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QCD Corrections to the Bottom and Charm Forward-Backward Asymmetries

OPAL COLLABORATION

Abstract

The forward-backward asymmetries of $e^+e^- \to c\bar{c}$ and $e^+e^- \to b\bar{b}$ are affected by significant QCD effects. To interpret the measured asymmetries in terms of the Standard Model parameters these effects have to be accounted for. In this note the necessary corrections are discussed, their influence on the experimental results is evaluated and corrections for the different OPAL asymmetry analyses are given.

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1 Introduction

OPAL has measured the charm and bottom forward-backward asymmetries using three different methods: leptons [1], jet-charge [2] and D mesons [3]. In these papers the asymmetries quoted are measured asymmetries, which include effects from QCD, QED etc. To arrive at the "bare" asymmetries, the measured asymmetries need to be corrected for these effects. Up to now the same corrections, based on analytical calculations and Monte Carlo simulations, have been applied to the lepton and the D meson analyses. Recently [4] it has been shown that the corrections might be significantly biased by the respective analysis methods employed, and that an individual evaluation of the corrections is needed for each analysis. This has already been done in [2] for the jet-charge asymmetry.

In this note the evaluation of the QCD corrections for the two OPAL analyses concerned [1,3] is described. It is intended as a complement to the existing OPAL papers and physics notes, and replaces the QCD corrections quoted there.

In the first part of this note a number of Monte Carlo studies are summarised and compared to analytical calculations. From this an underlying QCD correction is derived, before any experimental effects or biases are taken into account. A description of the evaluation of the experimental bias for the lepton and the D analysis follows and the values of the asymmetries, after applying the full QCD corrections, are given. These numbers are the ones which will be used in the LEP heavy flavour averages.

2 QCD Corrections

The forward-backward asymmetries in the Standard Model are defined relative to the angle between the incoming and the outgoing fermion in the reaction $\mathrm{e^+e^-} \rightarrow \mathrm{q\overline{q}}$. While the incoming fermion direction is very well known, the direction of the outgoing fermion is not measurable. It is approximated in [1, 3] by the thrust direction. This direction however is not exactly the same as the primary quark direction, mostly due to gluon bremsstrahlung and effects in the hadronisation process. In figure 1 some of the topologies in lowest order which might influence the asymmetry are shown. In particular the emission of a hard gluon into one hemisphere, with the two quarks recoiling into the other hemisphere, essentially destroys any information about the asymmetry for this event. There is no distinction possible between the quark and the antiquark, and the measured asymmetry in these events is zero. The measured asymmetries therefore have to be corrected for these effects to get the bare asymmetries.

In the Standard Model the differential cross-section $d\sigma/d\cos\theta$ in its most general form is given by [5]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}(\cos\theta)} = \frac{3}{8}(1+\cos^2\theta)\,\sigma_U + \frac{3}{4}\sin^2\theta\,\sigma_L + \frac{3}{4}\cos\theta\,\sigma_F\ , \tag{1}$$

where $\sigma_{U,L}$ are the unpolarised or longitudinally polarised cross-sections and σ_F is the difference between the right- and left-handed polarised cross-section. The angle θ is measured between the incoming and the outgoing fermion. This cross-section describes the decay of a spin one boson into two spin 1/2 fermions. If the fermions were massless no longitudinal component would be present. For massive fermions like heavy quarks discussed here a small longitudinal component is expected. In the presence of gluon radiation in the final state, as illustrated in

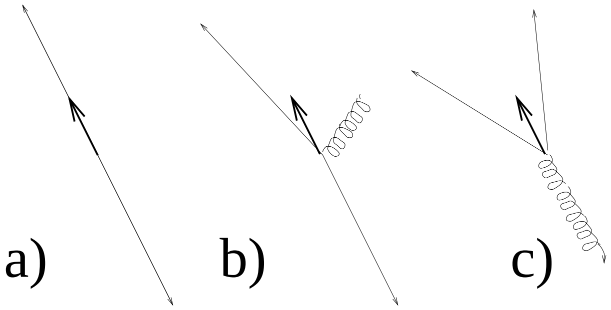

Figure 1: Pictorial representations of the different topologies which can influence the asymmetry. For each figure the direction of the tagged particles is indicated by the thin arrows, the gluon direction by the curled line, and the thrust direction by the thick arrow. (a) No gluon radiation: thrust direction and quark direction are equal; (b) "Soft" gluon radiation: The thrust direction is a good estimator of the primary $q\overline{q}$ direction, and the charge of the primary quark is unambiguously related to the charge of the tagged particle in the hemisphere; (c) "Hard" gluon radiation: The thrust direction is no longer a good estimator of the primary quark direction, and no charge information is contained in the hemisphere.

figure 1, the relative contributions of the different cross-sections are modified, and in particular the longitudinal cross-section increases and is no longer negligible.

Eq. 1 can be rewritten as

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{3}{6+2a}(1 + a\cos^2\theta) + A_{\mathrm{FB}}\cos\theta \tag{2}$$

where

$$\sigma = \sigma_U + \sigma_L \qquad a = \frac{\sigma_U - 2\sigma_L}{\sigma_U + 2\sigma_L} \tag{3}$$

and the forward-backward asymmetry is defined by

$$A_{\mathrm{FB}} = \frac{3}{4}\frac{\sigma_F}{\sigma_U + 2\sigma_L} \ . \tag{4}$$

In the presence of gluon radiation this defines an effective asymmetry, which is related to the bare asymmetry, A^0_{FB}, determined from the direction of the primary quark, by

$$A_{\mathrm{FB}} \equiv A^{\mathrm{theo}}_{\mathrm{FB}} = (1 - \delta^{\mathrm{theo}})A^0_{\mathrm{FB}}. \tag{5}$$

Eq. 5 is true quite generally as long as no experimental effects are taken into account. Experimental effects are, for example, the event selection, cuts, finite acceptances or the method itself used to determine the value of the asymmetry. Such Biases might change the observed asymmetry significantly relatively to $A^{\mathrm{theo}}_{\mathrm{FB}}$. This is parametrised by an experimental bias factor $\mathcal{B}$ defined by

$$A^{\mathrm{obs}}_{\mathrm{FB}} = (1 - \mathcal{B}\delta^{\mathrm{theo}})\, A^0_{\mathrm{FB}} \ . \tag{6}$$

It is this bias factor $\mathcal{B}$ which is different for the individual analyses. In the following two sections the theoretical QCD correction δ^{theo} will be derived, followed by a discussion of the experimental bias $\mathcal{B}$ for each of the three analyses.

3 Determination of the Theoretical QCD Correction

A number of analytic calculations exist which predict the QCD effects for the forward-backward asymmetries [5,6]. In most cases the correction quoted is that between the asymmetry measured using the primary quark direction and the asymmetry measured using the quark direction at the end of the perturbative phase ("after gluon radiation"). In [5] a correction is also quoted relative to the thrust direction, calculated using all partons before hadronisation. The calculations in [5] are in first order in α_s, while in [6] some results are given in order α_s^2. In this note the results from the first order calculations are used, while the results from [6] are used to estimate the error made by neglecting higher order effects.

The QCD corrections are also predicted by Monte Carlo programs. A comparison has been made between the analytic calculations and Monte Carlo models in order to estimate their reliability. These studies are done on the parton and on the hadron level, but without taking into account detector or experimental effects. The JETSET 7.4 Monte Carlo [7] and the HERWIG Monte Carlo [8] are used for the comparison. Both Monte Carlo models have been tuned to describe the OPAL data [9]. QCD corrections are calculated at three different stages of the fragmentation process and compared to analytical predictions, where possible:

- δ^{part}, calculated using the direction of the quark after gluon radiation;
- $\delta^{\mathrm{part,T}}$, calculated from the direction of the thrust axis using partons after gluon radiation;
- $\delta^{\mathrm{had,T}}$, calculated from the direction of the thrust axis after hadronisation, using all stable particles (including neutrinos).

Analytical calculations are only available for the first two cases, the third one is accessible only using Monte Carlo models.

The asymmetry for this study is determined as

$$A_{\mathrm{FB}} = \frac{n_f - n_b}{n_f + n_b}\ , \qquad (7)$$

where n_f, n_b are the number of events counted in the forward and backward hemisphere, respectively, at the appropriate stage in the fragmentation. The hemispheres are defined by the direction of the quark after gluon radiation, relative to the direction of the incoming electron. This definition has the advantage that it is independent of the exact shape of the underlying differential cross-section. The results are based on a Monte Carlo sample of approximately three million hadronic Z^0 decays into each of the flavours investigated. The number of events in the forward or backward hemisphere before and after QCD effects are highly correlated. These correlations are taken into account when calculating the error on the correction.

Events where a gluon splits into a pair of heavy quarks will also influence the observed asymmetry. Such events have a high probability to be tagged as a heavy flavour event, however they do not contribute to the asymmetry. These types of events are explicitly excluded from the calculation of the theoretical QCD corrections, and are also removed from the Monte Carlo studies.

The results of these studies are summarised in table 1 for $\mathrm{e^+e^-} \rightarrow \mathrm{b\bar{b}}$ events and for $\mathrm{e^+e^-} \rightarrow \mathrm{c\bar{c}}$ events. In general the agreement between theory and Monte Carlo is much better

	calculation	JETSET	HERWIG
	δ_{QCD} for $e^+e^- \to b\bar{b}$		
δ^{part}	$+0.0350 \pm 0.0020$	$+0.0402 \pm 0.0018$	$+0.0411 \pm .0014$
$\delta^{\mathrm{part,T}}$	$+0.0311 \pm 0.0025$	$+0.0304 \pm 0.0019$	$+0.0286 \pm .0013$
$\delta^{\mathrm{had,T}}$		$+0.0302 \pm 0.0020$	$+0.0275 \pm .0014$
	δ_{QCD} for $e^+e^- \to c\bar{c}$		
δ^{part}	$+0.0440 \pm 0.0030$	$+0.0495 \pm 0.0018$	$+0.0401 \pm 0.0040$
$\delta^{\mathrm{part,T}}$	$+0.0359 \pm 0.0040$	$+0.0403 \pm 0.0018$	$+0.0211 \pm 0.0038$
$\delta^{\mathrm{had,T}}$		$+0.0380 \pm 0.0019$	$+0.0216 \pm 0.0039$

Table 1: Table of QCD corrections calculated at different levels of the parton shower development for $e^+e^- \to c\bar{c}$ and $e^+e^- \to b\bar{b}$ events. Shown are results from theory, from the JETSET and from the HERWIG Monte Carlo models. The agreement between JETSET and the calculation is in general better than the one between HERWIG and the calculation. This is particularly obvious for $e^+e^- \to c\bar{c}$ events. The error on the analytical calculation is an estimate of higher order contributions, the one on the Monte Carlo numbers reflects the size of the samples.

on the parton thrust level than on the quark level. This is true both for bottom and for charm events, though overall charm events seem to be less well described than bottom events, and HERWIG seems to be more different from the calculation than JETSET. In particular the discrepancy between JETSET and HERWIG are not yet fully understood, and will need further investigations. The error on the theoretical calculations is an estimate of the contributions from higher orders. It also takes quark mass effects, the uncertainty of α_s and the uncertainty of the mass scale into account. A detailed discussion may be found in [4].

The mean value for the QCD correction $\delta^{\mathrm{part,T}} \equiv \delta^{\mathrm{part,T}}_{\mathrm{calc}}$, without taking any detector effects into account, is taken from the analytical calculations for both $e^+e^- \to b\bar{b}$ and $e^+e^- \to c\bar{c}$ events, evaluated using the thrust on the parton level. An additional correction, $\Delta^{\mathrm{had}}_{\mathrm{MC}}$, is applied to this for the changes due to hadronisation, which is taken from Monte Carlo. The mean value chosen for this reflects also the influence from the particular tune of the Monte Carlo, derived from a comparison between the four LEP experiments [4]. The error quoted does not include a contribution due to the differences between JETSET and HERWIG. The final theoretical correction δ^{theo} is defined as $\delta^{\mathrm{theo}} = \delta^{\mathrm{part,T}}_{\mathrm{calc}} + \Delta^{\mathrm{had}}_{\mathrm{MC}}$

The error for this underlying correction is derived from the theoretical error and by assigning a 100% error to the last step, the correction because of the influence of the hadronisation. This procedure is the same as adopted by the four LEP collaborations [4]. The final numbers are tabulated in table 2.

4 Experimental bias

Experimental biases in the asymmetry measurement can be introduced by the detector and by the selection procedure used in the analysis. Detector effects are already considered in the OPAL analyses, and systematic errors are given for this. The most important detector effect is a possible charge asymmetry of the detector, an effect which is most significant for the jet-charge

correction	$e^+e^- \to b\overline{b}$	$e^+e^- \to c\overline{c}$
$\delta^{\rm part,T}_{\rm calc}$	$+0.0311 \pm 0.0025$	$+0.0359 \pm 0.0040$
$\Delta^{\rm had}_{\rm MC} = \delta^{\rm had,T}_{\rm MC} - \delta^{\rm part,T}_{\rm MC}$	-0.0025 ± 0.0025	-0.0038 ± 0.0038
$\delta^{\rm theo}$	$+0.0285 \pm 0.0036$	$+0.0321 \pm 0.0058$

Table 2: Table of theoretical QCD corrections evaluated on the hadron thrust level. The first number, $\delta^{\rm part,T}_{\rm calc}$, is taken from analytical calculations, while the hadronisation correction $\Delta^{\rm had}_{\rm MC}$ is derived using Monte Carlo.

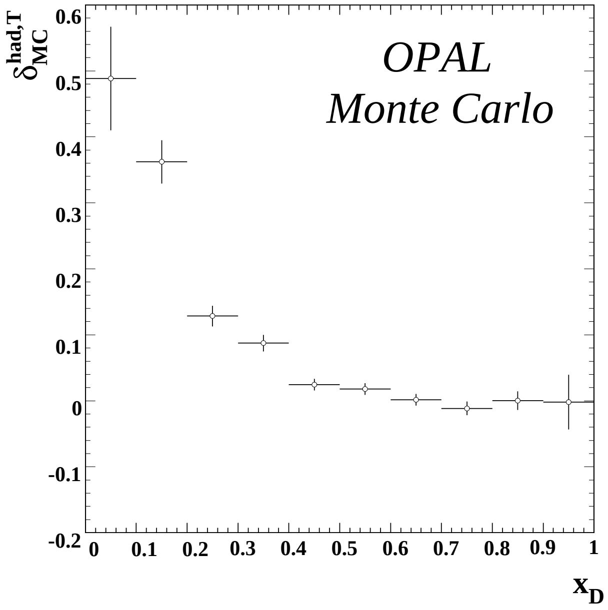

Figure 2: Dependence of the QCD correction of the charm forward-backward asymmetry in $e^+e^- \to c\overline{c}$ events tagged by a D mesons, as a function of the scaled energy of the D.

analysis [2]. Experimental selection biases can also change the size of the QCD corrections, in most cases decreasing the actual size of the correction. This depends entirely on the details of the analysis and the cuts used, and therefore has to be evaluated individually for each analysis.

The dominant part of the QCD corrections comes from events as shown in figure 1(c), where a hard gluon is emitted, recoiling against the $q\overline{q}$ system. As stated earlier such events have no measurable asymmetry if the thrust axis is used as a direction estimate. On the other hand the selection of such events is suppressed if for example high momentum leptons or D mesons are selected. The fraction of events of type (c) therefore will be reduced in the sample used to measure the asymmetry, and the QCD correction in this sample will be smaller than naively expected. This is illustrated in figure 2, where the QCD correction is shown as a function of the scaled energy $x_{\rm D} = E_{\rm D}/E_{\rm beam}$ of the selected D meson in $e^+e^- \to c\overline{c}$ events. For high energy D mesons the QCD corrections essentially dissapear.

Other important effects are introduced by the fitting method itself. In the OPAL analyses the asymmetry has been fitted using eq. 2 with $a = 1$. However Monte Carlo studies show that

a is different from one by around -5%. Fitting with this approximation biases the measured asymmetry, decreasing the QCD corrections by around 14%. It is therefore very important that the final bias is evaluated using as far as possible the same methods as were used in the actual analysis. This effect is absorbed into the experimental bias $\mathcal{B}$.

In the following two sections the evaluation of the experimental bias $\mathcal{B}$ is described for each analysis. In all cases a fit is done to the angular distribution of the thrust axis, closely following the original analyses. From this the observed asymmetry is extracted. A similar fit is performed to the angular distribution of the primary quark-anti quark pair, and the bare asymmetry is determined. Exactly the same sample of events is used for this second fit. From this the ratio $A_{\mathrm{FB}}^{\mathrm{obs}}/A_{\mathrm{FB}}^{0}$ is calculated. Also using Monte Carlo events, but this time without taking detector effects into account, the ratio $A_{\mathrm{FB}}^{\mathrm{had,T}}/A_{\mathrm{FB}}^{0}$ is calculated. Again the same events are used in calculating the ratios, thus minimising the statistical error. Note however that the events used to calculate the first and the second ratio are not identical. The experimental bias is defined in terms of these double ratios of asymmetries:

$$\mathcal{B} = \left(1 - \frac{A_{\mathrm{FB}}^{\mathrm{obs}}}{A_{\mathrm{FB}}^{0}}\right)_{\mathrm{MC}}^{\mathrm{det}} \Bigg/ \left(1 - \frac{A_{\mathrm{FB}}^{\mathrm{had,T}}}{A_{\mathrm{FB}}^{0}}\right)_{\mathrm{MC}} . \tag{8}$$

The superscript "det" indicates that these variables have been evaluated including experimental effects. The full QCD correction is determined by multiplying the analytical prediction for the QCD correction given in the previous section with the bias: $\delta_{QCD} = \mathcal{B}\delta^{\mathrm{theo}}$.

4.1 Lepton Analysis

The lepton selection and its separation into a prompt b $\rightarrow \ell$ component, a prompt c $\rightarrow \ell$ component, and background is made using several neural nets. Details of this procedure can be found in [1]. Instead of cutting on several quantities the analysis uses the output of two neural networks as discriminators. Individual events are weighted by these net outputs, and the asymmetry is extracted from a binned likelihood fit to the differential asymmetry distribution.

The experimental bias from this method is calculated by repeating these steps for Monte Carlo events. However the method has been slightly simplified:

- Since the analysis is only done on Monte Carlo, no background enters into the sample. The neural nets therefore are not needed for the flavour and signal to background separation, but only insofar as they bias the event selection. Therefore a coarser binning in the net output distributions has been used.
- Similarly the fit in $q\cos\theta$, where q is the charge of the lepton, has been simplified. The complete likelihood expression given in [1] is replaced by a simpler one without background contributions. The fit itself is done as a χ^2 fit instead of the likelihood fit.

It has been verified that these approximations do not significantly influence the results for the experimental bias within the quoted statistical errors.

In figure 3 the QCD correction including all experimental biases is shown for the lepton sample, as a function of the output of the neural net. The final QCD corrections is calculated by the weighted average of the QCD corrections as a function of the neural net outputs. The

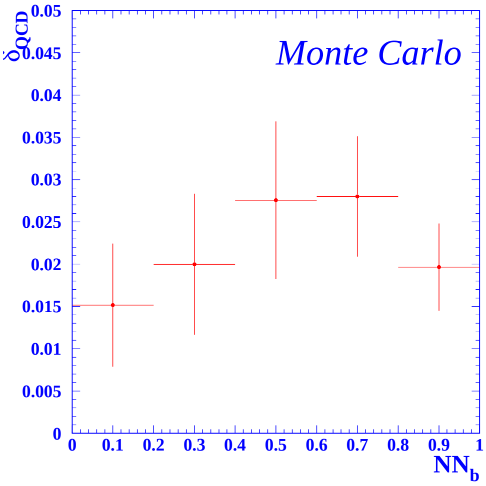

Figure 3: QCD correction including all experimental biases for the lepton sample in $e^+e^- \rightarrow b\bar{b}$ events as a function of the output of the neural network.

details of this weighting method can be found in [1]. The results are summarised in table 3. Typically the lepton analysis is sensitive to less than 70% of the underlying QCD corrections. The error on this bias factor is dominated by the statistical precision of the determination. Systematic errors of the hadronisation model etc. are already included in the original analyses.

4.2 The D Meson Analysis

Similar to the previous method, the analysis of the charm and bottom asymmetries using D mesons relies on an event weighting method to separate background, $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow c\bar{c}$ events.

The actual weighting and fitting method is a slightly simplified version of the one used in [3]. There a two dimensional weighting method has been used based on lifetime information and the scaled energy, x_D, of the candidate. Since here Monte Carlo events are used no separation between the different signal sources is needed, and only the energy dependence is relevant. Therefore weights have been calculated only as a function of x_D, and the bottom/charm separation is made using Monte Carlo information. It has been checked that this simplification introduces negligible effects in the determination of the QCD correction. In figure 4 the QCD corrections, including all detector biases, are shown as a function of x_D.

The final QCD correction is calculated as the weighted average of the QCD corrections shown in figure 4. The experimental bias is calculated in the same way as described before. The results are summarised in table 3. The error on the experimental bias is again dominated by the available number of Monte Carlo events.

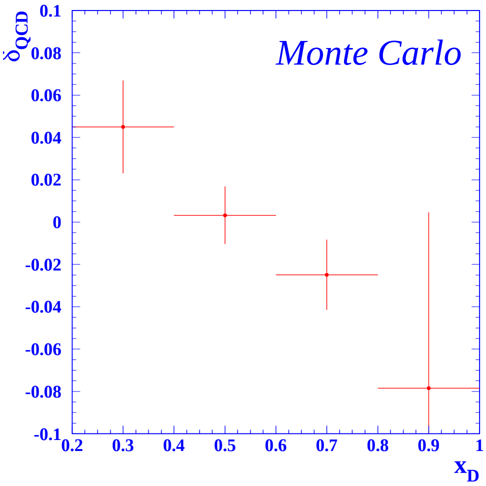

Figure 4: QCD correction including all experimental biases for the D meson sample in $e^+e^- \rightarrow c\bar{c}$ events as a function of the scaled energy of the D meson. The plot shown has not been rescaled to the analytically calculated QCD correction, but instead the JETSET prediction is used.

	jet-charge	lepton	D
	$e^+e^- \rightarrow b\bar{b}$		
bias $\mathcal{B}$	$+0.36 \pm 0.32$	$+0.692 \pm 0.133$	$+0.295 \pm 0.126$
δ_{QCD}	$+0.0103 \pm 0.0012 \pm 0.0050$	$+0.0197 \pm 0.0025 \pm 0.0038$	$+0.0084 \pm 0.0011 \pm 0.0036$
	$e^+e^- \rightarrow c\bar{c}$		
bias $\mathcal{B}$		$+0.366 \pm 0.081$	-0.061 ± 0.087
δ_{QCD}		$+0.0117 \pm 0.0021 \pm 0.0026$	$-0.0019 \pm 0.0004 \pm 0.0028$

Table 3: Table of experimental biases and final QCD corrections δ_{QCD} for $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow c\bar{c}$ events, for the three OPAL asymmetry analyses. The first error quoted is the theoretical error of the underlying QCD correction, the second one is dominated by the statistical precision with which the bias has been determined.

4.3 Jet-Charge Analysis

In the OPAL measurement of the bottom asymmetry using jet-charge, corrections are given to derive the quark level asymmetries from the measured ones. They include experimental biases as discussed in this note. The error quoted includes the theoretical uncertainty from the underlying QCD correction, and a 50% error on the difference between the quoted bias and the fully observed QCD correction ($\mathcal{B} = 1$) is assigned. The error on the experimental bias factor is again dominated by Monte Carlo statistics. The results are summarised in table 3.

5 Results and Conclusions

The final results for the bias and the QCD corrections are summarised in table 3. The QCD corrected asymmetries are given in table 4 for each of the three analyses, for the three energy ranges considered.

analysis	89.44 GeV	91.21 GeV	92.91 GeV
	$e^+e^- \rightarrow b\bar{b}$		
jet-charge	0.041 ± 0.021 ± 0.0024	0.1004 ± 0.0052 ± 0.0045	0.146 ± 0.002 ± 0.007
lepton	0.035 ± 0.017 ± 0.020	0.0910 ± 0.0044 ± 0.0020	0.107 ± 0.014 ± 0.004
D	0.087 ± 0.108 ± 0.029	0.095 ± 0.027 ± 0.022	-0.021 ± 0.090 ± 0.026
	$e^+e^- \rightarrow c\bar{c}$		
lepton	-0.069 ± 0.024 ± 0.005	0.0595 ± 0.0059 ± 0.0056	0.156 ± 0.020 ± 0.010
D	0.039 ± 0.051 ± 0.009	0.063 ± 0.012 ± 0.006	0.158 ± 0.041 ± 0.011

Table 4: Table of the final QCD corrected asymmetries for $e^+e^- \rightarrow b\bar{b}$ and $e^+e^- \rightarrow c\bar{c}$. The errors shown are the statistical error and the total systematic error.

References

[1] OPAL Collaboration, G. Alexander *et al.*, Z. Phys.**C70** (1996) 357;
OPAL Collaboration, *Updated measurement of the heavy quark forward-backward asymmetryies and the average B mixing using leptons in multihadronic events*, OPAL physics note PN-226 (1996).

[2] OPAL Collaboration, K. Ackerstaff *et al.*, *Measurement of the b quark forward-backward asymmetry around the* Z^0 *peak using jet charge and vertex charge*, CERN-PPE/97-06, submitted to Z. Phys..

[3] OPAL Collaboration, G. Alexander *et al.*, CERN PPE/96-101, accepted by Z. Phys..

[4] LEP Heavy Flavour electroweak working group, *Presentation of LEP Electroweak Heavy Flavour Results for Summer 1996 Conferences*, LEPHF 96-01.// Internal report by the LEP heavy flavour working group, under preparation, (1997).

[5] A. Djouadi, J. H. Kühn, P. Zerwas, Z. Phys.**C 46** (1990) 411;
P.Zerwas, B.Lampe, Z. Phys.**C 67** (1995) 123.

[6] G. Altarelli, B. Lampe, Nucl. Phys. **B 391** (1993) 3.

[7] T. Sjöstrand, Comp. Phys. Comm. **82** (1994) 74.

[8] G. Marchesini *et al.*, Comp. Phys. Comm. **67** (1992) 465.

[9] OPAL Collaboration, G. Alexander *et al.*, Z. Phys.**C 69** (1996) 543.