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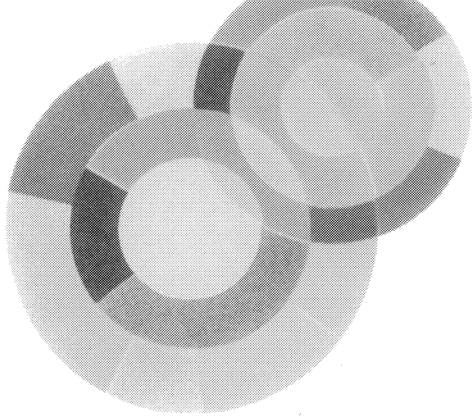

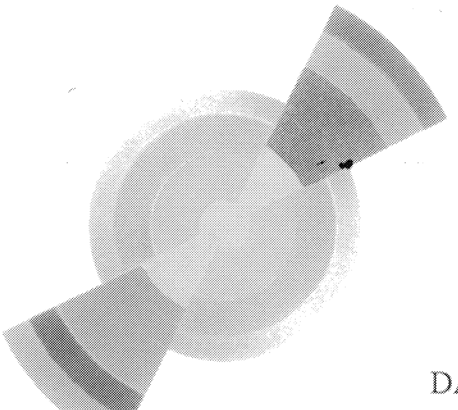

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Antimatter in General Relativity ?

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Abstract

Using the charge reversal and time reversal properties of the Kerr-Newman solution, a definition of antimatter in General Relativity is proposed, which would provide a parameter-free explanation of the apparent cosmological term evidenced by supernovae, CMB and LSS data. Tests on positronium and antihydrogen, which could be realized in the next few years next to the CERN Antiproton Decelerator facility, are briefly discussed

1 Introduction

It seems obvious that General Relativity is unable to distinguish gravitationally matter from antimatter. Central to the General Theory of Relativity, the Equivalence Principle seems to imply, through the Einstein equations relating the metric tensor to the stress-energy tensor that, for identical initial conditions, the trajectory must be independent of the nature of the test particles or antiparticles.

A number of authors have proposed various solutions appearing to circumvent this argument, notably by extensions of general relativity [1,2]. In the following, we follow a different approach by showing that charge-reversal (C) and time-reversal (T) properties exist in simple solutions of General Relativity that evoke strongly the transformation relating matter to antimatter. In particular, the Kerr-Newman geometry [3–5], which describes the geometry associated with a charged and spinning mass, and the simple geometry associated with a spinning cosmic string [6] can be used to show that General Relativity appears to propose, through the discrete C, P and T transformations, a rather natural, if surprising, definition of antimatter.

The three discrete symmetries C, P and T are fundamental to the definition of antimatter. In effect, according to the CPT theorem [7,8], considered to be valid in extremely general conditions —although this theorem has not been demonstrated in the case, notably, of curved spacetimes [9,10]— the CP transformation relating matter and antimatter is strongly associated to the discrete T time-reversal symmetry, whose violation has recently been demonstrated experimentally for the first time [11]. Conversely, CP violation, discovered experimentally in 1964 by Christenson et al. [12], introduces an asymmetry between matter and antimatter [13], which appears extremely limited and confined until now to the neutral meson systems. Therefore, to an excellent approximation, antimatter appears as the CP-transformation of matter. Similarly, if the CPT symmetry is exact, antimatter can be defined, at the same excellent approximation, as "matter going backwards in time" [14]. In a first part, we will recall the properties of the Kerr-Newman geometry, representing a charged spinning mass, to exhibit its charge and time-reversal properties.

2 A Kerr-Newman electron is also a positron

In order to test the existence of charge-reversal properties, it appears adapted to use the maximal analytic extension of the Kerr-Newman geometry which represents the geometry associated with a mass m (supposed for the moment positive), specific angular momentum $a = L/m$, and an electric charge e. We will use the fast Kerr geometry, i.e. respecting the condition $e^2 + a^2 > m^2$. In this case, the geometry has a simple topology and is not afflicted with a Cauchy horizon. Note that this condition is met for all elementary particles, with the notable exception of the scalar Higgs boson.

The disk limited by the annular singularity present in the Kerr-Newman geometry constitutes a "wormhole"between two asymptotically flat spacetimes isomorphic to $\mathbb{R}^4$, in which the top and bottom of the disk in the first spacetime are identified, respectively, to the top and bottom of the disk in the second spacetime [15]. An example is provided by the Kerr-Newman geometry with the m, a and e parameters of an electron:

$$m \sim 0.9 \times 10^{-30} kg, a = \hbar/2m, e \sim 1.6 \times 10^{-19}\text{C}$$

In this case, the radius of the ring is $\sim$ 100 fm.

The fast Kerr geometry is particularly simple since it involves no horizon. The angular momentum imposes an annular shape to the singularity, which appears naked but nevertheless almost invisible since the measure of initial conditions allowing to reach the ring singularity is zero. Brandon Carter has studied the topology of this solution [5], noting the striking analogy that a "Kerr-Newman particle"bears with real particles. In particular, the gyromagnetic factor of the Kerr-Newman electron is $g = 2$ and the geometric extension of the ring is of the order of the Compton wavelength of the electron, giving it a spatial extension compatible with its cross-section.

Another interesting feature concerns the charge conjugation (C) properties of this solution. By crossing the non singular interior of the ring, an observer will measure the charge and mass of the electron with a reversed sign. For a particle physicist, this means that the particle with the quantum numbers of an electron in the first spacetime has the quantum numbers of a positron in the second spacetime, linked to the first by the interior of the Kerr ring. It is important to note that if we assume that the initial "electron "has a positive mass, the "positron"has necessarily a negative mass $-m$, inducing repulsive gravity.

This results immediately from the symmetry properties of the metric and electromagnetic field tensor form of the Kerr-Newman solution, which can be expressed [5] in Boyer-Lindquist coordinates:

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)d\phi - adt\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

and :

$$F = 2e\rho^{-4}\left[\left(r^2 - a^2cos^2\theta\left(dr \wedge \left(dt - asin^2\theta d\phi\right)\right]\right. - 2ear\rho^{-4}\left[sin2\theta d\theta \wedge \left(adt - \left(r^2+a^2\right)d\phi\right)\right]$$

and where :

$$\Delta = r^2 - 2mr + a^2 + e^2$$

and

$$\rho^2 = r^2 + a^2cos^2\theta$$

Another significant and surprising feature of the Kerr-Newman solution is the fact that it is possible to go backward in time by exploring the second spacetime linked to the first by the interior of the Kerr ring. This feature was also studied by Brandon Carter, and is known as the "Carter time machine" [16]. Initially considered as a source of paradoxical situations, the solutions incorporating CTCs are now known to lead to consistent solutions [17–19]. Using regions with CTCs to define antimatter appears consistent with antimatter defined as matter going backwards in time, as suggested by the CPT theorem and the Feynman-Wheeler picture [14].

3 Conjugate points in the Kerr-Newman geometry

An important generic property of General Relativity is the existence of points of infinite magnification for the image of an object through the lensing created by a massive object. Used in recent years to detect massive compact halo objects (MACHOs) in our galactic neighborhood [20,21], this magnification, when it is infinite, has the consequence that the lensed object may appear infinitely more luminous and closer than its true position. This property is even stronger for the fast Kerr-Newman geometry, where Closed Timelike Curves (CTCs) exist between any two points. For a given point A in the neighborhood of the ring, there exists a set of points B such that the radar interaction between A and B —photons are emitted by A, scattered by B and received back by A— is instantaneous. The signal emitted comes back with zero time delay as seen by the emitter, and an object at location B will then appear to an observer as if it were at location A. These points can be explicitly constructed in the Kerr geometry in $2+1$ dimensions, where the spinning cosmic string is an exactly soluble model [6]. It is straightforward to demonstrate that the set of such points B lies on a portion of ellipse — a portion of ellipsoid in $3+1$ dimensions — (Fig. 1) since the time pitch associated with a 2π rotation around the spinning cosmic string can be written as $\Delta t = 8\pi a G$, where a is the specific angular momentum per unit length of the string [6].

From the existence of conjugate points in $2+1$ gravity, expected to be valid also in $3+1$ gravity from the existence of CTCs, there follows a (non local) definition of antiparticles in general relativity as the time-reversed *image* of particles observed through a Kerr ring. These Kerr rings could be present in all elementary particles, if they are string loops, and in the past singularity of the Big Bang. This (non-local) definition of antimatter and the symmetries exhibited by the Kerr-Newman solution imply as a consequence a gravitational repulsion between matter and antimatter, defined relatively to each other and not in an absolute way. The coupling of systems with opposite arrows of time is reminiscent of the dynamical systems studied by Schulman [22]. From the persistence of individual arrows of time for such weakly coupled systems, interactions of each system with the conjugate system are expected to appear as noise.

4 Explaining the cosmological constant coincidence

During the sixties and early seventies, several attempts have been made, using notably a conjectured repulsion in strong interactions [23], which failed to justify the survival of significant matter and antimatter domains in a symmetric

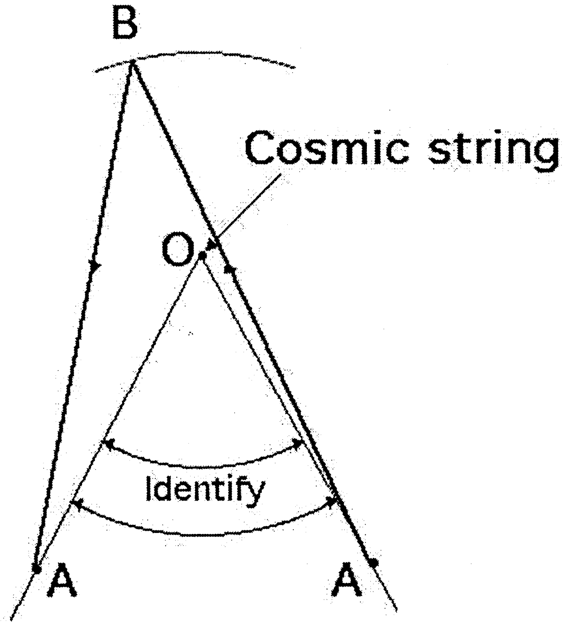

Fig. 1. An observer in A can use a spinning cosmic string to discuss at zero time delay with a set of points B located on a portion of ellipsoid. In the special case where the angle deficit created by a cosmic string is π, a point B exists such that a signal emitted by A and reemitted by B comes back to A at zero time delay *and* with a direction identical to that of the initial signal. The interaction between A and B is then diverging, and as a consequence B appears to be at the position of A

universe. The gravitational repulsion evidenced in the Kerr-Newman solution would, on the other hand, effectively lead to a symmetric matter-antimatter universe. It is fascinating to note that this gravitational repulsion between matter and antimatter as defined above appears to lead to a parameter-free explanation of the value of the "cosmological constant "observed in recent supernovae and CMB observations [24–26]:

$$\Omega_{tot} = \Omega_{matter} + \Omega_{\Lambda} \sim 1 \pm 0.02$$

where a nearly flat universe is composed of only 0.045 of ordinary baryonic matter, with a dark matter density $\Omega_{matter} \sim 0.30$ and an apparent cosmological density $\Omega_{\Lambda} \sim 0.70$. To justify this statement, let us consider the expression for the deceleration parameter q as a function of the scale factor a and its derivatives:

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2}$$

When observed on a scale larger than the matter or antimatter domains, this symmetric universe will appear flat with a parameter $q <\sim 0$ due to the repulsion of adjacent domains. Therefore, in a situation where the cosmological constant is zero, it is possible to parametrize the repulsive term by a cosmological constant respecting:

$$q = \Omega_{matter}/2 - \Omega_\Lambda <\sim 0 \rightarrow \Omega_\Lambda = \mathcal{O}(1)\Omega_{matter} \sim 0.5 \times \Omega_{matter}$$

Although, at any given epoch, this equality will locally be verified, the evolution of the matter density ρ_{matter} with time has for consequence that the derived value of the effective "cosmological constant "energy density will vary according to $(1+z)^3$ since the matter density varies approximately in this way after recombination. Therefore, the simple approximation we have used indicates that by using a large number of small-z supernovae, we would find a value of the cosmological constant $\Omega_\Lambda \sim \Omega_{matter}/2$, while for the sample used in the SN1a observations [24,25], it is simple to check that the supernovae have redshift parameters such that the average value of $\langle(1+z)^3\rangle \sim 3$, leading to an apparent $\Omega_\Lambda \sim 2 \times \Omega_{matter}$. Therefore, the apparent extraordinary coincidence between the matter and cosmological constant densities [27] is simply explained in this symmetric matter-antimatter universe. Ripalda, in a different theoretical context, has also noted that repulsive gravity would lead to a cosmological constant density of the same order as the matter density [28].

5 Critical discussion

Antigravity is usually considered as being impossible within the context of General Relativity. We recalled that, on the contrary, repulsive gravity is present in a large number of solutions in General Relativity and that the charge and time reversal properties of the solution provide a strong motivation for antigravity. In addition, it should be remembered that most if not all the impossibility arguments against antigravity have been shown to present loopholes (see Nieto and Goldman [2] for a critical discussion). In the following, we briefly discuss some of the impossibility arguments associated with the use of the $(mr < 0)$ part of the Kerr-Newman solution.

In particular, using the $(mr < 0)$ subspace in Kerr-Newman obviously violates the weak energy condition. On the other hand, various counterexamples are known for most of the expressions of positivity of energy (for a critical discussion, see e.g. Visser, Chap. 12, Ref. [29]). Therefore, while it seems probable that an instability will develop in some region of the maximal analytic extension of the Kerr-Newman solution, it does not mean, however, that the subspace defined by the condition $(mr < 0)$ has no physical content.

The two-body solution in General Relativity is presently not known and even appears as a long-term goal. We note here that this solution might involve the coupling of regions of space-time with opposite relative time arrows, which would restore the symmetry between the $(mr > 0)$ and the $(mr < 0)$ parts of the Kerr-Newmann solution.

Also, Penrose has conjectured that naked singularities are forbidden (the so-called "cosmic censorship" conjecture). It may seem that the cosmic censorship hypothesis is grossly violated in the Kerr-Newman solution, which should therefore be rejected as non-physical. In fact, the violation could be very mild since, as noted already by Carter [5], the Kerr and Kerr-Newman singularities can only be reached by an observer following a null trajectory with initial conditions of zero measure. In this sense, the singularity present in these solutions is nearly perfectly invisible, and it appears probable that in every realistic solution (the Schwarzschild solution being exceptional with its perfect spherical symmetry [30]), the singularities will be effectively invisible. Note that, in our past, a global singularity (the Big Bang) is visible, while time asymmetry and instability are manifest in almost every macroscopic physical phenomenon.

Until recently, it was believed that Closed Timelike Curves (CTCs) would lead to inconsistencies and should therefore be avoided at all cost. A more precise study [19,17,18] has shown that consistent histories can always be found despite the presence of CTCs. Surely enough, the existence of CTCs in a solution brings with it a loss of uniqueness and determinism in the solution, but it may be useful to remember that quantum physics has the same characters.

Similarly, coupling two systems with opposite arrows of time was initially considered as inconsistent, or leading at the very least to the destruction of the individual arrows of time of each of the two subsystems (see e.g. [22] and references therein). But Schulman [22] provided evidence, using simulations of simple dynamical systems, that the coupling of dynamical systems with opposite times would preserve the existence of individual time arrows. Note also that the initial low entropy of the universe could be justified much more easily in this context than in the standard cosmological model.

Finally, we note that the usual arguments invoked to exclude the existence of large domains of antimatter through the non observation of diffuse gamma-ray background [31] are not applicable since diffusion and annihilation at the border of matter and antimatter domains is prevented to a large extent by gravitational repulsion.

6 Experimental tests

There is presently no direct experimental test of the gravitational mass of antiparticles. Fairbank and Witteborn [32] made pioneering measurements on electrons but these measurements were inconclusive [33]. In fact, it can be shown that, under realistic conditions, the measurement of the gravitational mass of an electron is almost necessarily hidden in the Johnson noise of adjacent metallic surfaces used to shield the measurement from external fields. On the other hand, measurements on antiprotons, with a higher m/e ratio, could probably be realized using high bandwidth Single Electron Transistor (SET) electronics [34]. However, since the quark mass content of the proton is only of the order of one percent, the significance of a null measurement could remain ambiguous.

Measurements of the gravitational mass of antihydrogen and positronium atoms have been proposed [35–37]. Realized on neutral systems, these experiments could lead to the first precision measurements of the gravitational mass of simple antiparticle systems. These measurements could be realized within the next few years near the Antiproton Decelerator (AD) at CERN.

Precision measurements in cosmology, in particular on supernovae, could also represent an important test. By observing SN1a supernovae of high z (typically > 1), the difference of prediction between a cosmological constant term and a term directly linked to the matter content could be observed. Satellite experiments such as SNAP, by observing samples of a few thousand SN1a supernovae, could test our hypothesis with precision.

Finally, we note that our definition of antimatter in General Relativity could lead to a parameter-free explanation of CP-violation in the neutral meson system [38–42].

7 Conclusions

Charge (C) and time (T) reversal properties of the Kerr-Newman solutions suggest a natural definition of antimatter in General Relativity, strongly reminiscent of Dirac's definition of antimatter. Although, clearly, the cosmological consistency of our proposed definition of antimatter in general relativity has yet to be demonstrated, it provides a parameter-free explanation of the otherwise extraordinary coincidence of the cosmological constant energy density with the matter density, evidenced in the supernovae SN1a and CMB data.

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References

[1] J. Scherk, *Phys. Lett.* B **88** (1979) 265

[2] M. M. Nieto and T. Goldman, *Phys. Rep.* **205** (1991) 221 and references therein

[3] R.P. Kerr, *Phys. Rev. Lett.* **11** (1963) 238

[4] E. T. Newman, E. Couch, R. Chinnapared, A. Exton, A. Prakash and R. Torrence, *J. Math. Phys.* **6** (1965) 918

[5] B. Carter, *Phys. Rev.* **174** (1968) 1559

[6] S. Deser, R. Jackiw and G. t'Hooft, *Ann. Phys.* **152** (1984) 220

[7] W. Pauli, *Phys. Rev.* **56** (1940) 716

[8] G. Lüders, *Ann. Phys.* **2** (1957) 1

[9] T. Banks, *Nucl. Phys.* B **249** (1985) 332

[10] R.M. Wald, *Phys. Rev.* D **21** (1980) 2742

[11] A. Angelopoulos, et al., *Phys. Lett.* B **444** (1998) 43

[12] J. H. Christenson, J. W. Cronin, V. L. Fitch and R. Turlay, *Phys. Rev. Lett.* **13** (1964) 138

[13] J.J. Sakurai and A. Wattenberg, *Phys. Rev.* **161** (1967) 1449

[14] J.A. Wheeler, *Ann. Phys.* **2** (1957) 604

[15] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge Univ. Press, Cambridge, UK, 1973)

[16] B. O'Neill, *The Geometry of Kerr Black Holes* (A.K. Peters, Wellesley, Massachusetts, 1995)

[17] J. Friedman, M. S. Morris, I. D. Novikov, F. Echeverria, G. Klinkhammer, K. S. Thorne and U. Yurtsever, *Phys. Rev.* D **42** (1990) 1915

[18] J. Friedman and M. S. Morris, *Phys. Rev. Lett.* **66** (1991) 401

[19] F. Echeverria, G. Klinkhammer and K. S. Thorne, *Phys. Rev.* D44 (1991) 1077

[20] C. Alcock et al., (MACHO Collaboration) *Nature* **365** (1993) 621

[21] E. Aubourg et al., (EROS Collaboration) *Nature* **365** (1993) 623

[22] L.S. Schulman, *Phys. Rev. Lett.* **83** (1999) 5419

[23] R. Omnes, *Phys. Rev. Lett.* **23** (1969) 38 ; *Phys. Rep.* **3** (1970) 1

[24] S. Perlmutter et al., *Nature* **391** (1998) 51

[25] A. G. Riess et al., *Ap. J.* **116** (1998) 1009

[26] M. Tegmark et al., astro-ph/0310723, to appear in *Phys. Rev.* D

[27] S. Weinberg, *Rev. Mod. Phys.* **61** (1989) 1

[28] J.M. Ripalda, gr-qc/9906012

[29] M. Visser, *Lorentzian wormholes, From Einstein to Hawking*, (Springer, New York, 1996), Chapter 12

[30] W. Israel, *Phys. Rev.* **164** (1967) 1776

[31] A.G. Cohen, A. De Rujula, S.L. Glashow, *Ap. J.* **495** (1998) 539

[32] F.C. Witteborn and W.M. Fairbank, *Phys. Rev. Lett.* **19** (1967) 1049

[33] J.M. Lockhart, F.C. Witteborn and WM. Fairbank, *Phys. Rev. Lett.* **67** (1991) erratum

[34] V. Bouchiat, G. Chardin, M. Devoret and D. Esteve, *Hyperfine Interactions* **109** (1997) 345

[35] M. Charlton et al., *Phys. Rep.* **241** (1994) 65

[36] C.L. Cesar, *Hyperfine Interactions* **109** (1997) 293

[37] A.P. Mills, M. Leventhal, *Nucl. Instr. Meth.* B **192** (2002) 102

[38] G. Chardin and J.M. Rax, *Phys. Lett.* B **282** (1992) 256

[39] G. Chardin, *Hyperfine Interactions* **109** (1997) 83

[40] T. Goldman, M.M. Nieto and V. D. Sandberg, *Mod. Phys. Lett.* A **7** (1992) 3455

[41] A.W. Overhauser, Physics Department, Purdue University, March 2000, private communication

[42] S.H. Aronson, G.J. Bock, H.-Y. Cheng and E. Fischbach, *Phys. Rev.* D **28** (1983) 495, see note 69