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FREE ENERGIES OF STATIC THREE QUARK SYSTEMS

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We study the behaviour of free energies of baryonic systems composed of three heavy quarks on the lattice in SU(3) pure gauge theory at finite temperature. For all temperatures above T_c we find that the connected part of the singlet (decuplet) free energy of the three quark system is given by the sum of the connected parts of the free energies of qq-triplets (-sextets). Using renormalized free energies we can compare free energies in different colour channels as well as those of qq- and qqq-systems on an unique energy scale.

1. Introduction

While existing studies on static baryonic systems focus on zero temperature simulations[1,2] or used maximal abelian gauge at finite temperature[3], we have calculated the free energies in different colour channels of heavy three quark systems at finite temperature using Coulomb gauge.

Here we restrict ourselves to the analysis of equilateral triangles above the critical temperature on a $32^3 \times 8$ lattice in SU(3) pure gauge theory. The qq-triplet and -sextet free energies have been calculated recently in Ref. 4 and also by us in this work.

2. Colour Channels of the Three Quark System

The state of a three quark system as the product of the irreducible representation of three quarks in colour space can be decomposed into symmetry states

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8' \oplus 10, \tag{1}$$

where 3 is the irreducible representation of a quark in colour space and 1 denotes the singlet, 8 the first octet, $8'$ the second octet and 10 the decuplet state. The singlet is totally anti-symmetric, the first octet anti-symmetric in the first and second, the second octet in the second and third component and the decuplet is totally symmetric.

The derivation of the representation of free energies in these colour channels in terms of expectation values of Polyakov loop correlation functions is similiar to that for two quark systems[5], but more elaborate. Denoting the Polyakov-loop at $\mathbf{x}_i$ by L_i and $\beta = 1/T$, we find

$$\begin{aligned}\exp\left(-\beta F^{1}_{qqq}\right) = \frac{1}{6}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 - 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3)\\ - 9\,\mathrm{Tr}\,L_2\mathrm{Tr}\,(L_1L_3) - 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2)\\ + 3\,\mathrm{Tr}\,(L_1L_2L_3) + 3\,\mathrm{Tr}\,(L_1L_3L_2)\Big\rangle \end{aligned} \tag{2}$$

$$\begin{aligned}\exp\left(-\beta F^{8}_{qqq}\right) = \frac{1}{24}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 + 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3)\\ - 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2) - 3\,\mathrm{Tr}\,(L_1L_3L_2)\Big\rangle \end{aligned} \tag{3}$$

$$\begin{aligned}\exp\left(-\beta F^{8'}_{qqq}\right) = \frac{1}{24}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 + 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2)\\ - 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3) - 3\,\mathrm{Tr}\,(L_1L_2L_3)\Big\rangle \end{aligned} \tag{4}$$

$$\begin{aligned}\exp\left(-\beta F^{10}_{qqq}\right) = \frac{1}{60}\Big\langle 27\,\mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3 + 9\,\mathrm{Tr}\,L_1\mathrm{Tr}\,(L_2L_3)\\ +9\,\mathrm{Tr}\,L_2\mathrm{Tr}\,(L_1L_3) + 9\,\mathrm{Tr}\,L_3\mathrm{Tr}\,(L_1L_2)\\ + 3\,\mathrm{Tr}\,(L_1L_2L_3) + 3\,\mathrm{Tr}\,(L_1L_3L_2)\Big\rangle. \end{aligned} \tag{5}$$

With this we obtain for the average free energy of the three quark system F^{av}_{qqq} the relation

$$\exp\big(-\beta F^{\mathrm{av}}_{qqq}\big) = \big\langle \mathrm{Tr}\,L_1\mathrm{Tr}\,L_2\mathrm{Tr}\,L_3\big\rangle \tag{6}$$

$$\begin{aligned}= \frac{1}{27}\exp\left(-\beta F^{1}_{qqq}\right) + \frac{8}{27}\exp\left(-\beta F^{8}_{qqq}\right)\\ +\frac{8}{27}\exp\big(-\beta F^{8'}_{qqq}\big) + \frac{10}{27}\exp\left(-\beta F^{10}_{qqq}\right). \end{aligned} \tag{7}$$

For the free energies of the qq-system we used the operators given in Ref. 5. These operators as well as those defined in (2)-(5) are gauge dependent and thus have to be evaluated in a fixed gauge. We used Coulomb gauge for our calculations.

3. Perturbation Theory and Renormalisation

Table 1 summarizes the Casimirs c_s found for the free energies in the different colour channels of the quark systems $q\bar{q}$, qq and qqq. Using this one obtains the leading order perturbative behaviour of the free energy in the symmetry state s as well as that of the average free energy for small distances or high temperatures

$$\beta F^s(R) = c_s \frac{\alpha\beta}{R} \quad \text{and} \quad \beta F^{\text{av}}(R) = 1 + c_{\text{av}} \frac{\alpha^2\beta^2}{R^2}, \tag{8}$$

where $\alpha = g^2/4\pi$. Here R is the usual Euclidian distance. For qqq-systems we restrict ourselves to equilateral triangles. In this case R denotes the edge length, which is an appropriate distance measure. In the case of the two octet free energies it is convenient to calculate the average of both. For small distances, the average free energies behave like the left most colour channels up to a T-dependent constant. In the following we will show results for

Table 1. Casimirs c_s and c_{av} for the leading order behaviour of F^s and F^{av}.

system	average	singlet	triplet	sextet	octet	decuplet
$q\bar{q}$	$-4/9$	$-4/3$			$+1/6$	
qq	$-4/9$		$-2/3$	$+1/3$		
qqq	$-4/3$	-2			$-1/2^\star$	$+1$

*This simple form only holds for equilateral triangles.

renormalized free energies. These are obtained from renormalized Polyakov loops. The relevant renormalization constants have been determined in Ref. 6 from an analysis of $q\bar{q}$ free energies and can directly be used also for the qqq-systems.

4. Colour Channels above T_c in qq- and qqq-systems

We compare the behaviour of the free energy in different colour channels for temperatures $T/T_c = 3, 6, 9$ on a $32^3 \times 8$ lattice and for equilateral triangular configurations in Figure 1a-c.

One can see clearly that the singlet free energies are strongly, the octet weaker attractive and the decuplet free energies are repulsive. Together with (7) this results in weakly attractive average free energies. All free energies at a given temperature approach a common T-dependent constant for large $R\sqrt{\sigma}$.

In Figure 1d we show the connected qqq-singlet free energies for equilateral triangles

$$\Delta F^1_{qqq}(R,T) = F^1_{qqq}(R,T) - F^1_{qqq}(\infty,T) \tag{9}$$

and compare it to the corresponding connected qq-triplet free energies, scaled by a factor of 3. Over the entire RT-interval we find that $\Delta F^1_{qqq}(R,T)$ and $3\Delta F^3_{qq}(R,T)$ coincide within errors for all temperatures, which suggests

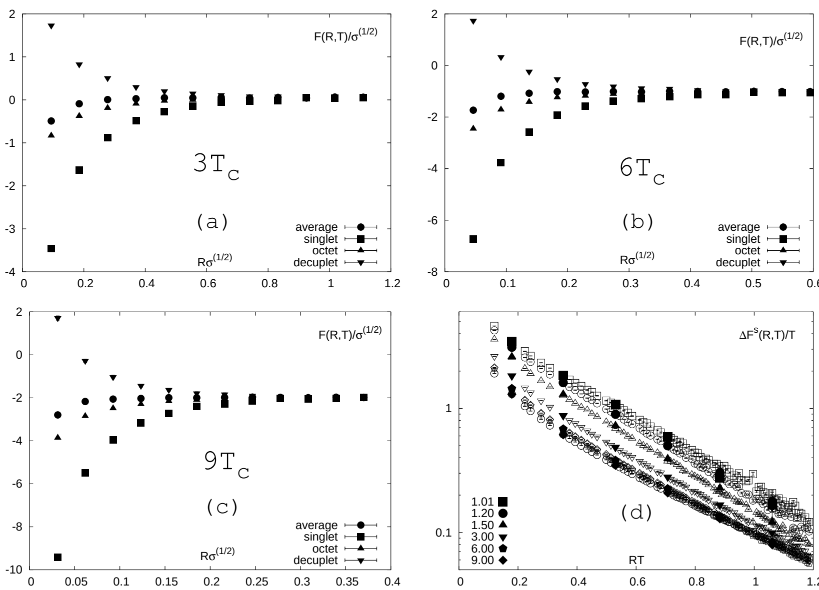

Figure 1. (a)-(c) Free energies of the qqq-system in different colour channels for temperatures $T/T_c = 3, 6, 9$. (d) Free Energies of the qq-(open symbols) and qqq-(full symbols) systems above T_c on a $32^3 \times 8$ lattice. Shown are $\Delta F^1_{qqq}(R,T)$ and $3\Delta F^3_{qq}(R,T)$ logarithmically.

that above T_c the interactions of the quarks in the qqq-singlet state can be decomposed into the pairwise interaction of three qq-pairs in a triplet state. The screening masses of both free energy channels are equal within errors. On the other hand we find that $F^1_{qqq}(\infty, T) = \frac{3}{2} F^3_{qq}(\infty, T)$, which shows that at large distances the static quark sources are screened independently by a gluon cloud. We find an analogous relation for the qqq-decuplet and qq-sextet free energies above T_c, the qqq-octet free energies show, however, some small deviations.

In Figure 2a we show $F^1_{qqq}(R,T)$ for several temperatures above T_c obtained on a $32^3 \times 8$ lattice renormalized by the procedure mentioned in section 3. For small distances we observe that for all temperatures the qqq-singlet free energies coincide, thus becoming T-independent.

5. Conclusion and Outlook

We calculated the free energies in different colour channels of heavy three quark systems at finite temperature. While the connected part of the qqq-

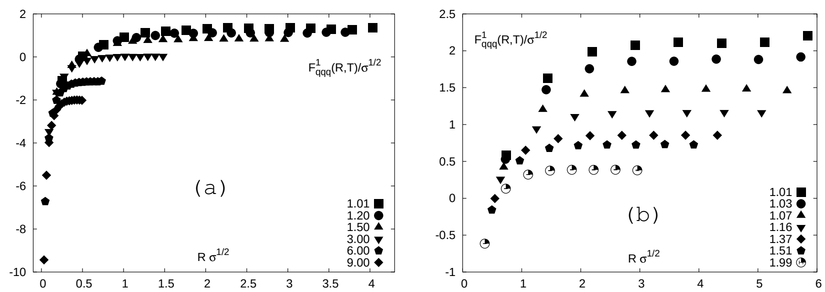

Figure 2. $F^1_{qqq}(R,T)$ (a) in pure gauge on a $32^3 \times 8$ lattice, (b) in 2-flavour QCD on a $16^3 \times 4$ lattice at $m/T = 0.4$.

singlet (-decuplet) free energies are found to be decomposable into three qq-triplet (-sextet) free energies for all distances calculated above T_c, the asymptotic large distance value of the free energies can be understood in terms of three independently screened quark sources. The qqq-octet free energies show deviations.

The approach used here for SU(3) gauge theory can also be applied to full QCD. In Figure 2b we show first results for $F^1_{qqq}(R)$ obtained on a $16^3 \times 4$ lattice in 2-flavour QCD. As in Figure 2a, the qqq-singlet becomes T-independent for small distances. Like the results in pure gauge simulations, we find that qqq-singlet (decuplet) free energies can be decomposed into qq-triplet (-sextet) free energies.

In the future we plan to perform a more detailed comparison between qq- and $q\bar{q}$-free energies in pure gauge and full QCD. Moreover, we are presently increasing the statistics for three quark free energies below T_c to be able to decide which flux tube geometry is realised close to T_c.

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