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PHYSICS I

ELECTRONICS EXPERIMENTS COMMITTEE

PROPOSAL FOR INVESTIGATING ELECTRON K_o COHERENT REGENERATION: DETERMINATION OF THE K_o FORM FACTOR

by

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Collaboration

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1. INTRODUCTION

It is possible that interactions of $|K_0\rangle$ and $|K_0\rangle$ states[1] with electromagnetic field have equal and opposite signs. In the $|K_S\rangle, |K_L\rangle$ representation this interaction does not have diagonal elements because of the neutrality of $|K_L\rangle$ and $|K_S\rangle$ states*). However such an interaction can have non-diagonal elements producing transformations of $|K_L\rangle$ into $|K_S\rangle$ and vice versa.

The interaction is proportional to div $(\vec{E})$, that is to the charge density ρ at the point of the K_0 state. In particular if a neutral K beam passes through a region of electromagnetic field in the absence of matter there will be no $|K_L\rangle \leftrightarrow |K_S\rangle$ transition.

The Feynman diagram of lowest order scattering is given in Fig. 1.

This scattering process is somehow analogous to the neutron-electron scattering process[2,3].

The K_0 scattering differential cross-section in the K_0 electron centre-of-mass system for the graph 1 has been given by Feinberg[1] and also Zel'dovich[4] in the non-relativistic limit.

In the laboratory system the scattering cross-section becomes strongly peaked in the forward direction, the maximum scattering angle being about

$$\vartheta_{MAX} \quad \left(\frac{m_e}{m_K}\right) = 10^{-3} \text{ rad} .$$

The total cross-section on electrons for K_0 particles of 3 GeV/c from Feinberg's formula is found to be about 6×10^{-34} cm^2 for a K_0 electromagnetic radius $r_{K_0} = 1.0$ fermi. This is several orders of magnitude smaller than the corresponding nuclear scattering amplitude.

If the incident wave is a linear combination of K and $\bar{K}$, in the scattered wave the ratio between K and $\bar{K}$ amplitudes changes sign. Therefore

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*) All CP violating mixing in the eigenstates are neglected.

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if the incident wave is a K_L current the outgoing scattered wave is a pure K_S current.

The differential cross-section for K_0 electron scattering is in the centre-of-mass system:

$$\frac{d\sigma}{d\Omega}(\Theta)\Big|_{c.m.} = \frac{e^4\lambda^2}{8\pi^2}\frac{1}{(E_e + E_K)^2}\,[2k^2E_eE_K + 2E_K^2E_e^2$$

$$- M_{K^0}(E_K^2 - M_K^2) + \cos\Theta\,(k^4 + k^2E_K^2 + 2k^2E_eE_K)] \qquad (1)$$

where: K is the centre-of-mass momentum, E_e, E_K are the centre-of-mass total energies, M_K is the K_0 mass, λ is proportional to the K_0 form factors, that is

$$F(q^2) = 0 + \lambda q^2 + \text{other terms of higher order} . \qquad (2)$$

The parameter λ can be related to the r.m.s. radius of the K_0:

$$\lambda = \frac{1}{6}\langle R^2\rangle . \qquad (3)$$

Units are such as $\hbar = c = 1$ and $e^2 = 4\pi\alpha = 4\pi/137$. Adding $\hbar$ and c wherever necessary and transforming formula (1) in the laboratory system, we get for the differential cross-section in the forward direction and zero momentum transfer:

$$\frac{d\sigma}{d\Omega}(0)\Big|_{lab.} = 4\alpha^2\, r^4\left(\frac{\hbar}{M_\pi c}\right)^2\left(\frac{M_K}{M_\pi}\right)^2\gamma_K^2 \qquad (4)$$

where r is defined as:

$$\sqrt{\lambda} = r\left(\frac{\hbar}{M_\pi c}\right) \qquad (5)$$

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The regeneration scattering amplitude is then:

$$|f(0) - \overline{f(0)}| = 4\alpha r^2 \left(\frac{\hbar}{M_\pi c}\right)\left(\frac{M_K}{M_\pi}\right)\gamma_K = 0.146\ r^2\gamma_K \text{ fermi} \qquad (6)$$

and for an atom of atomic number Z the coherent electron regeneration amplitude will be Z times formula (6).

Let us briefly consider how one can estimate λ. An optimistic choice would be to set the r.m.s. radius of the K_0 equal to the pion Compton wavelength

$$\sqrt{\lambda} = r\,\frac{\hbar}{M_\pi c} = \frac{1}{\sqrt{6}}\,\frac{\hbar}{M_\pi c}\ ,$$

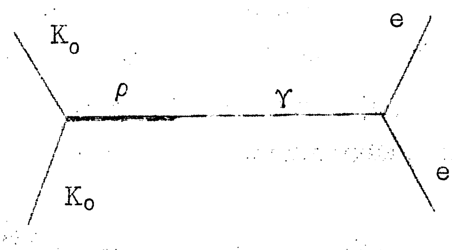
that is $r = 1/\sqrt{6} = 0.409$. Then for 3.0 GeV/c K^0, ($\gamma_K = 6.0$) we get

$$|f(0) - \overline{f(0)}| = 0.146 \text{ fermi}\ .$$

A more conservative choice could be

$$\sqrt{\lambda} = \frac{\hbar}{M_\rho c^2}\ .$$

Since the K-ρ coupling is known to be strong [e.g. via $K^*(1300) \rightarrow K+\rho$] and the γ-ρ coupling is strong (e.g. nucleon form factors) one might expect graphs like:



giving the above indicated choice for $\sqrt{\lambda}$. Then $r = M_\rho/M_\pi = 0.184$ and

$$|f(0) - \overline{f(0)}| = 0.0294 \text{ fermi} \qquad (\gamma_K = 6)\ .$$

The computed scattering amplitudes for electron regeneration are compared to the known total regeneration scattering amplitude in Table 1 below: (p_K = 3.0 GeV/c)

Table 1

Regeneration scattering amplitudes (in fermi)

Element	$f(0) - \overline{f(0)}$ Observed	$f(0) - \overline{f(0)}$ Electrons $r = 0.409 \left(\langle r \rangle \frac{\hbar}{M_\pi c}\right)$	$f(0) - \overline{f(0)}$ Electrons $r = 0.184 \left(\sqrt{\lambda} = \frac{\hbar}{M_\rho c}\right)$
C(Z=6)	7.1	0.87	0.176
Cu(Z=29)	22.0	4.21	0.852
W(Z=74)	40.0	10.7	2.16

2. EXPERIMENTAL METHOD

The presence of electrons can produce both single scattering and coherent regeneration effects. The single scattering events are confined in a cone of about 10^{-6} sr, and cannot be resolved from the forward direction with our present detector. The coherent regeneration contribution is interfering with the nuclear regeneration amplitude. Zel'dovich[4)] and more recently Placci and Zavattini[5)] have remarked that those effects could well be substantially large.

Following a proposal by Zel'dovich[4)] we would like to search for electron regeneration comparing nuclear diffraction and transmission regeneration. Good[6)] has shown that the ratio between the transmission regeneration intensity[*)] $|\rho|^2$ and the diffraction regeneration per unit of solid angle $d\sigma/d\Omega$ is independent of the scattering amplitude:

$$R = \frac{|\rho|^2}{\left(\frac{d\sigma}{d\Omega}\right)} = \frac{2\pi\Lambda}{k\left(\delta^2 + \frac{1}{4}\right)} \; \frac{1 + e^{-\ell} - 2e^{-\ell/2}\cos\delta}{1 - e^{-\ell}} \qquad (7)$$

*) ρ = (real const.) $\times\, i[f(0) - \overline{f(0)}]$

where: $\ell = d/\Lambda$, the regenerator thickness in decay lifetime units Λ,
δ = mass difference in lifetime units
k = is the K_0 wave number.

Important corrections to Good's formula for the finite size regenerator are due to the interference between regeneration and scattering and to double scattering effects. They have also been computed by Good. Good's correction is shown in Fig. 2 with some data recently obtained by our group.

For a sufficiently thin regenerator, the correction is rather modest and presumably it can be reliably computed. Therefore Good's relation (7) can be used to predict the nuclear transmission regeneration intensity $|\rho_{\text{nucleus}}|^2$ by extrapolation to zero angle of the diffraction regeneration per unit of solid angle $d\sigma/d\Omega$. The electron diffraction regeneration contribution is absent if diffraction angles appreciably larger than $\vartheta_{MAX} \simeq (m_e/m_k) \simeq 10^3$ rad are considered.

The transmission regeneration intensity $|\rho_t|^2$ is

$$|\rho_t|^2 = |\rho \text{ electrons} + \rho \text{ nuclear}|^2 .$$

Let $\Phi_t = \arg(\rho_t)$ be the phase of the observed transmission regeneration amplitude. Then:

$$|\rho_e| = |\rho_t| \cdot \left[\sin \Phi_t \pm \sqrt{\sin^2 \Phi_t + \frac{|\rho \text{ nuclear}|^2 - |\rho_t|^2}{|\rho_t|^2}} \right] \tag{8}$$

There are two solutions for each set of experimental numbers $|\rho_t|^2$, $|\rho \text{ nuclear}|^2$ and $\sin \Phi_t$.

The procedure is shown graphically in Fig. 3. For a very small electron regeneration amplitude ($\rho_e/\rho_t << 1$), formula (8) becomes:

$$\frac{|\rho_t|^2 - |\rho \text{ nuclear}|^2}{|\rho_t|^2} \simeq \pm 2 \sin \Phi_t \frac{|\rho_e|}{|\rho_t|} \tag{9}$$

where the sign ± depends on the relative sign of the imaginary parts of the regeneration amplitude from nucleon and electrons. In the case of copper and $p_K \sim 1\text{-}3$ GeV/c, we know[7)] that $\Phi_t \simeq 30°$. For r = 0.41, $|\rho_e/\rho_t| \simeq 0.14$ and we expect about 14% deviation from Good's formula.

The effects depend quadratically on the K_0 form factor r. A substantially smaller value for r, for instance r = 0.20 would therefore be at the borderline for detection.

The ratio $|\rho_e/\rho_t|$ is expected to grow larger for heavier nuclei, being about $|\rho_e/\rho_t| \sim 0.27$ for tungsten and r = 0.41. However a reduction in the observed effect is expected if Φ_t could become substantially smaller. We have investigated the dependence of regeneration phase on nuclear size with an optical model calculation. Calculations indicate a very small phase change between copper and tungsten $|\Phi_{Cu} - \Phi_W| \lesssim 5°$ for a variety of optical parameters. Therefore it is very likely that a substantially larger effect would take place for heavier nuclei.

Finally it is relevant to remark[4)] that because the regenerator is electrically neutral, one might think that one should include also the electromagnetic interactions between K^0 and protons which are of opposite sign and therefore would compensate the electron interaction.

Actually Zel'dovich[4)] has remarked that the electromagnetic effects due to the protons are indistinguishable from the nuclear effects. Therefore it there were electromagnetic interaction between K^0 and protons but not with electrons, Good's formula would be exact. Since $|\rho$ nuclear$|$ is taken from experiment, deviations from Good's formula are due only to electrons.

3. PROPOSED EXPERIMENT

We propose to use the wire chamber spectrometer presently in operation on the B_{13} neutral beam (S49). No major change in the apparatus is required. The counting rate of the present detector is of about 3-5 trigger/pulse, adequate to collect sufficient statistics in a relatively short time. Evidence supporting the possibility of separating transmission and single scattering regeneration events is shown in Fig. 4.

The main limitation to the sensitivity of the experiment is expected to come from systematic rather than statistical errors. A number of possible effects could produce deviations to Good's formula even in the absence of electron regeneration:

A) Leptonic decays or other many-body processes giving accidentally an apparent K_0 invariant mass if fitted as $K \to 2\pi$ decays. This background is presently reduced to a negligible level by electron and muon identification in a gas Čerenkov counter and in a muon detector. Furthermore this background is not sharply peaked around the K_0 mass and it can be subtracted out by looking at adjacent invariant mass bins and to events collected with no regenerator in the beam. So we anticipate no problem from this kind of event.

B) $K \rightarrow 2\pi^{\pm}$ decays associated with inelastic events. Inelastic events clearly do not contribute to transmission regeneration but they could fall into the diffraction regeneration bins. There are two distinct types of such events:

i) $K_0(\overline{K_0})$ events produced by associated production of incident neutrons and gamma rays. From known $K^{\pm}$ production cross-sections and the neutron content of the neutral beam at 10° production angle it is concluded that the contribution of such events is negligibly small.

ii) inelastic regenerative K_L scattering, that is events in which an incident K_L wave is regenerated leaving the residual nucleus in an excited state. The probability for this effect to happen at a very small angle of scattering is presumably very small. In the case of charged pion scattering on nuclei one knows that the probability of exciting nuclear levels tends to zero when the scattering angle go[illegible] to zero. We intend to verify this statement by detecting a sample of events with a large NaI gamma detector of nuclear de-excitation gamma rays on a side of the regenerator.

It is felt that the sensitivity of the presently proposed search for electron regeneration is finally limited to the extent in which inelastic events can be understood and subtracted out.

4. MACHINE TIME REQUEST

In order to carry out a complete search for regeneration from electrons, elements of different Z should be investigated. We would like to use carbon, copper and lead regenerators. For each element several different regenerator thicknesses have to be measured in order to extrapolate Good's formula to zero regenerator thickness. In addition some runs with no regenerator in the beam and an attenuation measurement of K_L in copper, carbon and lead are required for the analysis. We estimate about 75 hours for each element and 75 hours of calibration are sufficient to perform the experiment. In addition some parasitic machine time is required in order to tune up the NaI counter for detection of nuclear de-excitation gamma rays.

REFERENCES

1) Feinberg, Phys.Rev. 103, 1381 (1958).

2) E. Fermi and L. Marshall, Phys.Rev. 72, 1139 (1947).

3) Havens, Raby and Rainwater, Phys.Rev. 72, 634 (1947).

4) Zel'dovich, Soviet Physics - JETP 36, 984 (1959).

5) Placci and Zavattini, Note 1001, unpublished (1968).

6) R.H. Good et al., Phys.Rev. 124, 1223 (1961).

7) Melhop et al., unpublished;
J. Steinberger et al., private communication.

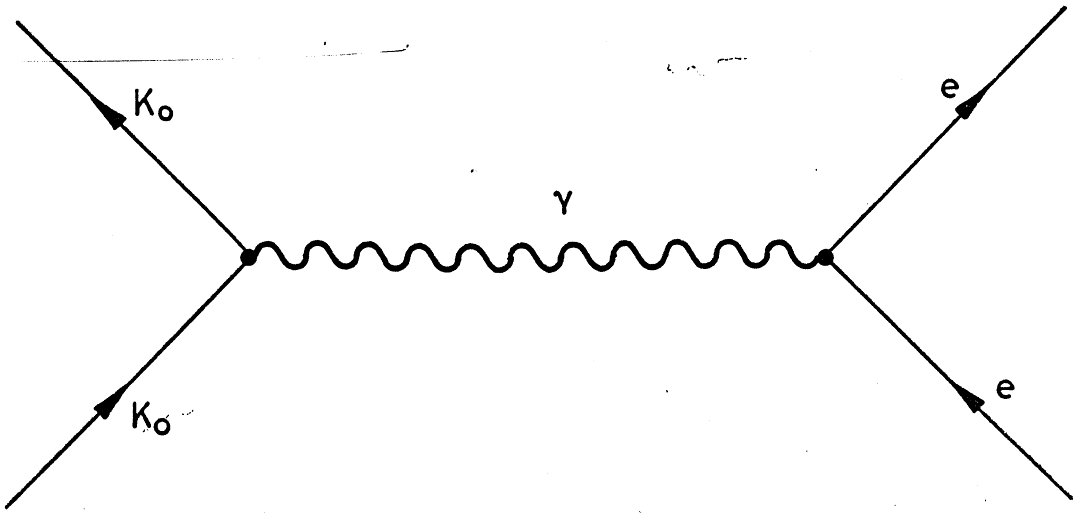
Figure captions

Fig. 1 : Feynman diagram for one photon contribution to electromagnetic K_0-electron scattering. This graph has been used by Feinberg[1] to predict the elastic scattering cross-section.

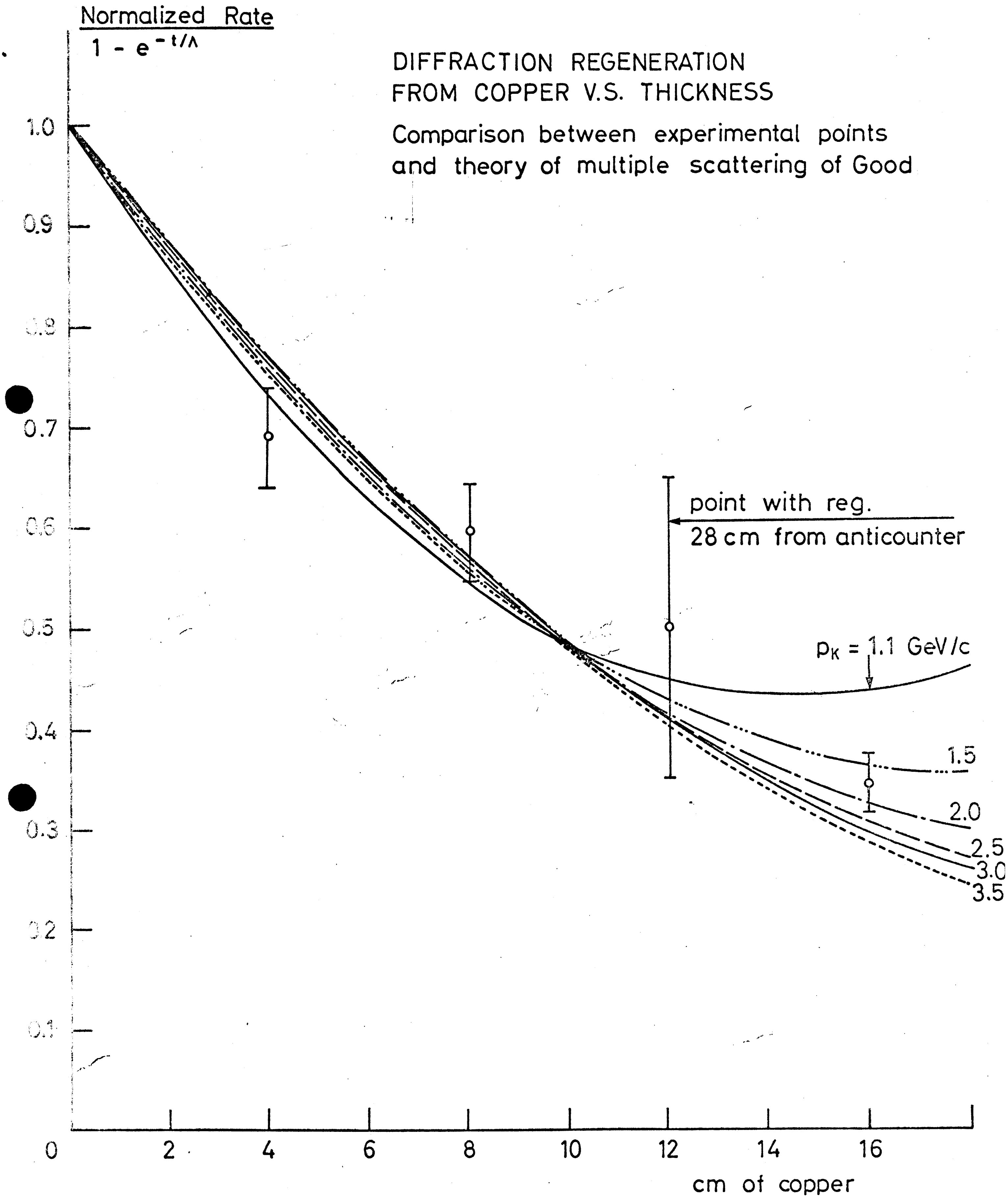
Fig. 2 : Diffraction regeneration scattering intensity versus thickness compared with the predictions of the theory of Good[6]. In the absence of Good's correction the normalized rate should be independent of regenerator thickness.

Fig. 3 : Vector diagram showing composition of electron and nuclear regenerations. The electron regeneration amplitude is real.

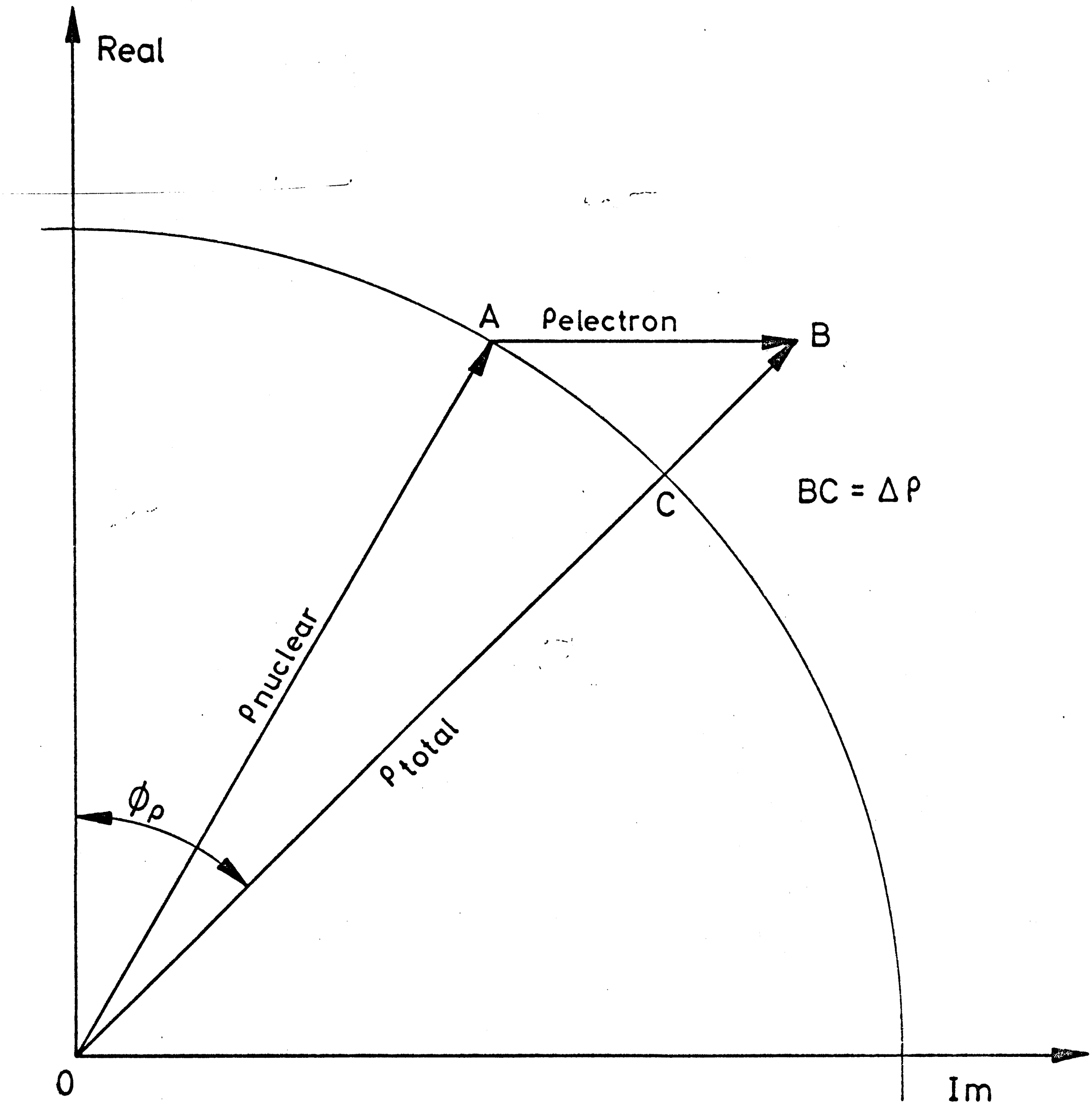
Fig. 4 : Angular distribution of $K \rightarrow 2\pi$ events detected after a thick copper regenerator. Separation between single scattering and transmission regenerator counts is demonstrated.



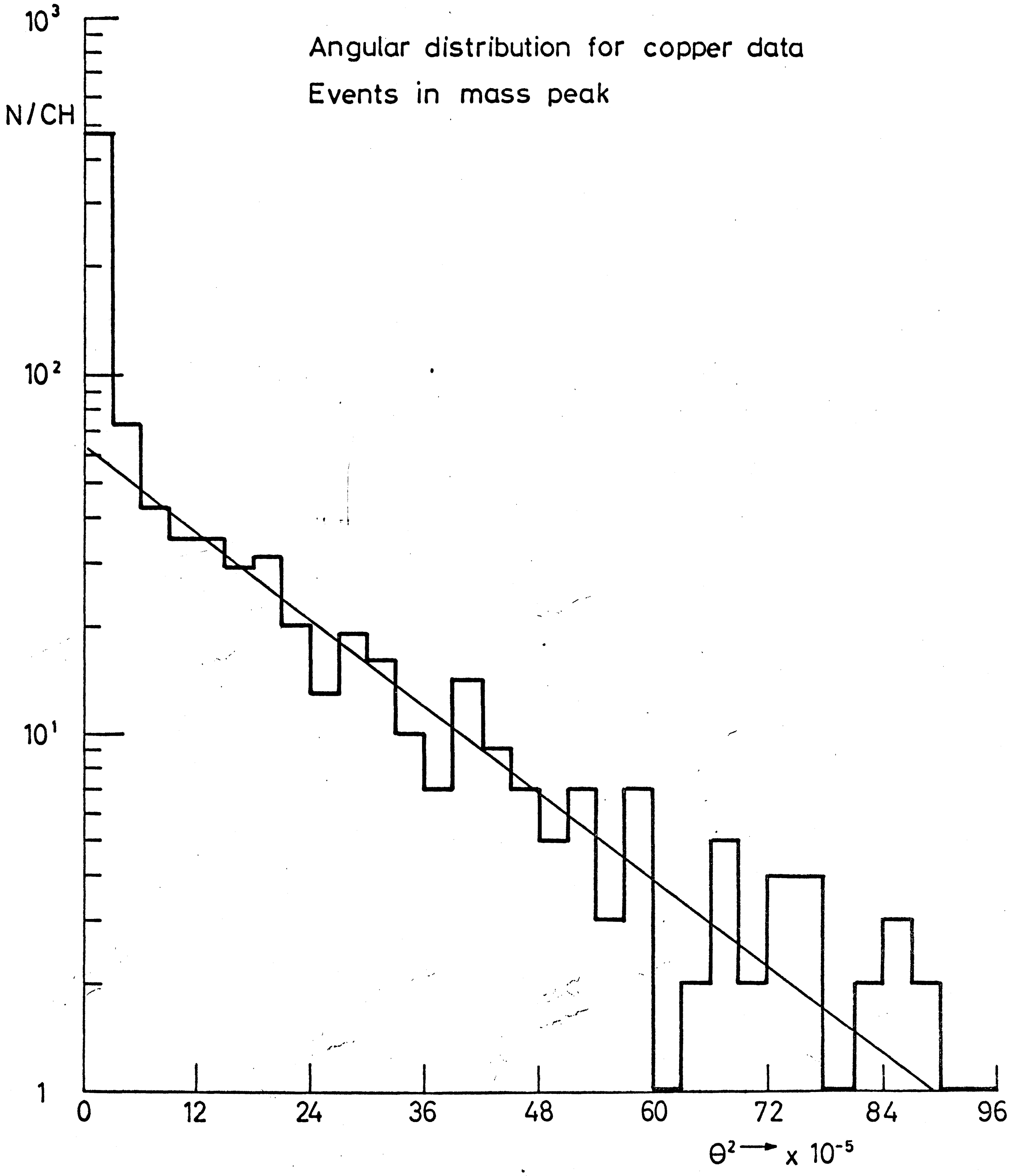
Feynman diagram for K -electron scattering



Normalized Rate
1 - e^{-t/Λ}
DIFFRACTION REGENERATION
FROM COPPER V.S. THICKNESS
Comparison between experimental points
and theory of multiple scattering of Good
point with reg.
28 cm from anticounter
p_K = 1.1 GeV/c
1.5
2.0
2.5
3.0
3.5
1.0
0.9
0.7
0.5
0.4
0.3
0
2
4
6
8
10
12
14
16
cm of copper



Real
A
ρelectron
B
C
BC = Δρ
ρnuclear
ρtotal
ϕ_ρ
0
Im


Angular distribution for copper data
Events in mass peak
N/CH
10³
10²
10¹
1
0
12
24
36
48
60
72
84
96
θ² → x 10⁻⁵