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ASYMPTOTIC BEHAVIOUR OF A DUAL TWO-POMERON GRAPH IN THE MISSING MASS RESONANCE REGION

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A B S T R A C T

The diffraction dissociation contribution to the inclusive cross-section is studied on the basis of a non-planar dual two-loop amplitude. The discussion is restricted to the resonance region in missing mass. Pomeron form factors describing coupling to a scalar particle and a resonance of spin j are defined. In the fixed angle limit a universal exponential behaviour dominating the tree B_6 - contribution has been found.

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1. INTRODUCTION

In the last years considerable interest has been devoted to the application of the dual resonance model to inclusive reactions, in particular, for studying their scaling behaviour [1)-4)]. A disadvantage of this approach was, however, that the Veneziano six-point amplitude used involves only ordinary trajectories, corresponding to secondary Regge poles, and not to the leading Pomeron trajectory. Although the latter one could be taken into account using modified Veneziano amplitudes [4)], it was desirable to describe the Pomeron in the language of the dual loop approach by a non-planar one-loop graph. A first attempt in this direction has been made by Alessandrini, Amati [5)] who showed that a non-planar one loop six-point amplitude leads to results very similar to the non-Pomeron case. In particular, they found the same exponential fall-off of the single particle distribution for large values of the transverse momentum. In this note we investigate a contribution of diffraction dissociation to inclusive reactions which is directly connected with the Pomeron singularity. Its contribution to the inclusive reaction $a+b \rightarrow c+x$ will be described by a six-point amplitude with resonances in the missing mass channel and Pomeron or Regge cut singularities in the $a\bar{c}$ or $b\bar{c}$ channels, respectively. Factorizing the corresponding two-loop amplitude in the missing mass channel we compute, at fixed missing mass, the Regge asymptotic and the fixed angle behaviour of the inclusive (or exclusive) reaction. As a result we define the Pomeron form-factor for the coupling to a scalar particle and a spin j resonance. Further we can show that the fixed angle limit yields a universal exponential behaviour of the cross-section which dominates the contribution of the tree six-point function, the exponent being the square of an expression calculated for the box amplitude with four scalar particles [6)].

The paper is organized as follows. In Section 2 we define the two-loop amplitude and rewrite it in a form suitable for the following calculations. In Section 3, we compute the Regge asymptotic behaviour and in Section 4 the fixed angle limit.

2. DEFINITION OF THE AMPLITUDE

Let us consider the inclusive process

$$a + b \longrightarrow c + X \tag{2.1}$$

near the resonances in the missing mass channel. The differential cross-section reads for s large

$$s\,\frac{d^2\sigma^{incl.}}{dt\,dM^2} = \frac{1}{s}\,disc_{M^2}\,T_{33} = \sum_j s\,\frac{d\sigma^{excl.}}{dt}(ab\to cj)\,\delta(M^2-m_j^2) \tag{2.2}$$

Here T_{33} denotes the three-to-three forward scattering amplitude of the process $a+b+\bar{c}\to a+b+\bar{c}$ and $d\sigma^{excl.}(ab\to cj)/dt$, is the differential cross-section of the (exclusive) process of resonance production $a+b\to c+j$. The Mandelstam variables used are defined by

$$\begin{aligned}
s &= (p_a+p_b)^2 \\
u &= (p_b-p_c)^2 \\
t &= (p_a-p_c)^2 \\
M^2 &= (p_a+p_b-p_c)^2 = P^2 \\
&(s+t+u = M^2+m_a^2+m_b^2+m_c^2)
\end{aligned} \tag{2.3}$$

In the following we are particularly interested in resonance production associated with diffraction dissociation which is usually described by a Pomeron contained, let us say, in the $a\bar{c}$ channel. Therefore, we need a model for a six-point amplitude T_{33} containing resonances in the missing mass variable M^2 and Pomeron singularities in the t variables.

In the dual loop theory the Pomeron is connected with the non-planar self-energy operator $\Sigma_T(p)$. To get a two-Pomeron six-point amplitude we have to join the Reggeons of the Pomeron-like amplitudes $A(a)$, $A(a^+)$ of Fig. 1 with a propagator, leaving the expression

$$T_{33} = \langle 0|A(a)\,D\cdot A(a^+)|0\rangle \tag{2.4}$$

D denotes here the untwisted propagator

$$D(p) = \int_0^1 dx\; x^{R-\alpha(p^2)-1}(1-x)^{\alpha_0-1} \tag{2.5}$$

or twisted propagator

$$\mathcal{D}_T(p) = \int_0^1 dx\; x^{R-\alpha(p^2)-1}\,\Omega(p)\,(1-x)^{W(p)+\alpha_0-1} \tag{2.6}$$

respectively, where

$$R = \sum_{n=1}^{\infty} n\, a^{+(n)} a^{(n)} \quad , \qquad \alpha(s) = \alpha_0 + \tfrac{1}{2}s = \frac{s-m^2}{2}$$

and Ω , W(p) are the twist and gauge operators defined in Refs. 7), 8).
We shall follow now the conventions and notations of Gross and Schwarz [8].
Attaching a projected twisted propagator as well as a symmetric vertex on both sides of their non-planar self-energy operator $\sum_T(p)$ we obtain

$$\begin{aligned}
A(a) = 4\pi^2 g^2 &\int_0^1 du\; u^{-\alpha(t)-1}(1-u)^{\alpha_0-1}\int_0^1 dv\, v^{-\alpha(t)-1}(1-v)^{\alpha_0-1-\alpha(M^2)} \\
\times &\int_0^1 d\omega \int_\omega^1 \frac{dx}{x}\; \frac{\omega^{-\alpha_0-1}}{\ln^2\omega}\,[f(\omega)]^{-4} \\
\times &\exp\Big\{ \frac{s}{2}(1|(u)(1-\omega)E_T(1-\omega)(v)|1) \\
&\quad - \frac{t}{2}\Big[(1|F_T(1-\omega)[(u)+(v)]|1) - 2\ln\Big(\frac{\psi_T(x)}{1-\omega}\Big)\Big] \\
&\quad + m^2\big[(1|(u)(1-\omega)E(1-\omega)(u)|1) \\
&\qquad + (1|(v)(1-\omega)E(1-\omega)(v)|1) \\
&\qquad - (1|(u)(1-\omega)E_T(1-\omega)(v)|1)\big] \\
&\quad + \frac{M^2 m^2}{2}\big[2(1|(v)(1-\omega)E(1-\omega)(v)|1) \\
&\qquad - (1|(u)(1-\omega)E_T(1-\omega)(v)|1) \\
&\qquad - (1|F_T(1-\omega)(v)|1)\big]\Big\} \\
\times &\exp\{(a|h) + (a|H|a)\}
\end{aligned} \tag{2.7}$$

with

$$
\begin{aligned}
|h) = {} & (1-v)M_+(v)(1-\omega)E_T(1-\omega)(u)|1)\,(-p_c) \\
& + 2(1-v)M_+(v)(1-\omega)E(1-\omega)(v)|1)\,(-p) \\
& + (1-v)\left[\,|1) + M_+(v)(1-\omega)\,F_T^t\,|1)\right](p_a - p_c)
\end{aligned}
\qquad (2.8)
$$

$$
H = (1-v)\,M_+(v)\,(1-\omega)\,E\,(1-\omega)(v\rangle\,M_+\,(1-v)
$$

Here $f(w)$ is the partition function, E, E_T, F_T are elliptic matrices, $\Psi_T(x)$ is a function related to the Jacobi's theta function and M_+ a simple number matrix [8].

We now want to calculate the contribution of resonances with $\alpha(M_j^2) = j$ to $\mathrm{disc}_{M^2} T_{33}$. For this we factorize Eq. (2.4) by using the diagonalizing operator τ of Ref. 9)

$$
\tau = (-1)^R\, e^{-\frac{1}{2}L_+}\, \gamma^W \quad , \quad \gamma = \sqrt{1-x} \qquad (2.9)
$$

which satisfies

$$
\begin{aligned}
\tau\, \Omega\, (1-x)^W\, \tau^{-1} &= (-1)^R \\
\tau^+ (4f)^{L_0}\, \tau &= x^{L_0} \quad , \quad x = \frac{4f}{(1+f)^2}
\end{aligned}
\qquad (2.10)
$$

Using the technique of canonical forms [7] and performing a variable transformation

$$
v' = \frac{v}{v(1-\gamma) + \gamma} \qquad (2.11)
$$

in the integrand of $A\ (a^+)$, the following "gauge invariance" of the Pomeron amplitude can be proved

$$
\gamma^W A(a^+)|0\rangle = A(a^+)|0\rangle \qquad (2.12)
$$

With Eqs. (2.10), (2.12) we then get

$$T_{33} = \sum_{\ell_0=0}^{\infty} \sum_{\{\ell\}} a(\ell,\ell_0) \frac{\langle 0|\bar{A}(a)|\{\ell\}\rangle\langle\{\ell\}|\bar{A}(a^+)|0\rangle}{\ell_0+\ell-\alpha(M^2)} \tag{2.13}$$

where

$$a(\ell,\ell_0) = (-1)^{\ell_0}\binom{2\alpha_0-1}{\ell_0}\left(1-\frac{\ell_0}{2\alpha_0-\ell_0}\right)\cdot 4^{\ell-\alpha(M^2)} \times \begin{cases} 1 & \text{for } D \\ (-1)^{\ell} & \text{for } D_T \end{cases}$$

The operatorial amplitude $\bar{A}(a)$ has again the representation (2.7) with the only change

$$\begin{aligned} |h) &\longrightarrow |\bar{h}) = C\,|h) + |\tfrac{1}{2})\,p \\ H &\longrightarrow \bar{H} = CHC^{t} \end{aligned} \tag{2.14}$$

where

$$C_{mn} = (-1)^n \left(\tfrac{1}{2}\right)^{m-n} \sqrt{\frac{n}{m}} \binom{m}{n}$$

Using D and D_T we could then introduce also a signaturized trajectory in the missing mass channel. Note that spurious states cannot couple to the Pomeron graph as follows from Eq. (2.12).

The matrix element of the operatorial part of $\bar{A}(a)$ between vacuum and occupation number states can be calculated according to the formula $\left(D_f^m \equiv \frac{\partial^m}{\partial f^m} \right)$

$$\begin{aligned} \langle 0|\, e^{(a|\bar{h}) + (a|\bar{H}|a)}\, |\{\ell\}\rangle &= \frac{1}{\sqrt{\ell_1!\,\ell_2!\ldots\,\ell_n!\ldots}} \times \\ &\times \ldots \sum_{m_n=0}^{\ell_n} \ldots \sum_{m_2=0}^{\ell_2} \sum_{m_1=0}^{\ell_1} \ldots \binom{\ell_n}{m_n} \ldots \binom{\ell_2}{m_2}\binom{\ell_1}{m_1} \\ &\times \ldots D_{f_n}^{m_n}\left[\ldots D_{f_2}^{m_2} D_{f_1}^{m_1}\, e^{(f|\bar{H}|f)}\right]_{f_i=0} \\ &\times \left(\bar{h}^{(1)}\right)^{\ell_1-m_1} \left(\bar{h}^{(2)}\right)^{\ell_2-m_2} \ldots \left(\bar{h}^{(n)}\right)^{\ell_n-m_n} \ldots \end{aligned} \tag{2.15}$$

Remembering Lorentz indices, the symbolical notation $\binom{\ell_n}{m_n}(\bar{h}^{(n)})^{\ell_n}$ means an abbreviation for $\binom{\ell_n}{m_n}$ tensors

$$\bar{h}^{(n)}_{\mu_1}\,\bar{h}^{(n)}_{\mu_2}\,\ldots\,\bar{h}^{(n)}_{\mu_{\ell_n}}$$

symmetrized in any pair of indices.

At fixed $j = \ell_0 + \ell$ only a finite number of ℓ_n's are different from zero. Thus we see that the matrix elements in Eq. (2.13) are polynomials in the external momenta which have to be contracted in their Lorentz indices. We finally obtain for the contribution of the resonances at $\alpha(M_j^2) = j$

$$\mathrm{disc}_{\alpha(M_j^2)=j}\, T_{33} = \sum_{\ell_0+\sum_n n\ell_n = j} a(\ell,\ell_0)\; A^{j}_{(\mu)}(s,t,M^2)\, A^{*j}_{(\mu)}(s,t,M^2) \tag{2.16}$$

where we have written

$$\langle 0|\bar{A}(a)|\{\ell\}\rangle = A^{j}_{(\mu)}(s,t,M^2)$$

Let us for further applications isolate the contribution of the parent resonance with spin j in Eq. (2.16). Using analogous arguments as in the tree case [7] one sees that the state with maximum angular momentum j inside the energy level $\alpha(M_j^2) = j$ is connected with the occupation numbers $\ell_1 = j$, $\ell_i = 0$ for $i \neq 1$ and $m_1 = 0$. The spin coupling factor of this contribution to Eq. (2.16) reads

$$\begin{aligned}
\frac{1}{j!}\Big[&\{C(1-v)\,M_+(v)\,(1-\omega)E_T(1-\omega)(u)|1)\}_1\,(-p_c) \\
&+ 2\,\{C(1-v)\,M_+(v)\,(1-\omega)\,E\,(1-\omega)(v)|1) - \tfrac{1}{2}|\tfrac{1}{2})\}_1\,(-p) \\
&+ \{C(1-v)[\,|1) + M_+(v)(1-\omega)\,\mathcal{F}_T^{\,t}|1)\}_1\,(p_a - p_c)\Big]^{j}_{(\mu)} \\
\times\; &\Big[\;\ldots\;\Big]^{*j}_{(\mu)}
\end{aligned} \tag{2.17}$$

The contribution of the parent resonance with pure spin j can now be easily obtained by inserting the covariant projection operator $P^{j}_{(\mu),(\nu)}$ for spin j

$$P^{j}_{(\mu),(\nu)} = \sum_{\lambda=-j}^{j} \mathcal{E}^{(j)}_{(\mu)}(\lambda,p)\, \mathcal{E}^{(j)*}_{(\nu)}(\lambda,p) \qquad (2.18)$$

between the tensors of Eq. (2.17) where $\mathcal{E}^{(j)}_{(\mu)}(\lambda,p)$ is the corresponding spin j helicity eigentensor. Recall that the projection operator is symmetric and traceless in the vector indices $(\mu)=\mu_1,\dots,\mu_j$ and $(\nu)=\nu_1,\dots,\nu_j$ separately, and dotted into p, gives zero.

3. REGGE LIMIT

Now let us proceed to calculate the Regge asymptotic behaviour as $s\to\infty$ at t and M^2 fixed of the inclusive (or exclusive) cross-section *). To do this it is useful to perform Jacobi's imaginary transformation for the variables x, w of the self-energy part, defined by

$$\ln\omega = \frac{2\pi^2}{\ln q} \quad , \qquad \frac{2\pi \ln x}{\ln\omega} = \sigma \qquad (3.1)$$

The asymptotic behaviour of Eq. (2.16) is then determined by the behaviour of the integrand of Eq. (2.7) [combined with Eq. (2.17)] near the critical points of the function $(1|(u)(1-\omega)E_T(1-\omega)(v)|1)$ which multiplies the asymptotic variable s. A critical point means either a saddle point in all variables (critical point of the first kind) or a point where the hypersurface $(1|(u)(1-\omega)E_T(1-\omega)(v)|1)=\text{const.}$ becomes tangent to some bounding hypersurface (critical point of the second kind or end-point singularity) [6]. One can see that in our case there exist only end-point singularities given by $q=0$ (all u, v, σ), $u=0$ or 1, (all q, σ, v) and $v=0, 1$ (all q, u, σ) with $(1|(u)(1-\omega)E_T(1-\omega)(v)|1)\equiv 0$ **). Only the $q=0$

---

*) Strictly speaking this limit has to be performed inside the strip of convergence of the amplitude parallel to the imaginary s axis. The result should then be continued analytically in other directions of the complex s plane.

**) This can be most easily seen by representing the above matrix element of E_T as a quotient of functions $\Psi_T(x)$ (see Section 4) and applying the symmetry arguments of Ref. 6).

end-point is connected with the Pomeron singularity leading to an asymptotic behaviour $s^{\alpha_p(t)}$, the other contributions being connected with renormalization of the corresponding tree-graph. For the following, we shall consider only the end-point $q=0$. In order to compute the leading term of the asymptotic series we have now to expand the integrand in powers of q and carry out the q integration. For this we recall the definitions [8)]

$$[E_T]_{mn} = -\frac{\sqrt{mn}}{m!\,n!}\left(-\frac{\partial}{\partial\lambda}\right)^m\left(-\frac{\partial}{\partial\mu}\right)^n \ln\left[\psi_T(x\lambda\mu)\right]\Big|_{\lambda=\mu=1} \qquad (3.2)$$

$$[E]_{mn} = \frac{1}{2}\frac{\sqrt{mn}}{m!\,n!}\left(-\frac{\partial}{\partial x}\right)^m\left(-\frac{\partial}{\partial y}\right)^n \ln\left[\frac{y-x}{\psi(\frac{x}{y})}\right]\Big|_{x=y=1} \qquad (3.3)$$

$$[(1|F_T]_m = \frac{\sqrt{m}}{m!}\left(-\frac{\partial}{\partial\lambda}\right)^m \ln\left[\frac{\psi(\lambda)}{\psi_T(x\cdot\lambda)(1-\lambda)}\right]\Big|_{\lambda=1} \qquad (3.4)$$

Using the Jacobi transformed functions $\psi(x)$, $\psi_T(x)$ [10)]

$$\log\psi(x) = \log\left[-\frac{2\pi}{\log q}\right] + \log\sin\frac{\sigma}{2} - \sum_{r=1}^{\infty}\frac{q^{2r}}{r(1-q^{2r})}\left[e^{ir\sigma}+e^{-ir\sigma}-2\right] \qquad (3.5)$$

$$\log\psi_T(x) = \log\left[\frac{q^{-\frac{1}{4}}(-\pi^2)}{\log q}\right] - \sum_{r=1}^{\infty}\frac{q^r}{r(1-q^{2r})}\left[e^{ir\sigma}+e^{-ir\sigma}-2q^r\right] \qquad (3.6)$$

we obtain as $q \to 0$ $(w \to 1)$

$$\{(1-\omega)E_T(1-\omega)\}_{mn} \sim -\frac{\sqrt{mn}}{m!\,n!}\, q\left[e^{i\sigma}(2\pi i)^{m+n} + e^{-i\sigma}(-2\pi i)^{m+n}\right] + O(q^2) \tag{3.7}$$

$$\{(1-\omega)E(1-\omega)\}_{mn} \sim \frac{1}{2}\Big\{\sum_{k\neq 0}\left(\frac{1}{k}\right)M_+\left(-\frac{1}{k}\right)\Big\}_{mn} + O(q^2) \tag{3.8}$$

$$\{(1|\mathcal{F}_T(1-\omega)\}_m \sim \Big\{(1|\Big[-\sum_{k\neq 0}\left(\frac{1}{k}\right)\Big]\Big\}_m + O(q) \tag{3.9}$$

Taking into account Eqs. (3.7)-(3.9) as well as the formula

$$\log\frac{\sin\pi z}{\pi z} = \sum_{n\neq 0}\log\left(1-\frac{z}{n}\right) \tag{3.10}$$

we get further

$$(1|(u)(1-\omega)E_T(1-\omega)(v)|1) \sim 8q\sin\pi u\sin\pi v\cos(\sigma+\pi(u+v)) + O(q^2) \tag{3.11}$$

$$(1|(u)(1-\omega)E(1-\omega)(u)|1) \sim \log\frac{\sin\pi u}{\pi u} + O(q^2) \tag{3.12}$$

$$(1|\mathcal{F}_T(1-\omega)(u)|1) \sim \log\frac{\sin\pi u}{\pi u} + O(q) \tag{3.13}$$

Similarly, the terms of Eq. (2.17) have the expansion

$$\{C(1-v)M_+(v)(1-\omega)E_T(1-\omega)(u)|1)\}_1 \sim$$

$$\sim\left[-2\pi v(1-v)\right]\cdot 4q\sin\pi u\cos(\sigma+\pi(u+v)+\pi v) \tag{3.14}$$

$$2\{C(1-v)M_+(v)(1-\omega)E(1-\omega)(v)|1)\}_1 \sim$$

$$\sim \pi v(1-v)\,\mathrm{ctg}\,\pi v - (1-v)$$

and

$$\left\{ C\,(1-v)\left[\,(1) + M_+(v)(1-\omega)\,\mathcal{F}_\tau^{\prime t}(1)\right]\right\}_1 \sim \left[-2\pi v(1-v)\right] \frac{ctg\,\pi v}{2} \qquad (3.15)$$

Taking into account the well-known relation

$$\frac{4\pi^2}{ln^2\omega}\left[f(\omega)\right]^{-4} = \omega^{\frac{1}{6}}\, q^{-\frac{1}{3}}\left[f(q^2)\right]^{-4} \sim q^{-\frac{1}{3}} \qquad (3.16)$$

we now obtain by the use of the above formulas, and restricting us, for illustration, to the contribution of the parent spin j resonance

$$\begin{aligned}
\varepsilon^{(j)}_{(\mu)} A^{j}_{(\mu)}(s,t,M^2) = &-\frac{2^{1-t}}{\sqrt{j!}}\,\pi^3 g^2 \\
&\times \int_0^1 du\; u^{-1-\alpha(t)}(1-u)^{-\frac{m^2}{2}-1}\left(\frac{\sin\pi u}{\pi u}\right)^{m^2-\frac{t}{2}} \\
&\times \int_0^1 dv\; v^{-1-\alpha(t)}(1-v)^{-\frac{m^2}{2}-1-j}\left(\frac{\sin\pi v}{\pi v}\right)^{m^2-\frac{t}{2}+j} \\
&\times \int_0^1 \frac{dq}{ln^3 q}\, q^{-\alpha_p(t)-1}\int_0^{2\pi} d\sigma \\
&\times \exp\left\{\frac{s}{2}\cdot q\cdot 8\sin\pi u\cdot\sin\pi v\,\cos(\sigma+\pi(u+v))\right\} \\
&\times \left[-2\pi v(1-v)\right]^{j} \\
&\times \Big[4q\sin\pi u\cos(\sigma+\pi(u+v)+\pi v)(-p_c) \\
&\qquad + \tfrac{1}{2}\,ctg\,\pi v\cdot(p_a-p_c)\Big]^{j}_{(\mu)}\cdot\varepsilon^{(j)}_{(\mu)}(\lambda,p)
\end{aligned} \qquad (3.17)$$

where $\alpha_p(t) = 1/3 + t/4$. The expansion of the bracket of Eq. (3.17) into tensor products of the vectors p_c and $(p_a - p_c)$ yields further

$$\varepsilon^{(j)}_{(\mu)} A^j_{(\mu)}(s,t,M^2) = \sum_{n=0}^{j} A^j_n(s,t,M^2) \left[\binom{j}{n} (p_c)^n (p_a - p_c)^{j-n} \right] \varepsilon^{(j)}_{(\mu)(\mu)}(\lambda, p) \tag{3.18}$$

The new bracket has again to be understood as a symbolical notation for the sum of $\binom{j}{n}$ direct products of n factors p_c and $(j-n)$ factors $(p_a - p_c)$ symmetrized in any pair of Lorentz indices; $A^j_n(s,t,M^2)$ are the invariant amplitudes of the resonance production process $a + b \to c + (j)$. Performing the different integrations and using

$$\begin{aligned} &\int_0^{2\pi} d\sigma \left[-\cos(\sigma + \pi(u+v)) \right]^{\alpha_p - k} \left[\sin(\sigma + \pi(u+v)) \right]^k \\ &= \int_0^{2\pi} d\sigma' \left[-\cos\sigma' \right]^{\alpha_p - k} \left[\sin\sigma' \right]^k \\ &= \frac{1}{2}(1 + e^{i\pi\alpha_p})(1 + (-1)^k)\, B_4\left(\frac{\alpha_p - k + 1}{2}, \frac{k+1}{2} \right) \end{aligned} \tag{3.19}$$

we have finally

$$\begin{aligned} A^j_n(s,t,M^2) &= \frac{(-1)^{j+1}}{\sqrt{j!}}\, g^2\, \pi^{5+t} \cdot 2^{2+3\alpha_p(t) - t + 2m^2} \\ &\times \Gamma(n - \alpha_p(t))\, \frac{\left(\frac{s}{2}\right)^{\alpha_p(t) - n}}{\ln^3 s}\, (1 + e^{i\pi\alpha_p(t)}) \\ &\times f_p(t) \sum_{k=0}^{n} \binom{n}{k} f^j_{P(k)}(t)\, [1 + (-1)^k]\, B_4\left(\frac{\alpha_p(t) + 1 - k}{2}, \frac{k+1}{2} \right) \end{aligned} \tag{3.20}$$

The Pomeron form-factors for coupling to two scalar particles or to one scalar and one spin j particle are defined by

$$f_P(t) = (2\pi)^{-(1+m^2)} \int_0^1 du \, [u(1-u)]^{-\frac{m^2}{2}-1} (\sin \pi u)^{m^2-\frac{t}{2}+\alpha_P(t)} \tag{3.21}$$

$$f^j_{P(k)}(t) = (2\pi)^{-(1+m^2)} \int_0^1 dv \, [v(1-v)]^{-\frac{m^2}{2}-1} (\sin \pi v)^{m^2-\frac{t}{2}+\alpha_P(t)} \times \rho^j_{(k)}(v) \tag{3.22}$$

with

$$\rho^j_{(k)}(v) = (\sin \pi v)^k \, (\cos \pi v)^{j-k}$$

The factors in front of the integrals have been introduced in order to compare with the Pomeron form-factor for scalar particle coupling of Ref. 6). If we write $f^j_{p(k)}(t)$ as a sum of poles

$$f^j_{P(k)}(t) = \sum_{m=0}^{\infty} \frac{R^j_{m(k)}}{\alpha_p(t) - \alpha_R(t) + k - m} \tag{3.23}$$

we see that spin coupling leads to a shifted f^o dominance of the form-factor.

The asymptotic contribution of the parent resonance of spin j to the cross-section Eq. (2.2) can then easily be computed by contracting tensors

$$\left[\binom{j}{n} (p_c)^n (p_a - p_c)^{j-n} \right]_{(\mu)}$$

of Eq. (3.18) with the projection operator for spin j expressed in terms of $g_{\mu\nu}$ and p_μ [11].

4. FIXED ANGLE LIMIT

It has been shown [6] that the Pomeron amplitude with four external scalar particles reproduces in the fixed angle limit $s \to \infty$, $t = -(1-\cos\Theta/2)s \to -\infty$ the exponential behaviour $\exp(-\alpha' sF(\cos\Theta))$ of the Veneziano amplitude with the difference, however, that the function $F(\cos\Theta)$ is multiplied by $-\alpha_p' s$ instead of $-\alpha' s$. Taking into account $\alpha_p' = (1/2)\alpha'$ the Pomeron amplitude will thus give the leading behaviour in the fixed angle limit. It is easy to show that this result will remain also valid for a four-point function with one excited leg so that the same fixed angle behaviour will likewise appear in the resonance contributions to the single particle distribution, Eq. (2.2). As expected, the fixed angle exponent will be found to be independent of the spin of the resonances, its form being connected only with the duality properties of the underlying graph.

To derive this result we go back to the expression (2.16) recalling that the matrix element of the operatorial part of $\bar{A}(a)$ leads only to a polynomial in the asymptotic variables s and t (M^2 fixed!). The exponential behaviour of Eq. (2.16) in the above limit will then be determined by the saddle points in all variables of the function

$$\begin{aligned} V_\kappa &= -(1|(u)(1-\omega)E_T(1-\omega)(v)|1) \\ &\quad -\kappa\left[(1|F_T(1-\omega)[(u)+(v)]|1) - 2\ln\left(\frac{\psi_T(x)}{1-\omega}\right)\right] \\ \kappa &= \frac{1-\cos\theta}{2} \end{aligned} \tag{4.1}$$

For the further discussion it will be convenient to use the following representation of the matrix elements of E_T and F_T [8]

$$(1|(1-\lambda)E_T(1-\mu)|1) = \log\left[\frac{\psi_T(x\lambda)\,\psi_T(x\mu)}{\psi_T(x)\,\psi_T(x\lambda\mu)}\right] \tag{4.2}$$

$$(1|F_T(1-\lambda)|1) = \log\left[\frac{\psi_T(x)\,\psi(\lambda)}{\psi_T(x\lambda)\,(1-\lambda)}\right] \tag{4.3}$$

Let us put

$$\begin{aligned} \lambda &= 1-(1-\omega)u \\ \mu &= 1-(1-\omega)v \end{aligned} \tag{4.4}$$

and introduce the auxiliary variables

$$\eta = \frac{2\pi \log \mu}{\log \omega}$$

$$\varphi = \frac{2\pi \log \frac{\omega}{\lambda}}{\log \omega} \qquad (4.5)$$

$$\tilde{\sigma} = \frac{\pi \log x^2 \mu \lambda / \omega}{\log \omega}$$

Expressing the functions $\psi_T(x), \psi(x)$ by Jacobi's theta function Θ_4, Θ_1 and using the above formulas we may then rewrite Eq. (4.1) in the form

$$V_\kappa = \log \left[\frac{\Theta_4(\tilde{\sigma} + \frac{\varphi - \eta}{2} | \tau)\, \Theta_4(\tilde{\sigma} - \frac{\varphi - \eta}{2} | \tau)}{\Theta_4(\tilde{\sigma} + \frac{\varphi + \eta}{2} | \tau)\, \Theta_4(\tilde{\sigma} - \frac{\varphi + \eta}{2} | \tau)} \right]$$

$$- \kappa \cdot \log \left[\frac{\Theta_1(\varphi | \tau)\, \Theta_1(-\eta | \tau)}{\Theta_4(\tilde{\sigma} + \frac{\varphi + \eta}{2} | \tau)\, \Theta_4(\tilde{\sigma} - \frac{\varphi + \eta}{2} | \tau)} \right] \qquad (4.6)$$

$$\left(\tau = -\frac{i}{\pi} \log q \right)$$

The expression (4.6) is identical with the expression of the Pomeron amplitude for external scalar particles found in [6]. It has been shown there using the properties of elliptic functions that there exists a saddle point in the integration region, given by $\tilde{\sigma} = \eta = \varphi = \pi$ or, expressed in the old variables, at $u = v = (1 + \omega_0^{\frac{1}{2}})^{-1}$, $x = \omega_0^{\frac{1}{2}}$ with q_0 determined by the equation

$$Y(q_0) \equiv \prod_{n=1}^{\infty} \left[\frac{1 - q_0^{2n-1}}{1 + q_0^{2n-1}} \right]^{-8} = \frac{1}{1-\kappa} \qquad (4.7)$$

Evaluating V_κ at this saddle point gives the result

$$V_\kappa(q_0) = \frac{1}{2}\left[\log Y(q_0) - \kappa \log (Y(q_0) - 1)\right] = -\frac{1}{2}\log\left[(\kappa)^{\kappa}(1-\kappa)^{1-\kappa}\right] \equiv \frac{1}{2}F(\cos\Theta) \tag{4.8}$$

Thus one obtains, finally $(\alpha' = \frac{1}{2})$

$$s\,\frac{d^2\sigma^{\text{incl.}}}{dt\;dM^2}\Big|_{\alpha(M_j^2)=j} \sim \left(e^{-\frac{s}{4}F(\cos\Theta)}\right)^2 \cdot R^2(s,\kappa,M^2) \tag{4.9}$$

where R is a polynomial in s. Factorizing a usual B_6 amplitude into the product of two B_4 amplitudes times a polynomial in s, t, M^2 one can get the same exponential behaviour as in Eq. (4.9) with 1/4 replaced by 1/2. The Pomeron contribution considered will thus dominate over the usual tree graph. The universal exponential behaviour of Eq. (4.9) holds also for each parent resonance separately, independent of its spin. These results have also been obtained by a phenomenological description of dual loop graphs [12].

It would be very interesting to compute the Regge and the fixed angle limit of the two-loop graph considered without the simplifying restriction of fixed missing mass. For $M^2 \to \infty$ one has to take into account then the contribution of an indefinitely increasing number of resonances, and to do this, one has to start with the two-loop amplitude in the non-factorized form of Ref. 13). To solve this problem the technique of domain variational methods for handling automorphic functions should be used. This question will be investigated further in connection with the 3-Pomeron vertex.

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NOTE ADDED IN PROOF

After completion of this work we found a paper of Morel and Quiros [14] where the Pomeron amplitude Eq. (3.17) has been studied, too, in connection with investigations of helicity conservation. These authors started from the general loop amplitude with external excited particles. It can be shown in the one-loop case that our expression is equivalent to their general formula. As can easily be seen from Eq. (3.22) we have also decoupling of spin-odd resonances, but, contrarily to them, we find, by factorization, the usual scalar particle Pomeron form-factor. Further, there are deviations in the spin-dependent form-factors arising from an incorrect treatment of the Jacobi transformation in Ref. 14).

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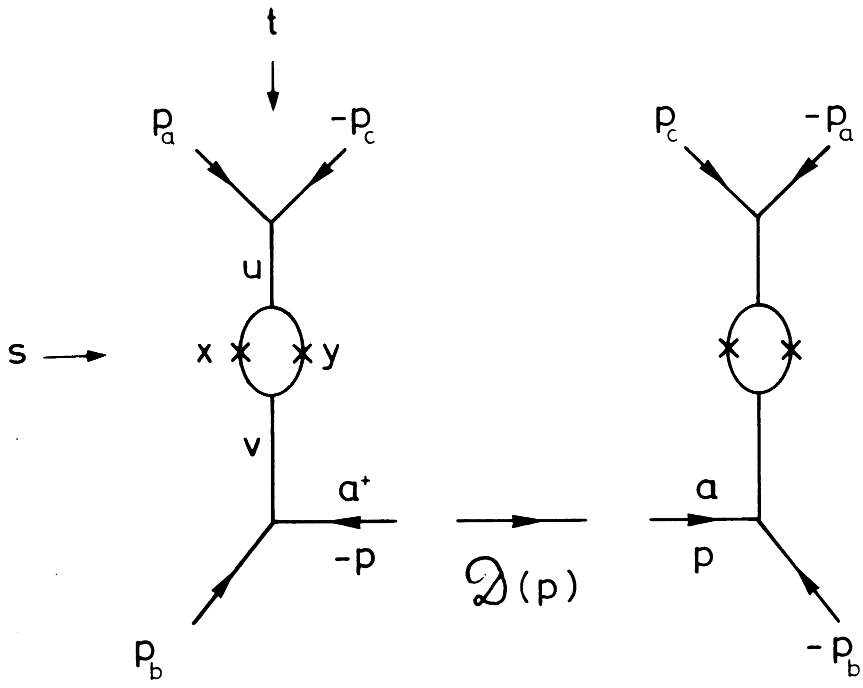

Fig. 1