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CAN PION SCATTERING YIELD USEFUL NUCLEAR STRUCTURE INFORMATION ?

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A B S T R A C T

It is pointed out that for excitations involving magnetic transitions, pion scattering from nuclei should in principle allow a separation of the orbital and spin contributions. This is not possible in electron scattering.

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The elastic and inelastic scattering of pions from light nuclei, especially ^{12}C, have been investigated intensively using Glauber theory [1),2)] or optical models [3)], and at least qualitative agreement with experiment [4)] has thus been obtained. Assuming that these models can be refined further, is there any hope of extracting relevant nuclear structure information from such experiments, complementary to that derived from electron or proton scattering, rather than just studying the albeit interesting question of the rôle of the pion in the nucleus ? It is the purpose of this note to point out that in principle, when looking at magnetic transitions such as the excitation of the $T = 1$, $J^P = 1^+$ level in ^{12}C at 15.1 MeV, the pion could be an extremely useful probe.

Apart from any simplicity due to its zero spin, the two distinctive features of the pion are its isospin 1, which permits both single and double charge exchange, and the strong resonances which are found in the pion-nucleon scattering. Thus in the vicinity of the Δ resonance ($T_{lab} \sim$ ~ 180 MeV), the pion-nucleon amplitude varies very fast with energy. It is this second feature, very different to the cases of electron or proton scattering, which we shall exploit here.

Consider first the excitation of this nuclear level by an incident electron. Reducing the electromagnetic current of the proton in the brick-wall frame [5)]

$$\vec{J} = \frac{e}{(2\pi)^3}\,\frac{1}{2m}\left[(\vec{p}+\vec{p}\,')\,F_1^{\,p}(q^2) + \vec{\sigma}\wedge\vec{q}\,\left(F_1^{\,p}(q^2)+F_2^{\,p}(q^2)\right)\right] \tag{1}$$

where $\vec{p}$ and $\vec{p}\,'$ are the initial and final nucleon momenta, and $\vec{q}$ the momentum transfer

$$\vec{q} = \vec{p}\,' - \vec{p} \tag{2}$$

F_1^p and F_2^p are the proton form factors normalized such that

$$\begin{aligned} F_1^{\,p}(0) &= 1 \\ F_2^{\,p}(0) &= \mu_p - 1 \end{aligned} \tag{3}$$

where μ_p is the total proton magnetic moment. This current is an operator acting on the spin of the proton.

To calculate the transition probability in Born approximation, $\vec{j}$ must be multiplied by $e^{iq\cdot r}$ and an expectation value taken between initial and final nuclear wave functions. If, for simplicity, we first assume that the excitation can be described by a $(p_{\frac{3}{2}})^{-1}(p_{\frac{1}{2}})$ simple particle-hole in j-j coupling, then this leads [6] to a matrix element proportional to

$$e\left[(\mu_p-\mu_n-\tfrac{1}{2})(1-q^2/6\alpha^2)+(\mu_p-\mu_n-2)\,q^2/24\alpha^2\right] q\, e^{-q^2/4\alpha^2} \tag{4}$$

where α is the harmonic oscillator parameter. In the above we have included the contributions from the neutrons, but we have not made explicit the q dependence of the nucleon form factors. In the long wave-length limit, the $\frac{1}{2}$ term in Eq. (4) comes only from the orbital motion of the nucleons in the p shell of the nucleus, and it is very small in comparison with the spin terms, $(\mu_p-\mu_n)\sim 3.7$. More realistic calculations using intermediate coupling cut down the strength of the transition appreciably, but do not change much the shape of the form factor. In addition to the electron scattering information, there is at least one more experimental constraint. The 15.1 MeV level in ^{12}C, together with the ground states of ^{12}B and ^{12}N form the famous triplet of the conserved vector current (CVC) test [7]. Assuming the validity of CVC, the β decay of ^{12}B to the ground state of ^{12}C allows one to determine the matrix element of σ (essentially at $q = 0$) between these two states. Using isospin invariance, this is related to the matrix element between the ground state and 15.1 MeV state in ^{12}C. This number, together with the form factor measurements, enables one to isolate the contribution of the orbital term near $q = 0$. Even in this case the errors are quite large, since it is the difference of two large numbers. For other nuclei, or other transitions, we just do not have this extra bonus from the β decay, and by purely electromagnetic means we cannot separate the two contributions to the magnetic form factors.

Now consider pion scattering, and first suppose that the pion-proton amplitude is purely due to the electromagnetic interaction. Then in the brick-wall frame

$$f=\frac{2\alpha}{q^2}\left[\omega F_1^p(q^2)-\frac{\vec{K}\cdot(\vec{p}+\vec{p}\,')}{2m}F_1^p(q^2)-\frac{i(\vec{\sigma}\wedge\vec{q})\cdot\vec{K}}{2m}\left(F_1^p(q^2)+F_2^p(q^2)\right)\right]F_\pi(q^2) \tag{5}$$

where w is the pion energy, and K the mean pion momentum

$$\vec{K} = \frac{1}{2}(\vec{k} + \vec{k}') \tag{6}$$

For completeness we have included the pion form factor $f_\pi(q^2)$. The first term in the above comes from the interaction with the charge, the others with the current [Eq. (1)], and for the pionic excitation of the 15.1 MeV level it is only these latter which contribute.

When the strong interactions are switched on, the pion-nucleon amplitude may be written as

$$\hat{f} = f + i\vec{\sigma}\cdot(\hat{q}\wedge\hat{K})\,g \tag{7}$$

Both the non-spin-flip and spin-flip amplitudes vary very fast with energy, so that it is not reasonable to neglect a priori the Fermi motion of the target nucleons. The square of the invariant energy S is related to the mean value S_o by

$$\begin{aligned} S &= S_o - 2\vec{p}\cdot\vec{K} + O(p^2) \\ &= S_o - (\vec{p}+\vec{p}')\cdot\vec{K} \end{aligned} \tag{8}$$

For the relevant transition we need only consider the energy variation of the spin-non-flip amplitude,

$$f(s) \approx f(S_o) - (\vec{p}+\vec{p}')\cdot\vec{K}\;\frac{\partial f}{\partial S}\bigg|_{q^2} \tag{9}$$

In terms of the pion energy this becomes

$$f(s) \approx f(S_o) - \frac{(\vec{p}+\vec{p}')\cdot\vec{K}}{2m}\;\frac{\partial f}{\partial w} \tag{10}$$

so that the full operator is

$$\hat{f} = f_o - \frac{(\vec{p}+\vec{p}')\cdot\vec{K}}{2m}\;\frac{\partial f}{\partial w} + i\vec{\sigma}\cdot(\hat{q}\wedge\hat{K})\,g \tag{11}$$

This, of course, has identically the same structure as in the electromagnetic case [Eq. (5)].

Now one reason why the orbital excitations are not too important in the electromagnetic case is that the spin-non-flip amplitudes there are varying only like ω. For pions on the other hand, the rapid variation of f can induce an orbital term which is of comparable magnitude to the spin term.

In single-scattering approximation, the amplitude for the charge exchange to the ground state of ^{12}B is

$$F_M(q) = \langle M \Big| \left[-\frac{(\vec{p}+\vec{p}\,')\cdot\vec{K}}{2m}\,\frac{\partial f^{c.e.}}{\partial \omega} + i\,\vec{\sigma}\cdot(\hat{q}\wedge\hat{K})\,g^{c.e.}\right] e^{i\vec{q}\cdot\vec{r}}\,\tau^- \Big| 0 \rangle \tag{12}$$

where M is the polarization of the final nuclear state, and the charge-exchange superscript denotes the charge-exchange pion-nucleon amplitudes. By isospin this is equal to the amplitude for exciting the 15.1 MeV level. Corrections due to isospin impurities are unimportant to the accuracy we are working. Taking q along the z direction and K the x, this reduces to

$$F_n(q) = -\frac{K}{2m}\,\frac{\partial f^{c.e.}}{\partial \omega}\langle M|(\vec{p}+\vec{p}\,')_x\, e^{i\vec{q}\cdot\vec{r}}\,\tau^-|0\rangle + i g^{c.e.}\langle n|\sigma_y\, e^{i\vec{q}\cdot\vec{r}}\,\tau^-|0\rangle \tag{13}$$

This may be evaluated quite simply in j-j coupling to give

$$F_{\pm 1} = -\frac{2\sqrt{2}}{3}\left[g^{c.e.}\left(1-q^2/8\alpha^2\right) + \frac{qK}{2m}\,\frac{\partial f^{c.e.}}{\partial \omega}\right] e^{-q^2/4\alpha^2} \tag{14}$$

with the longitudinal polarization zero. The experimental strength of this transition should be reduced by a factor 0.463, and perhaps the ratio of the orbital to the spin coefficients should also be weakened somewhat, though the errors on this are quite large [6),8)]. The oscillator parameter is determined [8)] to be $\alpha^2 = 0.282\ f^{-2}$.

Now we can only expect to be able to measure the influence of the orbital component if the second term in Eq. (14) is at least comparable to the first. In Table I are given the values of the two components $g^{c.e.}$ and $\zeta^{c.e.}$, defined by

$$\mathcal{Y}^{c.e.} \equiv \frac{K}{2m} \frac{\partial f^{c.e.}}{\partial \omega} \tag{15a}$$

$$\mathcal{G}^{c.e.} \equiv \frac{1}{q} g^{c.e.} \tag{15b}$$

in the forward direction as calculated from the phase shift analysis of Carter et al. [9]. In magnitude they are very similar, but below resonance the contributions add constructively, and above destructively. Because of this striking energy dependence it should be possible to pick out the coefficients of the two terms independently. With the values calculated as in the j-j limit, a single scattering calculation would predict essentially no pionic excitation of this line around 280 MeV incident energy.

Such a single scattering calculation is very unrealistic for pion-nucleus interactions near the 3-3 resonance, but the transition amplitude [Eq. (14)] may be used as a driving term in a distorted wave calculation. It is then assumed that the transition is basically a direct one, and that all the other multiple scatterings merely provide some kind of damping factor. This type of approach has been reasonably successful in describing the excitation of the first 2^+ and 3^- levels in ^{12}C, when the damping factor is calculated within the eikonal framework. Following the notation and method of Ref. 2), we parametrize all the pion-nucleon amplitudes in the form

$$f = f_o e^{-\beta^2 q^2/2} \tag{16}$$

for small momentum transfer. The values of these parameters for f, $\mathcal{Y}$ and $\mathcal{G}$ are given in Table I.

The results of the DWIA calculation are also shown in Table I, using the parametrization of Tibell [10], where for small (but not zero) momentum transfer

$$\frac{d\sigma}{d\Omega} \approx A q^2 e^{-\beta^2 q^2} \tag{17}$$

Values of A and β^2 are quoted for the case when only the spin-flip part of the transition is considered, as well as that from the sum.

The sensitivity found in the single-scattering calculation is maintained when the distortions are introduced. Thus at $q^2 = 0.8\ f^{-2}$ the ratio of the result including orbital motion effects to that without is 4.3, 1,1 and 0.0 at 120, 200 and 280 MeV, respectively.

In the extensive experiment on pion scattering from ^{12}C [4] evidence was found for the excitation of states in the 15 MeV region. For practical reasons the measurements here were only carried out at 200 MeV and above. Our predictions shown in Table I are at least an order of magnitude lower than these measurements. Now the experimental resolution was at least 1 MeV, so that from the compilation [11] of energy levels in this region, shown in Table II, it can be seen that this was insufficient to distinguish individual resonances. A rather broad enhancement was found in the vicinity of 15 MeV, with a full width typically of the order of 3 MeV, and after a background subtraction the resulting area led to the quoted cross-section [4]. It is clear that several states can contribute to this number. In the single-scattering calculation of Nishiyama and Ohtsubo [12], where the orbital motion terms were neglected, it was found that, of the isospin one levels, the 2^+ at 16.1 gave the most important contribution. Since this state has a B(E2) about 10% of the first 2^+ at 4.4 MeV, but with a rather similar transition radius [13], one would expect the pion excitation of the higher level to be only a few percent of the lower, which is insufficient to explain the data. In the recent high resolution experiment of 1 GeV protons scattered from ^{12}C [14], it seems that there is a lot of quasi-elastic $(p,p\alpha)$ scattering giving contributions already in this region of excitation energies. More seriously, they found a quite wide peak at 14.1 MeV, which is quite strongly excited, and except at very forward angles the 15.1 MeV level is not seen at all clearly. This situation is just the reverse of that which occurs at low proton energies where the one-pion exchange amplitude contribution to nucleon-nucleon scattering has just the right quantum numbers for exciting the 15.1 MeV level. The quantum numbers of the 14.1 MeV level have not yet been definitively determined, but the fact that it is so prominent in high energy proton scattering strongly suggests that it has zero isospin. On experimental grounds it cannot be ruled out that a significant fraction of the events observed in the pion scattering could be due to the excitation of this level [4]. It is interesting to note that in the pion experiment, no significant evidence was found for the excitation of the $T = 0$, $J^P = 1^+$ state at 12.7 MeV.

It is pertinent to ask now whether the 15.1 MeV level could be picked out cleanly, purely on the basis of energy resolution, in some of the experiments planned for the new high intensity accelerators. If not, then this would seem to be an ideal experiment to be done by detecting the γ ray from the decay in coincidence with the scattered pion [15)]. Just measuring the γ ray leads to large background problems [16)]. Looking at the charge-exchange to ^{12}B seems to be a harder way to get the same information.

Given that it can be done experimentally, it is possible that the theoretical tools used in the present paper are not sufficiently precise to extract the nuclear physics reliably. For example the importance of two-step processes in the excitation should be investigated. Assuming that theory and experiment can be made to agree in this case in ^{12}C, where we have a reasonable amount of nuclear structure information, then pions could provide a valuable tool for probing other magnetic transitions.

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T_{lab} (MeV)	120	200	280
$f(0)$ (f)	(0.72, 0.63)	(-0.38, 1.53)	(-0.76, 0.87)
β_f^2 (f^2)	1.59	0.80	0.47
$y(0)$ (f^2)	(0.04, 0.27)	(-0.40,-0.15)	(-0.08,-0.22)
β_y^2 (f^2)	0.93	0.93	0.76
$g(0)$ (f^2)	(0.38, 0.23)	(0.30, 0.41)	(0.09, 0.19)
β_g^2 (f^2)	0.56	0.30	0.25
A_{sum} (mb)	0.65	0.30	0.0
β^2_{sum} (f^2)	3.03	5.0	0.0
A_{spin} (mb)	0.40	0.27	0.15
β^2_{spin} (mb)	4.70	5.0	3.03

TABLE I : Input parameters of Eq. (16) for the DWIA calculation of the pionic excitation of the 15.1 MeV level in ^{12}C. The corresponding results are shown using the parametrization of Eq. (17).

Excitation Energy MeV	J^P ; T	Γ KeV
14.1	(4^+) ; 0	260
14.7		< 15
15.11	1^+ ; 1	0.039
16.11	2^+ ; 1	6
16.58	2^- ; (1)	300
17.23	1^- ; 1	1150
17.77	0^+ ; (1)	100

TABLE II : Energy levels of the ^{12}C system between 14 and 18 MeV taken from the compilation of Ajzenberg-Selove and Lauritsen [11].

REFERENCES

1) C. Schmit - Nuovo Cimento Letters 4, 459 (1970) ;
C. Wilkin - Nuovo Cimento Letters 4, 491 (1970) ;
H.K. Lee and H.McManus - Nuclear Phys. A167, 257 (1971).

2) C. Rogers and C. Wilkin - Nuovo Cimento Letters 1, 575 (1971).

3) M. Krell and S. Barmo - Nuclear Phys. B20, 461 (1970) ;
M.M. Sternheim and E.H. Auerbach - Phys.Rev.Letters 25, 1550 (1970) ;
R.H. Landau, S.C. Phatak and F. Tabakin - Ann.Phys. 78, 299 (1973) ;
G.W. Edwards and E. Rost - Phys.Rev.Letters 26, 785 (1971).

4) F. Binon et al. - Nuclear Phys. B17, 168 (1970) ;
J.P. Stroot - Private communication.

5) See, for example :
S. Gasiorowicz - "Elementary Particle Physics", J. Wiley and Sons Ltd., New York (1967).

6) L.L. Foldy and J.D. Walecka - Phys.Rev. 140, B1339 (1965).

7) M. Gell-Mann - Phys.Rev. 111, 362 (1958).

8) B.T. Chertok et al. - Phys.Rev. C8, 23 (1973).

9) J.R. Carter et al. - Nuclear Phys. B58, 378 (1973).

10) G. Tibell - Phys.Letters 28B, 638 (1969).

11) F. Ajzenberg-Selove and T. Lauritsen - Nuclear . A114, 1 (1968).

12) T. Nishiyama and H. Ohtsubo - Progr.Theor.Phys. 48, 188 (1972).

13) I.S. Gulkarov et al. - Soviet J.Nuclear Phys. 9, 666 (1973).

14) R. Bertini et al. - Phys.Letters B45, 119 (1973) ;
J. Thirion - Invited Paper at the Fifth International Conference on High Energy Physics and Nuclear Structure, Uppsala (June 1973) ;
J. Saudinos - Private communication.

15) A.B. Clegg - "High Energy Nuclear Reactions", Clarendon Press, Oxford (1965) ;
D. Scipione et al. - Phys.Letters 42B, 489 (1972).

16) H. Hilscher et al. - Nuclear Phys. A158, 593 (1970).