are not adequately determined by the moment analysis with present limited statistics.

The moment method allows the magnitudes of the amplitudes $a_\pm$, $b_\pm$ and $c_\pm$ to be determined and the results are shown in brackets in Table II. In principle the four relative phases can be determined from four of the moments on the right-hand column of Eq. (4), with the remaining two moments used as cross-checks and to resolve ambiguities. We had insufficient statistics to pursue this method which does not use the information contained in the moments optimally.

We preferred to express the probability distribution directly in terms of the moduli and relative phases of the amplitudes and to search for the set of amplitude components by maximum likelihood. This alternative method, besides exploiting the data fully, has the further advantage of constraining the observables (polarizations, etc.) to lie within their physical bounds. The results of the maximum likelihood search are given in Table II. The errors on the relative phases are much larger than on the moduli of $a_\pm$, $b_\pm$ and $c_\pm$.

From the amplitude determinations we reconstruct the frame invariants P_N, σ_U, P_U, Δ_+, Δ_- and Δ_0. where we may ascribe the signs of $-\phi^\pm_{bc}$ to $\Delta_\pm$. The agreement of these quantities (and $|a_\pm|^2$) when evaluated independently via s and t channel amplitudes is a check on the consistency and precision of our amplitude analysis. We also determine P^0_U and P^1_U and these are constrained to lie between ± 1, unlike the moment method determination of Table I.

To reconstruct the helicity amplitudes H, one needs the relative phase between amplitudes of opposite initial baryon transversity. This can be determined by experiments with a polarized target [6]. This decomposition into baryon helicity flip and non-flip components can also be made using exchange models. Furthermore, at $t' = 0$ one has trivially $a_+ = -a_- = +ic_+ = -ic_-$; $b_+ = b_-$ but since our $d\sigma/dt$ data do not show a clear forward spike, one cannot assume dominance of $n = 0$ amplitudes in the $0 < -t' < 0.2$ GeV^2 bin.

We can compare our results with simple model expectations and with data for related reactions. P_N and σ_N can be related to $\pi^- p \to K^0 \Lambda$ which has only natural parity exchange and where data [7] show a behaviour of P which is positive for $-t < 0.4$ GeV^2 and negative at larger $-t$. P_N has a consistently negative trend and this would imply that the same t channel mechanism is not present in both reactions - perhaps because of a contamination of P_N for $\pi^- p \to K^* \Lambda$ by multiple scattering or absorptive corrections due to unnatural parity exchange. From SU(3) and the duality diagram expectations one can relate the amplitude structure of $\pi^- p \to K^* \Lambda$ directly to that of $K^- p \to \phi\Lambda$ which also has a rotating phase structure. Existing data [5] are indeed very similar for these two reactions.

The large and statistically significant values of Δ_o imply at least two incoherent unnatural parity exchanges. Thus K exchange alone is insufficient - even with an exchange degenerate K_B added. One would need $K_A - K_Z$ exchange contributions in addition. These two types of unnatural parity exchange have different helicity couplings to baryons [+- for $K - K_B$ and ++ for $K_A - K_Z$ in an SU(3) limit] and are thus incoherent even if they have similar helicity amplitude phases ($e^{-i\pi\alpha}$ from duality diagrams).

With better statistics and with data at a range of energies and on related processes, it should be possible to make a quantitative analysis of the phases, energy dependence and spin structure of the amplitudes. This will illuminate the features of natural and unnatural parity hypercharge exchange in a detailed and model independent manner.

	$0 < -t' < 0.2$ (59 events)		$0.2 < -t' < 0.4$ (58 events)		$0.4 < -t' < 0.8$ (47 events)	
	t channel	s channel	t channel	s channel	t channel	s channel
$\rho_{11} - \rho_{oo}$	-0.10±0.15	-0.15±0.14	0.07±0.14	0.03±0.14	-0.07±0.17	0.42±0.14
$\mathrm{Re}\,\rho_{1o}$	-0.09±0.06	0.05±0.06	-0.07±0.06	0.06±0.05	-0.02±0.06	0.02±0.06
ρ_{1-1}	0.10±0.08	0.11±0.08	0.12±0.08	0.13±0.09	0.26±0.09	0.10±0.11
$P_\Lambda = -2\,\mathrm{Im}\,\rho_{+-}$	-0.07±0.38	-0.07±0.38	-0.15±0.36	-0.15±0.36	-0.49±0.36	-0.49±0.36
$\mathrm{Im}(\rho^{1o}_{++} - \rho^{1o}_{--})$	-0.11±0.15	0.17±0.18	-0.09±0.15	0.56±0.17	-0.69±0.18	0.27±0.17
$\mathrm{Im}\,\rho^{1-1}_{++}$	0.13±0.09	0.21±0.10	0.10±0.11	-0.04±0.12	-0.22±0.11	0.35±0.13
$\mathrm{Im}(\rho^{11}_{+-} - \rho^{oo}_{+-})$	0.27±0.22	0.13±0.21	0.30±0.19	-0.08±0.18	0.17±0.20	0.11±0.22
$\mathrm{Im}(\rho^{1o}_{+-} + \rho^{1o}_{-+})$	0.01±0.18	0.13±0.16	0.36±0.17	0.21±0.17	-0.42±0.19	0.35±0.16
$\mathrm{Im}(\rho^{1o}_{+-} - \rho^{1o}_{-+})$	-0.06±0.16	-0.07±0.16	-0.09±0.16	-0.10±0.16	-0.13±0.16	0.20±0.15
$\mathrm{Im}(\rho^{1-1}_{+-} + \rho^{1-1}_{-+})$	0.31±0.24	-0.08±0.20	0.64±0.23	-0.32±0.20	0.61±0.22	-0.73±0.26
$\mathrm{Im}(\rho^{1-1}_{+-} - \rho^{1-1}_{-+})$	-0.03±0.23	0.05±0.25	0.03±0.23	0.28±0.24	0.21±0.27	0.24±0.25
$\sigma_U = 1 - \sigma_N$	0.60±0.10		0.53±0.10		0.43±0.12	
P_N	-0.44±0.82		-0.59±0.72		-0.84±0.63	
P_U	0.18±0.66		0.25±0.67		-0.03±0.21	
Δ_o	0.83±0.17		0.92±0.13		0.62±0.51	
σ^o_U	0.40±0.10	0.43±0.09	0.29±0.09	0.31±0.10	0.38±0.11	0.05±0.09
σ^1_U	0.20±0.09	0.17±0.09	0.24±0.09	0.21±0.10	0.05±0.09	0.38±0.11
P^o_U	0.83±0.83	0.36±0.76	1.22±0.94	-0.51±0.84	0.15±0.74	-0.24±5.81
P^1_U	-1.11±1.36	-0.36±1.69	-0.93±1.03	1.36±1.18	-1.48±6.59	-0.00±0.77

- TABLE I -

	$0 < -t' < 0.2$		$0.2 < -t' < 0.4$		$0.4 < -t' < 0.8$	
	t channel	s channel	t channel	s channel	t channel	s channel
$\lvert a_+\rvert^2$	0.09±0.13	0.05±0.14	0.17±0.19	0.17±0.18	0.09±0.22	0.07±0.22
	(0.11±0.17)	(0.11±0.17)	(0.10±0.17)	(0.10±0.17)	(0.04±0.17)	(0.04±0.17)
$\lvert b_+\rvert^2$	0.11±0.12	0.15±0.16	0.03±0.09	0.20±0.15	0.16±0.20	0.03±0.14
	(0.03±0.16)	(0.14±0.17)	(-0.03±0.13)	(0.24±0.15)	(0.16±0.15)	(0.03±0.16)
$\lvert c_+\rvert^2$	0.17±0.14	0.05±0.09	0.17±0.15	0.00±0.31	0.05±0.13	0.17±0.21
	(0.21±0.14)	(0.11±0.15)	(0.23±0.14)	(-0.04±0.12)	(0.06±0.16)	(0.19±0.15)
$\lvert a_-\rvert^2$	0.26±0.15	0.33±0.17	0.32±0.18	0.32±0.17	0.47±0.23	0.48±0.22
	(0.29±0.16)	(0.28±0.16)	(0.38±0.18)	(0.37±0.18)	(0.53±0.19)	(0.53±0.19)
$\lvert b_-\rvert^2$	0.32±0.13	0.31±0.15	0.28±0.12	0.07±0.14	0.17±0.19	0.07±0.12
	(0.36±0.19	(0.29±0.18)	(0.32±0.16)	(0.08±0.14)	(0.22±0.16)	(0.02±0.16)
$\lvert c_-\rvert^2$	0.05±0.12	0.11±0.10	0.03±0.15	0.23±0.09	0.07±0.11	0.17±0.21
	(-0.01±0.14)	(0.05±0.15)	(0.01±0.12)	(0.25±0.15)	(-0.01±0.15)	(0.19±0.16)
ϕ^+_{bc}	-0.7 ±1.1	-3.0 ±6.8	-0.2 ±5.3	2.5 ±78.6	-1.2 ±2.0	-2.4 ±4.2
ϕ^+_{ca}	-0.8 ±2.1	0.3 ±6.7	-1.4 ±2.0	-2.9 ±79.6	-2.9 ±5.2	1.0 ±7.1
ϕ^+_{ab}	1.5 ±2.0	2.6 ±2.5	1.6 ±5.8	0.3 ± 2.1	-2.3 ±5.5	1.5 ±5.9
ϕ^-_{bc}	1.5 ±1.0	1.7 ±0.7	1.4 ±1.4	2.3 ± 1.6	1.9 ±1.2	1.2 ±1.5
ϕ^-_{ca}	1.3 ±1.2	-0.7 ±1.3	0.7 ±1.8	-1.8 ± 1.1	1.9 ±2.5	-0.5 ±2.7
ϕ^-_{ab}	-2.7 ±0.8	-1.0 ±1.0	-2.2 ±1.3	-0.4 ± 1.8	2.5 ±2.8	-0.7 ±2.2
$\sigma_U(=1-\sigma_N)$	0.65	0.62	0.50	0.51	0.44	0.45
P_N	-0.50	-0.72	-0.30	-0.31	-0.66	-0.74
P_U	0.12	0.33	0.23	0.22	0.07	0.10
Δ_+	0.63	0.17	0.15	-0.02	0.72	0.48
Δ_-	-0.66	-0.87	-0.61	-0.65	-0.86	-0.84
Δ_o	0.87	0.81	0.92	0.94	0.87	0.85
σ^o_U	0.44	0.46	0.30	0.27	0.33	0.11
σ^1_U	0.22	0.16	0.20	0.23	0.11	0.34
P^o_U	0.47	0.33	0.81	-0.47	0.03	0.38
P^1_U	-0.57	0.33	-0.66	1.00	0.18	0.01

- TABLE II -

TABLE CAPTIONS

TABLE I The top half of the Table shows the density matrix elements in both the t and s channel helicity frames calculated from moments of the decay correlations in $\pi^- p \to K^{*o}\Lambda$ at 3.9 GeV/c. The 0, ±1 suffices on the elements refer to the K^* helicity and the ± subscripts refer to a Λ of helicity $\pm\frac{1}{2}$. P_Λ is the polarization of the Λ. Also shown are four frame invariant quantities [defined by Eqs. (8)] and the K^* helicity 0 and 1 projections of σ_U and P_U.

TABLE II Amplitude moduli and phases determined from a maximum likelihood search to the decay distributions for $\pi^- p \to K^*\Lambda$ at 3.9 GeV/c. s and t channel frames were used and frame invariant quantities are $|a_\pm|^2$, σ_U, P_N, P_U, $\Delta_\pm$ and Δ_o. Relative phases ϕ^+_{ab}, etc., are defined as $\arg(b_+) - \arg(a_+)$ in the range $\pm\ \pi$ radians. For comparison we show in brackets the magnitudes calculated by the moment method.

REFERENCES

1) G. Fox - Preprint CALT-68-334 (1971).

2) A. Kotanski and K. Zalewski - Nuclear Phys. B4, 559 (1968).

3) A. Kotanski - Acta Phys.Polonica 29, 699 (1966) ; 30, 629 (1966).

4) M. Aguilar-Benitez, S.U. Chung, R.L. Eisner and N.P. Samios - Preprint BNL 16393 (1972).

5) M. Abramovich et al. - Nuclear Phys. B (to be published).

6) N. Byers and C.N. Yang - Phys.Rev. 135, B796 (1964).

7) M. Abramovich et al. - Nuclear Phys. B22, 477 (1971) ;
Kwan Wu Lai - private communication ;
D. Rust and D. Yovanovich - private communication.